Endogenous Trading Constraints with Incomplete Asset Markets*

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Abstract. This paper endogenizes the borrowing constraints on capital holdings in an infinite horizon incomplete markets model with production and idiosyncratic risk. In particular, it assumes that households can break their trading arrangements by going into financial autarky, in which case they are excluded from future asset trade. We study the economy with the loosest possible borrowing limits that prevent default in equilibrium. We find that these limits are significantly different from zero, which is the ad hoc value often assumed in the literature. Further, they get looser with a higher labour income, consistent with US data on credit limits. The analysis of the welfare effects of a revenue neutral tax reform also illustrates that it is very important to take into account the direct effects of tax policies and the indirect effects of capital accumulation on the credit limits, since they can have quantitatively important welfare implications.

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1. Introduction

The present work endogenizes the borrowing constraints in an infinite horizon incomplete markets model with production. This is done by introducing the possibility of default on financial liabilities. In particular, we assume that households can decide to break their trading contracts every period. In this case, they will be seized from any positive asset holdings and excluded for future asset trade forever. The endogenous trading limits are then set at the level where households are indifferent between honoring their debt or defaulting.

In a model with a continuum of agents, we first characterize the dependence of the limits on individual labour income both analytically and numerically. In particular, we show that they are monotonically increasing with labor income, while the limits as a proportion of labor income get tighter with income. Moreover, they are significantly different from zero, which is the ad hoc value that is often assumed in the standard literature with incomplete markets. Since we also show that these properties are consistent with the behaviour of credit limits in the US data, we believe that our framework provides a more satisfactory setup for quantitative analysis than standard models with exogenous and fixed borrowing constraints.

After characterizing the dependence of the borrowing limits on income, we use a calibrated version of the model to analyze the long run welfare implications of a revenue neutral tax reform that eliminates capital income taxes at the expense of a higher labour income tax. When such a reform is implemented, the fact that the endogenous limits also depend on aggregate capital can have important welfare consequences. Note that this is due to the fact that a change in tax policies affects the aggregate capital and thus the value of default and the ability to borrow. In particular, we show that the conclusion regarding the welfare effects of eliminating capital income taxes depends on wether one takes into account the effects of the reform on the borrowing constraints or not.

To get some intuition for why this is the case, consider a reform that makes borrowing cheaper due to capital accumulation effects and different tax policies. If the limits are endogenous, they will become looser, since the fact that borrowing is cheaper decreases the incentives to default. In turn, relatively poor borrowers will become more indebted and this will generate a lower aggregate welfare. In contrast, if the borrowing constraints are fixed, this effect disappears and one would conclude that the same reform is welfare improving. This simple application thus illustrates that it is very important to take into account the aggregate effects of a reform on the on the ability to borrow, an effect that is ignored when the borrowing limits are fixed.

Our work builds a bridge between several important strands of literature. *First*, it contributes to an increasingly growing literature where a number of authors have introduced limited enforceability of risk-sharing contracts, implicitly resulting in agent and state specific trading constraints. In particular, Kehoe and Levine (1993), Alvarez and Jermann (2000, 2001) and Krueger and Perri (2005a) introduce these type of limits in exchange economies, whereas Kehoe and Perri (2002, 2004) study a production economy where investors are interpreted as countries. These authors, however, introduce the possibility of default into an otherwise complete markets context.

Apart from the fact that they can solve a central planning problem and thus do not characterize the limits on assets explicitly, several authors have shown that these models generate too much risk sharing in the presence of capital accumulation. In addition, the lack of commitment leads to equilibrium allocations with imperfect risk that labels these models endogenous incomplete market economies.

Apart from the fact that they do not characterize the endogenous borrowing limits, one of the problem with the previous literature is that the previous result may not be robust to the introduction of capital accumulation into a closed economy model. For example, Abraham and Carceles-Poveda (2007b) show that the equilibrium in a two agent model with endogenous production exhibits full risk sharing in the long run for standard parameterizations.[1] Further, Krueger and Perri (2005b) show that a model with a continuum of agents and endogenous incomplete markets is not able to account for the increase in US wealth inequality due to the fact that there is too much risk sharing. Since the implications of models with full or close to full risk sharing are clearly at odds with the data, this provides a strong motivation to study limited commitment in economies with incomplete markets, where risk sharing is always limited. Whereas the number of assets traded in still exogenous in this case, the presence of limited commitment endogenizes the amount that households can borrow. In this sense, the degree of market incompleteness becomes partially endogenous.

We should also point out that the fact that a different tax policy can affect the incentives to default has been already noted by Krueger and Perri (2005a), who study the optimality of progressive income taxation in a model with endogenous incomplete markets. In their case, moving from a progressive labour income tax (which in principle should lead to a higher degree of risk sharing) to a proportional tax can actually increase welfare by decreasing the value of defaulting and by allowing therefore for a looser limit and for a higher level of risk sharing. As in the previous literature, however, markets are complete. In addition, they focus on proportional labour income taxation and do not have capital accumulation. In contrast, we study a model with incomplete markets and capital accumulation and we focus on capital income taxation. Moreover, a key differences is that we also take into account how a different aggregate capital (due to a different tax policy) affects the value of default indirectly, while the previous authors only capture the direct effect of a change in taxes on the value of default.

Second, our work is also related to the traditional incomplete market models where the borrowing limits on the traded assets are ad hoc. Some examples are Heaton and Lucas (1996), Telmer (1993), Aiyagari (1994, 1995), Huggett (1997) and Krusell and Smith (1997, 1998). Whereas the previous authors have often argued that the ad hoc trading constraints are tighter than the natural borrowing limits to avoid default in equilibrium, the present work is one of the few formalizing this argument. It therefore provides a deeper foundation of the

[1] A similar full risk sharing result is obtained by Cordoba (2007) in a production economy with a continuum of agents, complete markets and collateral constraints. Note that Kehoe and Perri (2002, 2004) obtain imperfect risk sharing in an open economy with complete markets and production. However, one of the key differences is that their idiosyncratic shocks are interpreted and calibrated as country specific *aggregate* productivity shocks, whereas they are shocks to *individual* labour productivity in our economy. In addition, Bai and Zhang (2005) calibrate a similar economy to the one of Kehoe and Perri differently, and they also find extensive risk sharing under complete markets.

default thresholds and trading limits. An exception is the work by Zhang (1997a, 1997b), where the author derives the endogenous borrowing limits resulting from the possibility of default in a Lucas type exchange economy with trade in one asset. In contrast to this, we allow for the possibility of capital accumulation, an assumption that can affect the incentives to default. In addition, whereas the previous author focuses on the fixed trading restrictions effectively faced by the households, we characterize the dependence of both the default thresholds and the effective borrowing limits on idiosyncratic income.

Here, it is important to note that the endogenous incomplete markets literature often argues that there is a positive relationship between income and default incentives due to the fact that a higher income shock increases the outside option of default. As stated earlier, we find that this does not necessarily lead to tighter default thresholds, which we define as the level of assets at which households are indifferent between defaulting of staying in the contract. This is due to the fact that agents are not fully insured under limited commitment, even if markets are complete. Thus, a higher income also increases the value of staying in the trading arrangement. In other words, whereas the higher autarky value would lead to tighter default thresholds, this incomplete markets effect alone suggests that these thresholds are getting looser with income. To our knowledge, we are the first ones to document that this last effect dominates in models with imperfect risk sharing and limited commitment.

Finally, our work is also related to an increasingly growing literature where default can be an equilibrium outcome. Among others, Dubey et al (2005) study economies with incomplete markets and arbitrary utility penalties upon default, while Chaterjee et al (2005) study household bankruptcy in a general equilibrium endowment economy with both a riskless asset and unsecured credit with the option of default.[2] Similarly to these papers, we also extend our setup with competitive financial intermediaries who rent capital to the representative firm and borrow from and lend to households. Further, when the intermediaries are allowed to set the borrowing limits, we show that no-default limits only arise in equilibrium if they cannot differentiate between borrowing and lending rates. While this assumption is restrictive and is therefore relaxed in the previous papers, our model is not intended to study equilibrium default. Instead, our aim is to endogeneize the limits in a way that is more consistent with data and this result simply illustrates that there exist conditions under which these limits could potentially arise in equilibrium.

We would like to point out that the presence of endogenous trading limits considerably complicates our computations, since we have to extend usual policy iteration algorithm to incorporate a state dependent and non rectangular grid for some of the endogenous states, introducing an additional fixed point problem. In spite of the computational difficulties, however, we believe the methods developed in the present work can be fruitfully applied to study a wide set of interesting incomplete market models with endogenous limits. An example is the recent work by Bai and Zhang (2005), where the authors show that such an economy can account better for the observed cross country correlations of savings and investment rates than the complete markets counterpart. As explained earlier, our results

[2] The welfare implications of a related model with incomplete markets and equilibrium default is also studied by Mateos-Planas and Seccia (2005).

also suggests that fiscal policy and social insurance programs can have significant effects on the level of the endogenous trading constraints. Given this, a welfare analysis of any policy reform should take these indirect effects into account.

The rest of the paper is organized as follows. Section 2 presents the general model with incomplete markets and it characterizes the endogenous trading limits that prevent equilibrium default. In addition, it describes a financial intermediation structure that supports the endogenous limits in equilibrium. Section 3 documents several facts regarding the relationship between credit limits and income in the data. Section 4 presents the numerical solution of the benchmark model and Section 5 analyzes the welfare implications of a tax reform. Moreover, it compares the results to a model with fixed zero limits. Finally, Section 6 summarizes and concludes.

