# Bayesian forecast combination for VAR models*


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**Abstract**

This paper proposes a Bayesian procedure for combining forecasts from multivariate forecasting models, e.g. VAR models. Standard applications of Bayesian model averaging suffer from a basic difficulty in this context, when additional variables are included and modelled the connection between the overall measure of fit for the model and the expected forecasting performance for the variables of interest is lost. We circumvent this problem by focusing on the predictive performance for the variables of interest and base the forecast combination on the predictive likelihood.

Specifically we consider forecast combination and, indirectly, model selection for VAR models when there is uncertainty about which variables to include in the model in addition to the forecast variables. For this purpose we consider all possible combinations of variables and lag lengths and the models that arise from these.

The procedure is evaluated in a small simulation study and found to perform competitively in applications to real world data.



**Keywords:** Bayesian model averaging, Predictive likelihood, GDP forecasts
**JEL-codes:** C11, C15, C32, C52, C53


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# 1 Introduction

The increasing availability of data has spurred the interest in forecasting procedures that can extract information from a large number of variables in an efficient manner. Examples include the diffusion indexes of Stock and Watson (2002*b*) and procedures based on combining forecasts from many models as in Jacobson and Karlsson (2004), see Stock and Watson (2006) for a recent review and additional references. While this development has clear implications for policy makers such as central banks (see e.g. Bernanke and Boivin (2003)) procedures of this type are not particularly widespread in central banks. Notable practitioners are Sveriges Riksbank, the Bank of England and the Bank of Canada. These central banks employ a wide variety of model approaches, ranging from simple univariate time series models to highly sophisticated multivariate non-linear models. While a great many models are used, the procedures are easy to manage and highly automated (see, for example, Andersson and Löf (2007) and Kapetanios, Labhard and Price (2007)).

One possible reason for the apparent lack of interest in the possibilities offered by these procedures is that the literature has largely focused on univariate forecasting procedures. This paper attempts to bridge this gap by proposing a Bayesian procedure for combining forecasts from multivariate forecasting models, e.g. VAR models. Standard applications of Bayesian model averaging suffer from a basic difficulty in this context, when additional variables are included and modelled the connection between the overall measure of fit for the model, the marginal likelihood, and the expected forecasting performance for the variables of interest is lost. It is easy to see that the (multivariate) marginal likelihood can change when a model is modified by adding, removing or exchanging variables without this having the corresponding effect on the predictive ability for the variable of interest.

We circumvent this problem by focusing on the predictive performance for the variables of interest and base the forecast combination on the predictive likelihood as proposed by Eklund and Karlsson (2007) in the context of univariate forecasting models. While the basic predictive likelihood is also multivariate it is meaningful to marginalize the predictive distribution with respect to the auxiliary variables yielding a univariate predictive distribution and corresponding predictive likelihood. Forecasts from different models can then be combined using weights based on the univariate predictive likelihood.

Specifically we consider forecast combination and, indirectly, model selection for VAR models when there is uncertainty about which additional variables to include in the model. Given a set of auxiliary variables that are expected to be useful for modelling and forecasting the variable of interest we consider the set of models that arise when taking all possible combinations of the auxiliary variables. The forecasts from these models are then combined using weights based on the predictive likelihood at the relevant forecast horizon.

In most cases the predictive likelihood will not be available in closed form. Instead we use MCMC methods to simulate the predictive distribution and estimate the density function from the MCMC output. In addition the MCMC output is used to obtain forecast intervals both for forecasts based on a single model and the combined forecast.

The procedure is evaluated in a simulation study and found to perform competitively in an application to forecasting the growth rate of US GDP.

# 2 Bayesian Forecast Combination

Bayesian forecast combination is a straightforward application of Bayesian model averaging (see Hoeting, Madigan, Raftery and Volinsky (1999) for an introduction to Bayesian model averaging and Min and Zellner (1993), Jacobson and Karlsson (2004) and Koop and Potter (2004) for applications of Bayesian model averaging to forecasting and Timmermann (2006) for a review of forecast combination). Suppose that the forecaster has a set, $\mathfrak{M} = \{\mathcal{M}_1, \ldots, \mathcal{M}_M\}$, of $M$ possible forecasting models available, each specified in terms of a likelihood function $L(\mathbf{y}|\theta_i, \mathcal{M}_i)$ and prior distribution for the parameters in the model, $p(\theta_i|\mathcal{M}_i)$. In addition the forecaster assigns prior probabilities, $p(\mathcal{M}_i)$, to each model, reflecting the forecasters prior confidence in the models. The posterior model probabilities can then be obtained by routine application of Bayes theorem

$$p(\mathcal{M}_i|\mathbf{y}) = \frac{m(\mathbf{y}|\mathcal{M}_i)\, p(\mathcal{M}_i)}{\sum_{j=1}^{M} m(\mathbf{y}|\mathcal{M}_j)\, p(\mathcal{M}_j)} \tag{1}$$

where

$$m(\mathbf{y}|