2. The Model

We consider an infinite horizon economy with endogenous production, idiosyncratic income shocks and sequential asset trade. Time is discrete and indexed by $t = 0, 1, 2....$

Households. The economy is populated by a government, a representative firm and a continuum (measure 1) of infinitely lived households that are indexed by $i \in I$. Households have identical additively separable preferences over sequences of consumption $c_i \equiv \{c_{it}\}_{t=0}^{\infty}$ of the form:

$$U(c_i) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}\right), \tag{1}$$

where $\beta \in (0, 1)$ is the subjective discount factor and E_0 denotes the expectation conditional on information at date $t = 0$. The period utility function $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}$ is assumed to be strictly increasing, strictly concave and continuously differentiable, with $\lim_{c_i \to 0} u'(c_i) = \infty$, and $\lim_{c_i \to \infty} u'(c_i) = 0$.

Each period, households can trade (borrow or save) in one asset (physical capital) to insure against uncertainty.[3] Further, the total amount of this asset accumulated by the households determines the aggregate capital stock.

The after-tax gross return on capital is denoted by $r_t\left(1 - \tau_k(k_{it})\right)$, where k_{it} represents the beginning of period individual asset holdings and τ_k is the tax rate on interest income. In our benchmark economy, capital income taxes depend on the level of assets in a very simple way:

$$\tau_k(k_{it}) = \begin{cases} \tau_k & \text{if } k_{it} \geq 0 \\ 0 & \text{if } k_{it} < 0 \end{cases}$$

This implies that only savers pay tax on their interest income. Apart from asset income, household $i \in I$ receives a stochastic labour endowment ϵ_i. This shock is is i.i.d. across households and it follows a Markov process with S_ϵ possible values and transition matrix $\Pi(\epsilon'|\epsilon)$. The after-tax individual labor income is equal to $w_t\left(1 - \tau_l\right)\epsilon_{it}$, where w is the aggregate wage rate and τ_l is the tax rate on labor income. The households' budget constraint can be expressed as:

$$c_{it} + k_{it+1} = w_t\left(1 - \tau_l\right)\epsilon_{it} + r_t\left(1 - \tau_k(k_{it})\right)k_{it} \tag{2}$$

[3] This framework can be easily extended to the presence of trade in more than one asset.

At each date, household $i \in I$ also faces a possibly endogenous and state-dependent trade restriction on the end of period capital holdings k_{it+1}. Throughout the paper, we assume that households cannot commit on the trading contracts and the borrowing constraint is endogenously determined at the level that prevents default in equilibrium. In case of default, we assume that they will be seized from their asset holdings and excluded from future asset trade, implying that their only source of income from the default period is their labor income. Following Livshits, MacGee and Tertilt (2006), we also assume that households face an additional penalty λ that reduces their labour income by $(1-\lambda)$ after default. This reflects aspects such as the fact that a fraction of income is garnished by creditors upon default or other costs of bankruptcy, such as the utility cost of filing (stigma), the increased cost of consumption and the fixed filing cost.[4]

Whereas these limits are simply imposed throughout the text, Appendix A illustrates how they could arise as an equilibrium outcome if competitive financial intermediaries were able to set them. In particular, the financial intermediation sector is the same as the one introduced by Ábrahám and Carceles (2007a), who study a similar environment but assume a complete set of financial assets. The appendix shows first that the loosest possible limits that prevent default constitute a symmetric Nash equilibrium. Moreover, it shows that there is no symmetric equilibrium with looser limits, implying that no default can arise in equilibrium.

It is important to note that this result is derived under the restriction that intermediaries cannot charge different interest rate on savers than borrowers. First, in the present setting with incomplete financial markets, this assumption is necessary to avoid default in equilibrium. In contrast, other authors, such as Dubey, Geanakoplos and Shubik (2005), allow for different saving and borrowing rates, implying that equilibrium default can be sustained. Second, under a complete set of securities and limited commitment (leading to endogenous incomplete markets), Ábrahám and Carceles (2007b) show that the endogenous borrowing limits on Arrow securities that avoid default arise as a symmetric Nash equilibrium without the above restriction on borrowing and lending rates. Finally, in a setup where current income is private information, Sanchez (2007) shows that it may happen that the only incentive compatible borrowing contracts that financial intermediaries can offer impose borrowing constraints that do not allow for default.

Production. At each date, the representative firm uses capital $K_t \in \mathbb{R}_+$ and labor $L_t \in (0,1)$ to produce a single good $y_t \in \mathbb{R}_+$ with the constant returns to scale technology:

$$y_t = Af(K_t, L_t), \tag{3}$$

where A is a technology parameter that represents total factor productivity. The production function $f(\cdot,\cdot) : \mathbb{R}_+^2 \to \mathbb{R}_+$ is assumed to be continuously differentiable on the interior of its domain, strictly increasing, strictly concave in K and homogeneous of degree one in K

[4] This punishment for default resembles the bankruptcy procedures under Chapter 7. Under this procedure, households are seized from any positive asset holdings but can keep at least part of their labour income. Whereas they are allowed to borrow after some periods, this becomes considerably more difficult and costly because their credit rating deteriorates significantly.

and L. Capital depreciates at the rate δ and we denote total output including undepreciated capital by:

$$F(K_t, L_t) = Af(K_t, L_t) + (1-\delta)K_t. \tag{4}$$

Each period, the firm rents capital and labor to maximize period profits:

$$F(K_t, L_t) - w_t L_t - r_t K_t, \tag{5}$$

leading to the following first order conditions:

$$w_t = f_L(K_t, L_t) \tag{6}$$

$$r_t = f_K(K_t, L_t) + 1 - \delta. \tag{7}$$

Government and Market Clearing. The government consumes the amount G_t in period t and it taxes labor and interest rate income at the rates τ_l and τ_k respectively. The government budget constraint is therefore equal to:

$$G_t = w_t \tau_l L_t + r_t \tau_k \widehat{K}_t. \tag{8}$$

Here $\widehat{K}_t > K_t$ is the capital income tax base, that is, the level of capital holdings of those who hold non-negative assets. As usual, labor and asset market clearing require that the sum of individual labor income shocks and individual capital holdings are equal to the total labor supply and aggregate capital stock respectively. Further, the good's market clearing condition requires that the sum of investment and aggregate consumption, including household and government consumption, is equal to the aggregate output.

Recursive Competitive Equilibrium. In the present framework, the aggregate state of the economy is given by the joint distribution Ψ of consumers over individual capital holdings k and idiosyncratic productivity status ϵ. Households perceive that Ψ evolves according to:

$$\Psi' = \Gamma[\Psi],$$

where Γ represents the transition function from the current aggregate state into tomorrow's wealth-productivity distribution. Since the individual state vector includes the individual labour productivity and capital holdings (ϵ, k), the relevant state variables for a household is summarized by the vector $(\epsilon, k; \Psi)$.

The outside option or autarky value V of a household with income shock ϵ can thus be expressed recursively as:

$$V(\epsilon; \Psi) = u(w(\Psi)\,(1-\tau_l)\,\epsilon\,(1-\lambda)) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) V(\epsilon'; \Gamma[\Psi]). \tag{9}$$

Equation (9) reflects that the autarky value is a function of the wealth-productivity distribution. Note that this is in contrast with some of the literature with complete markets and no commitment, where V is exogenous (see e.g. Alvarez and Jermann (2000, 2001)). As we will see later, this is due to the fact that the distribution determines aggregate capital accumulation, which in turn determines future wages and therefore the future value of financial autarky. On the other hand, since agents are seized from all their assets upon default, the autarky value is not a function of the individual capital holdings.

Second, the expression in (9) uses the fact that there is a continuum of agents and no default occurs in equilibrium. In such a setting, agents assume the aggregate state of the economy will follow the same law of motion $\Gamma[\Psi]$ if they default. Note that this is correct, since their individual deviation does not influence the aggregate variables and no-one defaults in equilibrium.

We are now ready to define the recursive competitive equilibrium. Since the aggregate labor supply is constant due to a law of large numbers, we write $w(\Psi) = w(K)$ and $r(\Psi) = r(K)$ in what follows.

Definition 2.1: Given a transition matrix Π and some initial distribution of shocks $\epsilon_0 \equiv (\epsilon_{i0})_{i \in I}$ and asset holdings $k_0 \equiv (k_{i0})_{i \in I}$, a *recursive competitive equilibrium* relative to the vector of taxes (τ_k, τ_l) and the borrowing limit $\underline{k}(\epsilon; \Psi)$ is defined by a law of motion Γ, a vector of factor prices $(r, w) = (r(K), w(K))$, a government consumption G, value functions $W = W(\epsilon, k; \Psi)$ and individual policy functions $(c, k') = (c(\epsilon, k; \Psi), k(\epsilon, k; \Psi))$ such that:

(i) *Utility Maximization:* For each $i \in I$, W and (c, k') solve the following problem given k_0, ϵ_0, Π, Γ and (r, w):

$$W(\epsilon, k; \Psi) = \max_{c,k'} \left\{ u(c) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) W(\epsilon', k'; \Psi') \right\} \tag{10}$$

$$\text{s.t. } c + k' = w(K)(1 - \tau_l)\epsilon + r(K)(1 - \tau_k(k))k$$

$$\tau_k(k) = \left\{ \begin{array}{cc} \tau_k & \text{if } k \geq 0 \\ 0 & \text{otherwise} \end{array} \right\}$$

$$\Psi' = \Gamma[\Psi]$$

$$k' \geq \underline{k}(\epsilon'; \Psi') \text{ for all } \epsilon'|\epsilon \text{ with } \Pi(\epsilon'|\epsilon) > 0.$$

(ii) *Profit Maximization:* Factor prices satisfy the firm's optimality conditions, i.e., $w(K) = f_L(K, L)$ and $r(K) = f_K(K, L) + 1 - \delta$.

(iii) *Balanced Budget:* The government budget constraint is satisfied, i.e., $G = w(K)\tau_l L + r(K)\tau_k \widehat{K}$, where

$$\widehat{K} = \int_{k \geq 0} k d\Psi(\epsilon, k).$$

.

(iv) *Market Clearing:*

$$\int k(\epsilon, k; \Psi) d\Psi(\epsilon, k) = K'$$

$$\int \epsilon d\Psi(\epsilon, k) = L$$

$$\int [c(\epsilon, k; \Psi) + k(\epsilon, k; \Psi)] d\Psi(\epsilon, k) + G = F(K, L) + (1 - \delta)K$$

(v) *Consistency:* Γ is consistent with the agents' optimal decisions, in the sense that it is generated by the optimal decision rules and by the law of motion of the shock.

(vi) *No default:* $\underline{k}(\epsilon;\Psi)$ is such that individuals are indifferent between trading and going into autarky, i.e.,

$$\underline{k}(\epsilon;\Psi) = \{\underline{k} : W(\epsilon,\underline{k};\Psi) = V(\epsilon;\Psi)\}. \tag{11}$$

Several remarks are worth noting. First, as reflected in condition (i), households are only allowed to hold levels of individual capital that are above a state-dependent lower bound for each continuation state with positive probability next period. This implies that the effective limit on capital holdings $\kappa(\epsilon;\Psi)$ faced by a household is the tightest among these state-dependent lower bounds. Using the recursive notation, the effective constraints on capital holdings can thus be expressed as:[5]

$$k' \geq \kappa(\epsilon;\Psi) \equiv \sup_{\epsilon':\Pi(\epsilon'|\epsilon)>0} \left\{\underline{k}\left(\epsilon';\Gamma[\Psi]\right)\right\}. \tag{12}$$

Second, the definition of the state-dependent lower bounds in (11) implies that we can think about $\underline{k}(\epsilon;\Psi)$ as a state-dependent default threshold, since it represents the level of capital holdings such that households are indifferent between defaulting or paying back their debt. Clearly, condition (vi) implies that we only consider equilibria where the trading limits are such that default is not possible. Whereas there are many borrowing limits that prevent default in equilibrium, we consider the loosest possible ones of such limits. In other words, we study the economy with limits that are not too tight, in the sense that they satisfy (11) and (12).

Finally, note that the default thresholds are very closely related to the endogenous borrowing limits on Arrow securities in the literature with complete markets and limited commitment (see Alvarez and Jermann (2000) and Ábrahám and Carceles (2007a) for the definition of these limits in endowment and production economies respectively).

Characterization of the Endogenous Default Thresholds. In what follows, we provide some theoretical results that show the existence of a unique lower bound $\underline{k}(\epsilon;\Psi)$ satisfying equation (11). Furthermore, we characterize the dependence of $\underline{k}(\epsilon;\Psi)$ on the labor income shock. All the proofs are relegated to Appendix B.

Proposition 2.1. *If u is unbounded below, equation (11) defines a unique, non-positive and finite default threshold $\underline{k}(\epsilon;\Psi)$ for every ϵ and Ψ.*

The existence of the default thresholds established by Proposition 2.1 is a consequence of the fact that $V(\epsilon;\Psi)$ is finite whenever ϵ and K is bounded away from zero, while $W(\epsilon,k;\Psi)$ goes to minus infinity as k goes to the natural borrowing limit. Further, uniqueness simply follows from the fact that $V(\epsilon;\Psi)$ does not depend on k while $W(\epsilon,k;\Psi)$ is strictly increasing in k. An important implication of uniqueness is the fact that the value of staying in the

[5] It is important to note that, if the probability of all future shock realization pairs is strictly positive for any current pair, the effective limit faced by the households will not be a function of the current shocks, since the trading restriction has to be satisfied for all possible continuation states. This will not be the case, however, in our calibrated example.

trading arrangement is always higher than the autarky value if the capital holdings are above the default threshold, that is,

$$W(\epsilon, k; \Psi) \geq V(\epsilon; \Psi) = W(\epsilon, \underline{k}; \Psi) \text{ for } \forall k \geq \underline{k}(\epsilon; \Psi).$$

The fact that the thresholds are finite is a consequence of the fact that $V(\epsilon; \Psi)$ is finite. Finally, the equilibrium default thresholds and effective limits have to be clearly non-positive. Intuitively, note that agents would not default with a positive level of asset holdings, since they could then afford a higher current consumption than in autarky and at least as high of a life-time utility as in autarky from next period on by staying in the trading arrangement.

To characterize the dependence of $\underline{k}(\epsilon; \Psi)$ on the labor income shock, we assume differentiability of both the trading and autarky values. Further, to make the exposition easier and to be able to express the differential effect of a change in ϵ on the lower bounds, we assume that it follows a continuous $AR(1)$ process that is given by:[6]

$$\epsilon' = \mu_\epsilon + \rho_e \epsilon + \varepsilon'_\epsilon \text{ with } \varepsilon'_\epsilon \sim N(0, \sigma^2_\epsilon).$$

Denote the individual policy functions by $k' = g_k(\epsilon, k; \Psi)$ and $c = g_c(\epsilon, k; \Psi)$. To express the effects of a change in ϵ, we can differentiate equation (11), obtaining that:

$$\frac{\partial \underline{k}(\epsilon; \Psi)}{\partial \epsilon} = -\frac{W_\epsilon(\epsilon, \underline{k}; \Psi) - V_\epsilon(\epsilon; \Psi)}{W_k(\epsilon, \underline{k}; \Psi)}. \tag{13}$$

In the previous equation, $W_\epsilon(\epsilon, \underline{k}; \Psi)$ and $V_\epsilon(\epsilon; \Psi)$ represent the derivatives of the two value functions, evaluated at $\underline{k}$, with respect to the income shock ϵ. Similarly, $W_k(\epsilon, \underline{k}; \Psi)$ represents the derivative of the trading value, evaluated at $\underline{k}$, with respect to k.

Since more individual capital holdings (ceteris paribus) expand the budget sets, and because the utility function is strictly increasing , it follows that $W_k(\epsilon, \underline{k}; \Psi) > 0$**.** Given this, the sign of the previous derivative is determined by wether a change in income increases the trading value W more or less than the autarky value V. If the trading value increases more than the autarky value after an increase in the income shock, the derivative will be negative. In this case, a higher income will lead to looser default thresholds. For our main characterization result, we will use the results of the following lemma.

Lemma 2.1. *At the default threshold, agents consume less in the trading arrangement than they would in autarky, i.e.,* $g_c(\epsilon, \underline{k}(\epsilon; \Psi); \Psi) \leq w(K)(1 - \tau_l)\epsilon(1 - \lambda)$.

The previous lemma implies that agents who are at the default threshold have a higher current consumption in autarky. Note that, if this was not the case, agents would be strictly better off by consuming $g_c(\epsilon, \underline{k}(\epsilon; \Psi); \Psi)$ at the threshold and by defaulting next period. However, this would contradict either the definition of the default threshold or the fact that W is the maximal life-time utility that households can achieve. We are now ready to state the proposition that shows the dependence of $\underline{k}(\epsilon; \Psi)$ on the labor income shock.

Proposition 2.2 *Assume that ϵ is i.i.d for each agent. ($\rho_\epsilon = 0$). Then, ceteris paribus, the higher IS the productivity shock of an agent, the looser are the default thresholds, i.e.* $\frac{\partial \underline{k}(\epsilon; \Psi)}{\partial \epsilon} \leq 0$.

[6] More precisely, we assume that it follows a truncated AR(1) processes to avoid negative values, while the truncation does not modify any of our results.

The result of Proposition 2.2 follows from Lemma 2.1 and from the fact that households are risk averse. As stated earlier, a higher income shock decreases the default thresholds if the trading value increases by more than the autarky value after an increase in income. First, since the income shock is i.i.d., a marginal change in income only affects current consumption. In particular, if c^{au} represents the consumption in autarky, we have that $W_\epsilon(\epsilon, k; \Psi) - V_\epsilon(\epsilon; \Psi) = w(K)\left(u'(c) - u'(c^{au})\right)$. Second, since the current consumption in autarky is higher, the concavity of u implies that $u'(c) - u'(c^{au}) \geq 0$ and $\frac{\partial \underline{k}(\epsilon;\Psi)}{\partial \epsilon} \leq 0$. In other words, the fact that current consumption in autarky is higher than in the trading arrangement implies that the change in the autarky value is lower after an increase in income, leading to looser default thresholds.

Several remarks are worth noting. First, some authors document that incentives to default are typically higher for lower income types (see e.g. Sullivan et. al. (1989)). Moreover, the ability to borrow is a positive function of income in the data, as we will show in section 3. Given this, the result of proposition 2.2 is a desirable property of the present setting.

Second, note that these findings may seem somewhat surprising, since it is often argued in the literature on optimal risk sharing with limited enforceability that agents with a higher income have more incentives to default. While this is also true in our model, in the sense that higher income shocks lead to a higher autarky value, Proposition 2.2. shows that this effect does not necessarily translate into tighter borrowing limits, since the value of staying in the trading arrangement increases by even more after an increase in income. Of course, a key aspect for obtaining this result is that markets are incomplete. Note, however that the same result would obtain in an environment with a complete set of financial assets and the possibility of default, since markets are endogenously incomplete whenever the borrowing constraints on Arrow securities are binding. Given this, the key arguments of the proof would be unchanged.

Finally, it is important to note that the assumption of i.i.d. shocks is crucial to obtain the previous result. In particular, if $\rho_\epsilon > 0$, the levels of future consumption also become important in determining the sign of the derivative of the limit in (13), and the above relationship may be weakened or reversed. As we will show numerically in section 4, however, the results of the proposition are robust to persistent shocks.

3. Credit Limits and Income in the Survey of Consumer Finances

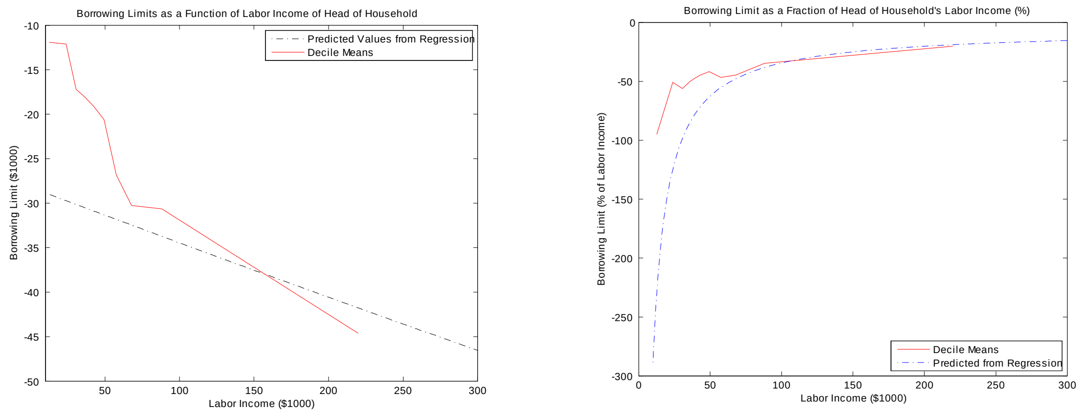
This section documents several facts about the relationship between credit limits and individual income in the data. These facts will help us to evaluate the empirical relevance of Proposition 2.2 as well as the predictions of a calibrated version of the model regarding the borrowing constraints.

Our data source is the 2004 Survey of Consumer Finances. We only consider heads of households that are working full time and have a positive labour income and credit card limit. To construct the income series, we use three different measures. The first is the annual labor income of the heads of households. Our income data is constructed survey questions regarding earnings and labor supply (number of weeks worked per year).[7] As to

[7] Using alternative defintions of labor income based upon W2 forms and total household income, we ob-

the borrowing limits, the best available information is based on question that asks the heads of households how much they can borrow on all their credit card accounts.

Figure 1: Limits as a Function of Labour Income in the Data



Limits

Limits as a Proportion of Income

The left panel of Figure 1 depicts the borrowing limits as a function of labor income. The right panel plots the borrowing limits as a proportion of labor income against labor income. The solid lines display data using deciles of the income distribution, by taking averages within a decile. The dashed lines are the predicted borrowing limits coming from a regression where a third order polynomial of income together with age, gender and education was used to explain the borrowing limit. The figures show the predicted limits for men and for the average age and educational level of the sample.

The left panel of the figure shows a clearly negative relationship between the level of income and the credit limits. In other words, higher income people have a higher ability to obtain unsecured credit. While this is consistent with the findings of Proposition 2.2, note that the latter relied on the fact that income shocks are i.i.d.. Since it is well documented in the literature that income shocks are persistent, we solve numerically a calibrated version of the model with persistent shocks to see if the results of the proposition are robust to this extension.

The right panel of figure shows that the negative relationship between credit limits and income is reversed when we plot the limits as a proportion of labor income. Essentially, this implies that people with a higher income can borrow a lower proportion of their income. In the next section, we will also test the predictions of the model against this fact.

4. Quantitative Results

This section solves a calibrated version of the model described in the previous sections and we study the stationary distribution of the economy studied in Section 2. Note that, in the steady state, all aggregate variables, including the asset distribution, government consumption, taxes, the aggregate capital and hence wages and the interest rate, are constant. We first

tained very similar results.

discuss the calibration and solution method for the benchmark economy and we study the properties of the endogenous borrowing limits, particularly the relationship between these limits and income. Further, the allocation is compared to the one resulting when the limits are exogenously fixed at zero. The latter is the typical assumption in the incomplete markets literature (see Aiyagari (1994, 1995) or Krusell and Smith (1997, 1998)).

4.1. Calibration and Solution Method. One of the main aims of the calibration is that the model steady state matches the earnings and wealth distribution in the US. In addition, we want to match some aggregate statistics, such as the labor share, the capital output ratio and the interest rate.

The time period is assumed to be one year and preferences are of the CRRA class, $u(c) = \frac{[c^{1-\mu}-1]}{1-\mu}$. For comparison with the existing literature, we set the risk aversion to $\mu = 2$. Further, the production function is Cobb Douglas, $f(K,L) = AK^{\alpha}L^{1-\alpha}$, where $\alpha = 0.36$ to match a labor share of 0.64 in the data. The depreciation rate is set to $\delta = 0.08$ obtain a capital output ratio of around 3 and the technology parameter A is chosen so that output is equal to one in the steady state of the deterministic economy.

As discussed in Livshits, MacGee and Tertilt (2006), bankruptcy filers face several types of punishment. Apart from the fact that filers cannot save or borrow, a fraction of earnings is garnished by creditors in the three year period of filing. In addition, there is a utility costs of filing or stigma, a fraction of consumption may be lost and a fixed cost of filing. To match key observations regarding the evolution of bankruptcy filings in the last decades, the authors choose a garnishment rate of 0.319 and set the other costs to zero. Since our penalty is imposed every year after default, we have chosen $\lambda = 0.1$ in the benchmark economy.

Table 1: Earnings Process

$$\epsilon = \begin{bmatrix} 0.1805 & 0.3625 & 0.8127 & 1.8098 & 3.8989 & 8.4002 & 18.0980 \end{bmatrix}$$

$$\Pi^* = \begin{bmatrix} 0.3173 & 0.2231 & 0.3128 & 0.0719 & 0.0453 & 0.0245 & 0.0051 \end{bmatrix}$$

$$\Pi(\epsilon'|\epsilon) = \begin{bmatrix} 0.9687 & 0.0313 & 0 & 0 & 0 & 0 & 0 \\ 0.0445 & 0.8620 & 0.0935 & 0 & 0 & 0 & 0 \\ 0 & 0.0667 & 0.9180 & 0.0153 & 0 & 0 & 0 \\ 0 & 0 & 0.0666 & 0.8669 & 0.0665 & 0 & 0 \\ 0 & 0 & 0 & 0.1054 & 0.8280 & 0.0666 & 0 \\ 0 & 0 & 0 & 0 & 0.1235 & 0.8320 & 0.0445 \\ 0 & 0 & 0 & 0 & 0 & 0.2113 & 0.7887 \end{bmatrix}$$

Regarding the process for idiosyncratic income shock, we have used a seven state income process which generates a Gini coefficient that is equal to 0.6 and thus the same concentration for income as in the data. The process, which is similar to the one used Diaz et. al (2003) and Davila et. al (2005), is also chosen so that it generates a realistic wealth distribution in the benchmark steady state. This, together with a discount factor of $\beta = 0.93$, matches an interest rate of 3.7% and the total financial assets hold by the lowest and highest quintiles of the US wealth distribution. Table 1 describes the earnings process. It displays the shock values, the stationary distribution and the transition matrix.

Table 2: The Wealth Distribution in the Benchmark Model and in The Data

	Quintiles					
Economy	Q1	Q2	Q3	Q4	Q5	% In Debt
Benchmark Model	-1.55	-0.66	2.63	7.89	91.70	35.62
Data (SCF 2004)						
Total net worth	-0.18	1.13	4.37	17.10	82.90	7.14
Total net financial assets	-1.55	0.09	1.61	8.66	91.19	19.97

Table 2 contains information about the wealth distribution in our benchmark model and in the 2004 Survey of consumer finances. Since the present paper is about unsecured credit, we have tried to match some key moments of the distribution of net financial assets. In contrast, most of the macroeconomic literature focuses on the wealth distribution based on the net worth (defined as the difference between total assets and total liabilities). Further, when calculating net financial assets, we exclude the value of residential property, vehicles and direct business ownership from the assets, and secured debt due to mortgages and vehicle loans from the liabilities. This level of assets represents better the amount of liquid assets that households can use to smooth out income shocks. Moreover, both residential properties and vehicles can be seen as durable consumption as much as investment.

As we see in the Table, according to the 2004 Survey of Consumer Finances, the lowest quintile of the wealth distribution, as measured by net financial assets, held -1.55% of total financial wealth, whereas 91.19 percent was held by the highest quintile. Our model matches this aspect of the distribution very well, since the assets held by the lowest and highest quintiles in the model are -1.55 and 91.70 respectively. Further, we also match reasonably well the asset holdings of the three medium quintiles. On the other hand, 19.97% of the population was in debt in the data, while our model implies that 35 percent of people are in debt. In other words, the model somewhat overestimates the population in debt.[8] Nevertheless, most models studying tax reforms, such as Aiyagari (1995) and Domeij and Heathcote (2004), assume no borrowing. As we show in the next section, this assumption may have important limitations, in the sense that a model with endogenous borrowing limits can lead to very different conclusions.

Finally, we choose a capital tax rate of $\tau_k = 0.40$, which is very close to the tax rate found by Domeij and Heathcote (2004) using the method of Mendoza et. al (1994). Furthermore, we chose G such that government expenditure is 22% of output in our benchmark economy, which is close to the government to output ratio in the US data. The level of labor income taxes that solves the budget constraint is then equal to $\tau_l = 0.28$.

[8] If we consider individuals with zero net financial wealth as agents in debt, then the proportion of individuals in debt in the data rises to 24.31%, which is closer to the pecentage in debt generated by our benchmark model.

Solution Method. To find the solution, we use a policy function iteration algorithm that is described in detail in Appendix C. Solving the model with endogenous trading limits involves several computational difficulties. First, our state space is endogenous, a problem that we address by incorporating an additional fixed point problem to find the state-dependent limits on the individual capital holdings. This also implies that our policy functions have to be calculated over a non-rectangular grid. Further, given that the limits in our model are endogenously determined at the level where the value function from staying in the contract is at least as large as the autarky value, it becomes clear that a good approximation of the value functions close to the limits is needed to obtain reliable results. To address this issue, we use a relatively high number of grid points, we interpolate the policy and value functions over this grid and we allow the limits to take values between grid points as well. In order to speed up the solution procedure we update the interest rate and the borrowing limits simultaneously.

4.2. Results for the Benchmark Economy. We summarize the aggregate statistics for the benchmark economy in Table 3. The two columns display some of the steady state statistics in the benchmark economy and the economy with no short selling constraints respectively. First, we note that the tighter limits in the economy with no borrowing imply higher precautionary savings and therefore a higher capital stock and a lower interest rate in this last case. Since the endogenous limits are considerably looser than the fixed limits of zero, risk sharing is more limited in this latter case. On the other hand, the economy with endogenous limits generates a higher wealth inequality, as shown by the higher coefficient of variation of the asset distribution. Moreover, in the steady state wealth distribution that is displayed in Table 2, we see that almost 1/3 of individuals are in debt when the borrowing limits are endogenous. Note that the last two facts explain why there are significant long run welfare gains (6.56% in consumption equivalent terms) from tightening the limits.

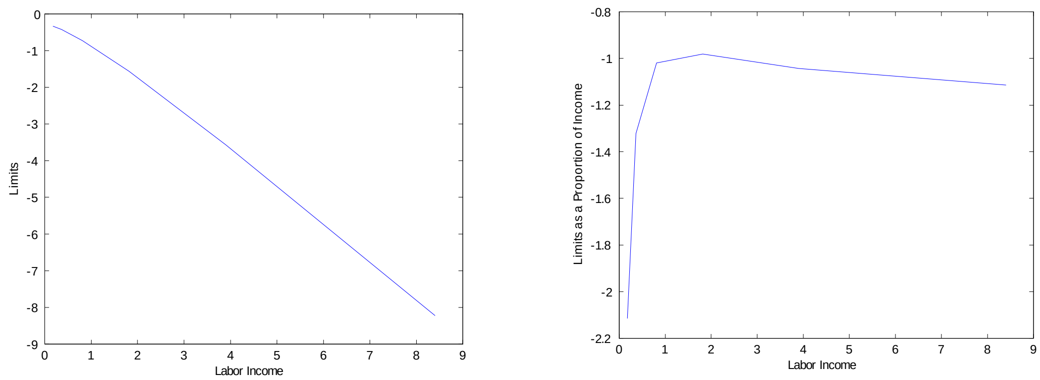
Table 3: Steady State of the Benchmark Economy

	Benchmark Economy with Endogenous Limits	Economy with no Borrowing
K	4.223	4.348
$\frac{K}{Y}$	3.076	3.134
$r\%$	3.700	3.483
$\frac{\sigma_k}{K}$	2.712	2.598
(τ_k, τ_l)	$(0.4, 0.28)$	$(0.4, 0.279)$
$\frac{G}{Y}$	0.224	0.222
W	-57.940 (100)	-54.366 (106.57)

The endogenous limits in the benchmark economy are displayed in Figure 2. The left panel of the figure shows the level of the endogenous borrowing limits as a function of income, while the right panel plots the limits as a proportion of income against income. The first thing we observe is that the endogenous limits exhibit a similar behaviour to the one in the data. In particular, they get looser with income. Note that these findings confirm that the results of Proposition 2.2 are robust to the presence of persistent shocks. Further, the limits as a proportion of income get tighter with a higher income, at least for low income levels. On the other hand, for higher income levels, the model can only capture the fact that limits are relatively constant as a proportion of income.

In addition, we observe that the limits are quite different from zero, even for lower income types. While this implies that the level of risk sharing is higher than in economies with no short selling constraints, it also implies that, in the long run, low income individuals can become more indebted. As we will see in the next section, these considerations will play an important role for the determination of aggregate welfare and for the design of optimal government policy. In particular, we will see the importance of modelling the borrowing limits appropriately by analyzing the welfare implications of a tax reform.

Figure 2: Limits as a Function of Labour Income in the Model



Limits

Limits as a Proportion of Income

5. Welfare Effects of Capital Income Taxation

This section analyzes the long run welfare implications of a revenue neutral tax reform that eliminates the capital income tax at the expense of a higher labor income tax. We first study the impact of the reform in the presence of a fixed no short selling constraint. The results for this case are displayed in Table 4.

Table 4: Tax Reform with Exogenous Limits

τ_k	τ_l	$\frac{K}{Y}$	r	$w(1-\tau_l)$	$r(1-\tau_k)$	W
0.4	0.279	3.13	3.483	0.6399	2.0902	-54.36 (100)
0	0.333	3.36	2.689	0.6158	2.6894	-54.41 (99.91)

The two rows of the table report the pre-reform and post-reform steady state respectively. The first three columns display the tax rates and the capital output ratio; the next three columns display the interest rate and the two factor prices after taxes; finally the last column displays the aggregate welfare with the consumption equivalent term in percentage in the brackets.

As reflected by the second and the last columns, the labor income tax increases from 0.279 to 0.333 when the capital income tax is eliminated, while there is a decrease in the aggregate welfare in spite of the fact that the aggregate capital stock is higher after the tax reform. To see why this is the case, note first that a higher aggregate capital lowers the interest rate r and increases the aggregate wage rate w. However, the change in the tax rates leads to a higher after-tax interest rate and a lower after-tax wage. This change benefits relatively rich agents, for whom capital income is more important, while it hurts relatively poor people, who primarily rely on labor income. By concavity, the welfare losses of the poor offset the welfare gains of the richer agents.

These findings are consistent with the ones in Aiyagari (1995), who first studied the optimal capital income taxation in a model with incomplete markets and borrowing constraints. In contrast to the seminal paper of Chamley (1986), who shows that the optimal capital income tax is zero in the long run for a wide class of infinite horizon models with complete markets, Aiyagari (1995) shows that in models with incomplete asset markets and fixed borrowing constraints the optimal capital income tax is always strictly positive. The precautionary savings motive that arises in the presence of uncertainty implies that capital is over accumulated above the optimal level, calling for a positive tax rate. Note that our previous results show that a key reason for the decrease in welfare when the capital tax rate is eliminated is the fact that relatively poor people are worse off after the reform due to a higher labor income tax. In addition, Domeij and Heathcote (2004) study the same reform with no borrowing and show that this result is also true along the transition path.

It is also important to note that a recent paper by Davila et al (2006) studies the welfare properties of the market and the constrained efficient allocations of a similar economy, obtaining the opposite result. In particular, when the income of the poor is mostly composed of labor earnings, the authors conclude that the aggregate capital achieved by the market allocation is too low. Note that this different conclusion can be explained by the fact that Davila et al (2006) allow for state contingent and agent specific taxes, as opposed to the fix tax rates in the present paper and in Aiyagari (1995) and Domeij and Heathcote (2004).

The results presented above assume an exogenous borrowing constraint that is equal to zero. Given this, they abstract from important effects that could modify and possibly overturn the main findings if the limits were endogenous. The reason is that, in this last case, discussed in the previous section of the paper, the tax policy will affect the relative value of default and consequently the borrowing constraints. Furthermore, a different new level of capital will also affect the borrowing constraints and the ability to self-insure indirectly. In sum, the optimal level of capital and thus the optimal capital income taxation might crucially depend on how the borrowing limits are modelled.

To evaluate the importance of these effects quantitatively, we study the same revenue neutral tax reform in the presence of endogenous borrowing limits. Here, we distinguish between two different scenarios. As in the benchmark economy, we assume first that only agents with positive assets pay capital income taxes. We label this economy as the asymmetric case. In contrast, both savers and borrowers pay taxes on capital income in the symmetric case. In this last case, this implies that the capital tax rate is actually a subsidy on borrowing. Table 5 reports the results of the reform for both cases and the benchmark penalty of $\lambda = 0.1$. In addition, the level of the endogenous limits is displayed in Table 6.

Table 5: Tax Reform with Endogenous Limits

Benchmark	$\frac{K}{Y}$	r	w	$w(1-\tau_l)$	$r(1-\tau_k)$	W	τ_l
$\tau_k = 0.4$	3.076	3.700	0.878	0.632	2.22	-57.94 (100)	0.280
$\tau_k = 0$ (int.)	3.320	2.843	0.916	0.608	2.84	-57.57 (100.63)	0.336
$\tau_k = 0$	3.310	2.875	0.915	0.606	2.88	-58.38 (99.23)	0.337
Symmetric	$\frac{K}{Y}$	r	w	$w(1-\tau_l)$	$r(1-\tau_k)$	W	τ_l
$\tau_k = 0.4$	3.026	3.895	0.870	0.626	2.337	-58.835 (100)	0.280
$\tau_k = 0$ (int.)	3.299	2.911	0.913	0.605	2.911	-59.480 (98.91)	0.337
$\tau_k = 0$	3.311	2.872	0.915	0.607	2.872	-58.317 (100.88)	0.336

Apart from the fact that agents are able to borrow if the borrowing limits are endogenous, an important difference with the fixed limit case is that the endogenous borrowing constraints also change after the reform. To disentangle this effect from the rest, we have also solved for the intermediate case (int) in which the reform is implemented but the endogenous limits are kept the same.

As under fixed limits, Table 5 reflects that the aggregate capital stock is higher after the capital income tax is eliminated, but the increase in lower than under fixed limits. As noted earlier, the reason is that there is now more risk sharing and thus less precautionary savings. In addition, we also have that $r(1-\tau_k)$ is higher and $w(1-\tau_l)$ is lower after the reform, while the increase in the labour income tax is similar to the one obtained under fixed limits. As we see in the previous to the last column of the table, the reform leads to a welfare improvement in the symmetric economy, while welfare is lower in the benchmark case.

Table 6: Effective Limits

Symmetric	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7
$\tau_k = 0.40$	-0.523	-0.523	-0.607	-0.908	-1.746	-3.780	-8.485
$\tau_k = 0$ (int)	-0.523	-0.523	-0.607	-0.908	-1.746	-3.780	-8.485
$\tau_k = 0$	-0.414	-0.414	-0.496	-0.793	-1.610	-3.583	-8.123
Benchmark							
$\tau_k = 0.40$	-0.3353	-0.3353	-0.4213	-0.7282	-1.5616	-3.5739	-8.2194
$\tau_k = 0$ (int)	-0.3353	-0.3353	-0.4213	-0.7282	-1.5616	-3.5739	-8.2194
$\tau_k = 0$	-0.4133	-0.4133	-0.4957	-0.7916	-1.6080	-3.578	-8.1123

To understand these welfare differences, it is important to disentangle the effect of a change in the borrowing limits after the reform. To do this, consider first the intermediate scenario in which the endogenous limits do not change. In the benchmark case, only savers paid the capital income tax and the after tax interest rate rises from 2.22% to 2.88% when the tax is eliminated. This implies that they save more and aggregate capital increases, leading to a drop in the borrowing interest rate. These two effects will lead to a welfare improvement. Note also that a higher capital leads to higher wages. However, this effect is more than offset by the increase in labor income taxes, leading to a lower welfare. Overall, the aggregate welfare turns out to be higher in the benchmark case, while the opposite happens in the symmetric case. In the latter scenario, relatively rich savers are still better of after the reform, while relatively poor borrowers are hurt by the increase in the net interest rate, since they can only borrow at a higher interest rate. As before, the fact that relatively poor people are worse off decreases aggregate welfare.

Finally, consider the situation where the endogenous limits also change when the tax reform is implemented. As we see, the limits become looser after the reform in the benchmark case, while the opposite happens if both borrowers and savers pay the capital income tax. Note that, in the first case, borrowing becomes cheaper after the reform. In turn, this decreases the incentives to default and leads to looser borrowing constraints. However, this also implies that relatively poor borrowers get more indebted and end up being worse off in the long run. As a result, aggregate welfare is actually lower. In contrast, borrowing becomes more expensive after the reform in the symmetric case, leading to higher incentives to default and to tighter limits. In turn, relatively poor borrowers hold on average less debt and are better off, leading to a higher aggregate welfare.

The previous results illustrate that it is very important to take into account the effects on the borrowing constraints for the optimal design of government policy. This effect is not taken into account when the borrowing limits are fixed. However, the previous example illustrates that one would obtain the opposite conclusion regarding the welfare effects of a revenue neutral tax reform if one takes into account the effects of the reform on the borrowing constraints or not. Moreover, Table 6 reflects that welfare effects are amplified by changes in the borrowing constraints that are not present when the limits are fixed. In sum, it is very important to model the borrowing constraints appropriately when evaluating the effects of government policy.

6. Conclusions

The present work studies an economy with incomplete markets, capital accumulation and the possibility of default on financial liabilities. In particular, we study competitive equilibria where the loosest possible limits that prevent default are imposed. Further, we characterize how these endogenous limits depend on labour income.

In the presence of iid shocks, we shows that the endogenous limits become looser with a higher individual labour income, while the limits as a fraction of income become tighter. We show that these facts are consistent with the data. Moreover, the results are robust to the presence of persistent shocks.

When the allocations with endogenous limits are compared to the ones resulting from fixed zero limits, we see that capital accumulation is higher in the latter case due to the fact that there is less risk sharing. Moreover, we find that the borrowing limits are considerably different from zero, a characteristic that plays a crucial role when analyzing government policy. In particular, we analyze the welfare implications of a revenue neutral tax reform that eliminates capital income taxes at the expense of higher labor income taxes.

Using a calibrated version of the model, we show that it is very important to model the borrowing limits appropriately and to take into account that they can change when the reform is implemented. In particular, our example shows that one would obtain the opposite conclusion by not taking the effects on the borrowing limits into account. In addition, consistent with the findings under fixed limits, we find that the reform reduces aggregate welfare in the more realistic case where borrowers do not pay capital income taxes. However, the effects are amplified when the borrowing limits are endogenous. One of the key reasons is that the relatively poor borrowers hold more debt are worse off due to a higher after-tax interest rate.

We believe that our findings can be applied to a variety of other interesting contexts with incomplete markets and endogenous limits. As an example, Krueger and Perri (2003) analyze consumption and wealth inequality in a context with complete markets and enforcement constraints, providing a possible explanation as of why the increase in earnings inequality has not been accompanied by an increase in consumption inequality in recent decades. As argued by the authors, borrowing limits might have become looser due to a change in the exogenous earning process, whereas our results suggest that growth (capital accumulation) could have the same effect. Note that this is also consistent with the decline in savings rates that we have observed during the same period. In addition, Aiyagari et. al. (2002) study optimal government debt and fiscal policy in a context where the government wants to accumulate assets to smooth taxes and insure against future expenditure shocks. This also implies that households will be in debt in the long-run. On the other hand, the possibility of default reflected in the endogenous borrowing limits could impose constraints on how much assets the government can accumulate. More generally, our results suggests that fiscal policy and social insurance programs can have significant effects on the level of the endogenous trading constraints, and the welfare analysis of any policy reform should therefore take this into account. This is particularly relevant for the study of social security reforms, where the level of the endogenous limits could considerably affect the financial viability and impact of the reform.

APPENDIX

Appendix A: The Financial Intermediation Sector

This section discusses the simplest possible intermediation structure under which the endogenous limits that are not too tight would arise in equilibrium.

Throughout the section, we assume that households can trade through risk neutral financial intermediaries in a risky asset with an endogenous return that we denote by R. In particular, if a household invests in the asset ($a_{it} > 0$), for each unit of consumption he or she gives up at t, the intermediary promises to pay back R_{t+1} units of the consumption good at $t+1$. Further, if the household borrows ($a_{it} < 0$), for each unit of consumption he or she receives, he or she promises to pay back R_{t+1} units to the intermediary at $t+1$. We assume that the intermediaries and the households take R as given. Further, the intermediaries cannot price discriminate, in the sense that they have to pay the same return to investors as they charge to borrowers.

Under these assumptions, the budget constraint and wealth accumulation equation of household $i \in I$ can be written as follows:

$$c_{it} + a_{it} = w_t \epsilon_{it} + R_t a_{it-1}.$$

The intermediaries live for two periods. At the beginning of the first period, they set the borrowing limits (α_{it}) on the agents' asset holdings. Here, we assume that the intermediaries are free to set these limits and we look for a symmetric equilibrium where all intermediaries set the same limits. Households are therefore subject to a trading constraint of the form:

$$a_{it} \geq \alpha_{it}.$$

The cash flows of the intermediaries can be described as follows. During the first period, they trade consumption goods with the households and collect a total amount of $A_t \geq 0$ goods. Further, they transform this (or some portion of it) into physical capital $k_t \leq A_t$, which is rented to the representative firm. In order to be consistent with the setup in the main text, we assume that this transformation is one-to-one. In the second period, they receive rental income of $r_{t+1}k_t$ from the firm and they have to honor the trading contracts with the households by paying back $R_{t+1}A_t$. Let $t+1|t$ be the set of possible contingencies after period t. If $q_{t,t+1}$ denotes the discount factor of the intermediaries between periods t and $t+1$, their profit function can be written as:

$$A_t - k_t + \sum_{t+1|t} q_{t,t+1} \left[r_{t+1}k_t - R_{t+1}A_t \right]. \tag{14}$$

The intermediaries can commit to repay back their debt to the households at any possible contingency, but they cannot be forced to lend to or borrow from any household. Further, we assume that $A_t > 0$, implying that intermediaries cannot be solely making pure arbitrage profits but have to mediate between the households and the production sector.[9]

We first discuss the restrictions that need to be satisfied in a symmetric equilibrium where all intermediaries hold the same portfolio. First, an equilibrium implies that $k_t = K_t$, where K_t is the demand for capital of the representative firm. Second, the following condition has to hold:

$$\sum_{t+1|t} q_{t,t+1} r_{t+1} \leq \sum_{t+1|t} q_{t,t+1} R_{t+1}. \tag{15}$$

To see why this is the case, note that the intermediaries could otherwise make arbitrarily large profits by demanding arbitrarily large funds A_t and by setting $k_t = A_t$. Third, since the intermediary can commit to repay back its debt to the households at each date, solvency requires that $R_{t+1}A_t \leq r_{t+1}k_t$. This, together with the fact that $k_t \leq A_t$, implies that $R_{t+1} \leq r_{t+1}$. Finally, combining the last condition with (15), it becomes clear that the only possible equilibrium is to have $R_{t+1} = r_{t+1}$ and $k_t = A_t = K_t$. Given this, equation (14)

[9] This assumption guarantees the existence of symmetric equilibria where all intermediaries hold the same portfolio.

implies that all intermediaries make zero profits, regardless of the particular value of the discount factor q.

While there are many allocations with different borrowing constraints that satisfy the above restrictions, we consider the one with the loosest limits. As before, the limits are defined in two steps. First, we define default thresholds that are *not too tight*, in the sense that some agent would default under some continuation state if they were loosened by some $\varepsilon > 0$. Note that we have just shown that the return on a_{it} is the same as the return on physical capital. Given this, we can use the value functions W and V to define the thresholds if we replace the individual capital holdings k with the asset holdings a. Second, we can obtain the borrowing limits set by the intermediaries at a given date by defining the tightest of all these default thresholds for that date.

In order to show that these limits actually arise in equilibrium, we have to prove that no intermediary can make positive profits by loosening the trading limits relative to the limits that are not too tight. This is established by the following proposition, also showing that there does not exist any symmetric equilibrium with limits that allow for default.

Proposition A.1. *(i) The allocation with default thresholds that are not too tight is a (symmetric) equilibrium. Further, (ii) a symmetric equilibrium with default does not exist.*

Proposition 2.3. implies that the limits set by a competitive intermediation sector are such that no household has an incentive to default in a symmetric equilibrium. Further, among these equilibria, we chose the one with the loosest possible limits or limits that are not too tight, since this allows for the highest possible risk sharing.

Appendix B: Proofs

Proof of Proposition 2.1. First, since the periodic utility function $u(\cdot)$ is continuous in consumption, $W(\epsilon, k; \Psi)$ satisfies the same property in k. It is also clear that the value function has to be increasing in k, since everything that is feasible under a given $(\epsilon, k; \Psi)$ has to be feasible under $(\epsilon, \widetilde{k}; \Psi)$ for all $\widetilde{k} \geq k$. This implies that $W(\epsilon, \widetilde{k}; \Psi) \geq W(\epsilon, k; \Psi)$ for $\widetilde{k} \geq k$. Let $\underline{k}^N(\epsilon, \Psi) < 0$ be the appropriately defined natural borrowing limit. This limit is defined as the level of debt such that households are just able to repay their debt under every possible contingency and still have non-negative consumption.[10] Further, define the (possibly infinite) supremum of the utility function as follows: $\overline{U} \equiv \sup_{c \in \mathbb{R}_+} u(c) \leq \infty$. Then, the strict monotonicity of the period utility function implies that:

$$\lim_{k \to \underline{k}^N(\epsilon, \Psi)} W(\epsilon, k; \Psi) = -\infty \text{ and } \lim_{k \to \infty} W(\epsilon, k; \Psi) = \frac{\overline{U}}{1 - \beta}.$$

The first equality follows from the fact that u is unbounded below and from the fact that, at the natural borrowing limit, households would end up consuming zero along some history with positive probability. In addition, since our assumptions imply that the shock ϵ and

[10] See Santos and Woodford (1997) for a specification of such limits in a general incomplete markets context. As shown by the authors, the natural limits represent an appropriately defined present value of the individual labor income, which has to be finite in equilibrium.

the aggregate capital K are positive and finite, it also follows that $-\infty < V(\epsilon;\Psi) < \frac{\overline{U}.}{1-\beta}$. Given this, the intermediate value theorem implies that there exists a unique and finite $\underline{k} > \underline{k}^N(\epsilon,\Psi) > -\infty$ for $\forall(\epsilon,\Psi)$ such that $W(\epsilon,\underline{k};\Psi) = V(\epsilon;\Psi)$.

Second, since $W(\epsilon,\underline{k};\Psi) = V(\epsilon;\Psi)$ and $W(\epsilon,0;\Psi) \geq V(\epsilon;\Psi)$, the fact that W is increasing in k implies that $\underline{k}(\epsilon;\Psi) \leq 0$ and therefore $\sup_{(\epsilon;\Psi)}\{\underline{k}(\epsilon;\Psi)\} \leq 0$. In other words, the equilibrium default threshold and effective limits are non-positive. ■

Proof of Lemma 2.1. Suppose that $g_c(\epsilon,\underline{k}(\epsilon;\Psi);\Psi) > w(K)(1-\tau_l)\epsilon(1-\lambda)$. In this case, it follows that:

$$u(g_c(\epsilon,\underline{k}(\epsilon;\Psi);\Psi)) + \beta\sum_{\epsilon'}\Pi(\epsilon'|\epsilon)V(\epsilon';\Psi') > V(\epsilon;\Psi) = W(\epsilon,\underline{k}(\epsilon;\Psi);\Psi).$$

The first inequality follows from the definition of the autarky value in (9), whereas the last equality follows from the definition of the lower bounds in (11). However, the previous inequality contradicts the fact that W is the maximal life-time utility that agents can achieve, since it implies that they could improve welfare by consuming $g_c(\epsilon,\underline{k}(\epsilon;\Psi);\Psi)$ at the default threshold and by defaulting at every future contingency. It therefore follows that $g_c(\epsilon,\underline{k}(\epsilon;\Psi);\Psi) \leq w(K)(1-\tau_l)\epsilon(1-\lambda)$.■

Proof of Proposition 2.2. To prove the proposition, we can use the definitions of the two value functions to derive the following expression for the nominator of (13):

$$W_\epsilon(\epsilon_t,\underline{k}_t;\Psi_t) - V_\epsilon(\epsilon_t;\Psi_t) = E_t\sum_{\tau=t}^{\infty}(\beta\rho_\epsilon)^{\tau-t}\,w(K)(u'(c_\tau) - u'(c_\tau^{au})), \tag{16}$$

where $c_\tau^{au} = w(K_\tau)(1-\tau_l)\epsilon_\tau(1-\lambda)$ and $c_\tau = w(K_\tau)(1-\tau_l)\epsilon_\tau + r(K_\tau)(1-\tau_k)k_\tau - k_{\tau+1}$ represent the consumptions in autarky and in the trading arrangement respectively. If $\rho_\epsilon = 0$, the sign of the left hand side of the previous equation is equal to the sign of $u'(c_t) - u'(c_t^{au})$. Further, the concavity of u and lemma 2.1 imply that $c_t \leq c_t^{au}$ and $u'(c_t) - u'(c_t^{au}) \geq 0$. It therefore follows that $\frac{\partial\underline{k}(\epsilon;\Psi)}{\partial\epsilon} \leq 0$.■

Proof of Proposition A.1. (i) We first prove that the allocation with limits that are not too tight is a symmetric equilibrium. In particular, we first consider a symmetric equilibrium with the loosest possible limits that avoid default in equilibrium and then show that no intermediary can achieve positive or zero profits by loosening these limits. To do this, assume without loss of generality that the participation constraint is binding for agents of a certain type i at some given contingency $\widetilde{t}+1$ after period t. This implies that:

$$W(\epsilon_{1\widetilde{t}+1}, a_{it};\Psi_{\widetilde{t}+1}) = V(\epsilon_{1\widetilde{t}+1};\Psi_{\widetilde{t}+1}), \tag{17}$$

or that $a_{it} = \alpha_{it}$.

We now show that no intermediary can build a portfolio $\overline{A}_t > 0$ at date t which gives him at least zero profits and which involves more lending $\overline{a}_{it}(s^t) < a_{it}$ to the type i agents. To see this, note first that the intermediaries need to satisfy the following condition to be solvent at each future contingency $t+1|t$:

$$\overline{R}_{t+1}\overline{A}_t \leq r_{t+1}\overline{k}_t,$$

where $\overline{R}_{t+1}$ is the return offered by the intermediary for continuation history $t+1|t$ and $\overline{k}_t$ is the total capital rented to the firm at t. Second, since $\overline{k}_t \leq \overline{A}_t$, it has to be the case that $\overline{R}_{t+1} \leq r_{t+1} = R_{t+1}$ for all $t+1|t$, where the last equality follows from the fact that R and r satisfy condition (15) for a symmetric equilibrium with no default. Finally, note that (17) implies the agents of type i will default in contingency $\widetilde{t}+1|t$ under the new portfolio due to the fact that W is decreasing in a and $\overline{a}_{it} < a_{it}$. It therefore follows that $\overline{R}_{\widetilde{t}+1} < r_{\widetilde{t}+1} = R_{\widetilde{t}+1}$. Since this is a strictly worse deal for other agent types who do not default, it follows the intermediary will not be able to build this portfolio.

To see that the last condition $\overline{R}_{\widetilde{t}+1} < r_{\widetilde{t}+1} = R_{\widetilde{t}+1}$ has to hold, note that the intermediary has at most $r_{\widetilde{t}+1}\overline{A}_t$ goods available at contingency $\widetilde{t}+1$. On the other hand, he has to pay out $\overline{R}_{\widetilde{t}+1}\left(\overline{A}_t - \overline{a}_{it}\right)$ to the other agent types. Since the type i agents will only default if $\overline{a}_{it} < 0$, however, this can only be done if $\overline{R}_{\widetilde{t}+1} < r_{\widetilde{t}+1} = R_{\widetilde{t}+1}$. Alternatively, the intermediary could decrease $\overline{R}_{\widetilde{t}+1}$ so as to make the type i agents not want to default any more, but this would directly imply that $\overline{R}_{\widetilde{t}+1} < r_{\widetilde{t}+1} = R_{\widetilde{t}+1}$. The other agent types will therefore not be willing to accept the deal in either case.

(*ii*) We now prove that symmetric equilibria with default cannot exist. To do this, assume first that the limits are such that, at a given date t, there exists at least a continuation state $\widehat{t}+1|t$ (or more generally a set of states) where agents of type i will default. We now show that this equilibrium cannot exist, since the intermediaries will be able to make positive profits by not "lending" to the types with positive default probabilities.

We prove it by contradiction, assuming that such an equilibrium exists. First, note that the following condition has to hold in such an equilibrium:

$$\sum_{t+1|t} q_{t,t+1} r_{t+1} \leq \sum_{t+1|t} q_{t,t+1} R_{t+1} - q_{t,\widehat{t}+1} R_{\widehat{t}+1} \frac{a_{it}}{A_t}, \tag{18}$$

where a_{it} are the asset holdings of the type i agents and A_t are the total funds collected by the intermediaries. Further, r and R represent the prices in the symmetric equilibrium with default. Note that, if the previous condition was not satisfied, the intermediaries could set $k_t = A_t$ and make arbitrarily large profits.

Second, since $k_t \leq A_t$ at all nodes and the intermediaries have to be solvent at every possible contingency, the only equilibrium with $A_t > 0$ is given when $k_t = A_t$ and (18) satisfied with equality. In this case, the intermediaries make zero profits. On the other hand, since the agents of type i do not want to default unless $a_{i\widetilde{t}+1} < 0$, it also follows that:

$$\sum_{t+1|t} q_{t,t+1} r_{t+1} > \sum_{t+1|t} q_{t,t+1} R_{t+1}. \tag{19}$$

Condition (19) implies that an intermediary could make positive profits under the current prices by only accepting deposits from agents that do not default and by not lending to the type i agents, i.e., by setting $a_{it} = 0$. Under this profitable deviation, any intermediary offering the original contract would be driven out of the market. Further, there would not be any lending to the type i agents, contradicting the existence of an equilibrium with default of the agents of type i.■

Appendix C: Numerical Algorithm

The general algorithm used to solve for the steady state given a vector of taxes (τ_k, τ_l) is an extension of the one in Aiyagari (1994).

First, we define a grid on the endogenous state space, given by the individual level of capital k. Note that the grid on the exogenous shock ϵ is implicitly defined by our Markov chain assumption. *Second*, we initialize the capital return r and compute the corresponding levels of K and w from the optimality conditions of the firm, Moreover, we define some initial default thresholds $\underline{k}(\epsilon)$. *Third*, given the grid on k and ϵ, our procedure finds continuous equilibrium policy functions for the individual consumptions $c(\epsilon, k)$ and end of period asset holdings $k(\epsilon, k)$ such that all the conditions of the recursive competitive equilibrium defined earlier are satisfied for the given set of default thresholds, $\underline{k}(\epsilon)$. Since the aggregate capital is constant in the stationary state, the thresholds only depend on the labor income shock. Similarly, the policy functions only depend on the individual state vector (ϵ, k).

Here, it is important to note that the algorithm is different for the symmetric and asymmetric cases. In the symmetric case, we have to make sure that the following Euler equation coming from the household's optimization problem is satisfied:

$$u'(c) \geq \beta E u'\left(c'\right)\left(1 + r\left(1 - \tau_k\right)\right).$$

If ω is the individual wealth and k_l and k_u are the upper and lower bounds for individual assets, there are three different cases in the symmetric scenario: $k' = k_l$, $k' = k_u$ and $k' \in (k_l, k_u)$. On the other hand, the asymmetric scenario is more complicated, since only the previous Euler condition does not always hold, since only the borrowers pay taxes. In this case, we distinguish between the following cases, which are characterized by different Euler conditions and by different values of the capital income tax rate: $k' = k_l$, $k' = k_u$, $k' \in (k_l, 0)$, $k' \in (0, k_u)$ and $k' = 0$.

Fourth, using the equilibrium policy functions, the value functions $W = W(\epsilon, k)$ and $V = V(\epsilon)$ are calculated recursively. Finally, the endogenous default thresholds are loosened if $W(\epsilon, \underline{k}(\epsilon)) > V(\epsilon)$ and they are tightened otherwise. All the previous objects are approximated with continuous functions using linear interpolation over the finite and endogenous grid, and the procedure is repeated until convergence.

More precisely, given a set of default thresholds $\underline{k}$, let h be the vector consisting of the policy functions of interest, i.e., $h = [c, k']$. Further, let T be a non-linear operator such that $T[h\ W\ V\ \underline{k}\ ]$ satisfies the equilibrium system of equations and the participation constraints determining the limits. The solution to our problem is then a fixed point of T, i.e., a vector $[h\ W\ V\ \underline{k}]$ such that $[h\ W\ V\ \underline{k}] = T[h\ W\ V\ \underline{k}]$. To approximate the fixed point, we follow the steps below.

Step 1: Guess an initial vector $[h_0\ W_0\ V_0\ \underline{k}_0]$, where $h_0 = [c_0, k'_0]$.

Step 2: For each iteration $n \geq 1$, use the previous guess $[h_{n-1}\ W_{n-1}\ V_{n-1}]$ and $\underline{k}_{n-1}$ to compute the new vector $[h_n\ W_n\ V_n]$ that satisfies the equilibrium conditions. Further, use the new vector $[h_n\ W_n\ V_n]$ to find the new lower bound $\underline{k}_n$ such that $W_n(\epsilon, \underline{k}_n) \approx V_n(\epsilon, \underline{k}_n)$, and update the grid accordingly.

Step 3: Use $[h_n\ W_n\ V_n]$ and $\underline{k}_n$ as the next initial guess and iterate until $[h_n\ W_n\ V_n\ \underline{k}_n]$ converges.

The incomplete markets allocation with ad hoc fixed limits is solved with a version of the procedure that does not update the limits.

Fifth, we need to calculate the stationary employment-wealth distribution. To do this, we make use the fact that it can be obtained by defining a Markov process for the distribution and by calculating the stationary distribution of this Markov chain. In particular, we approximate the process for (k, ϵ) with a Markov chain, from which we can obtain the stationary probability vector $\lambda(k_n, \epsilon_m)$, where:

$$\lambda(k_n, \epsilon_m) = \Pr(k_t = k_n, \epsilon_t = \epsilon_m)$$

Note that $\lambda(k, \epsilon)$ represents the fraction of time that a household indexed by (k, ϵ) spends in state (k, ϵ). Alternatively, it is the fraction of households with asset holdings k and labor endowment ϵ (there is an equivalence between the cross sectional and time series properties of the distribution). Given this, if the individual state of the economy is given by the vector:

$$x = \{(\epsilon_1, k_1), ..., (\epsilon_1, k_n), (\epsilon_2, k_1), ..., (\epsilon_2, k_n), ..., (\epsilon_m, k_1), ..., (\epsilon_m, k_n)\}^T$$

the policy function $k' = k(\epsilon, k)$ and the shock transition matrix Π induce a Markov chain on x via the following formula:

$$\begin{aligned} \Pi &= \Pr[(k_{t+1} = k', \epsilon_{t+1} = \epsilon')|(k_t = k, \epsilon_t = \epsilon)] \\ &= \Pr[(k_{t+1} = k'|\epsilon_t = \epsilon, k_t = k)] \Pr[\epsilon_{t+1} = \epsilon'|\epsilon_t = \epsilon] = \Upsilon(k', k, \epsilon)\Pi \end{aligned}$$

where $\Upsilon(k', k, \epsilon)$ is an indicator function which is equal to one if $k' = k(\epsilon, k)$ and zero otherwise. Further, the stationary probability vector of this Markov chain is given by:

$$p_\infty \Pi = p_\infty = \{\lambda(\epsilon_1, k_1), ..., \lambda(\epsilon_1, k_n), \lambda(\epsilon_2, k_1), ..., \lambda(\epsilon_2, k_n), ..., \lambda(\epsilon_m, k_1), ..., \lambda(\epsilon_m, k_n)\}^T$$

while the aggregate asset holdings are equal to $p_\infty^T x$. This is compared to the aggregate capital demanded by the firm. Unless they are sufficiently close, the interest rate is updated and the previous procedure is updated until convergence.

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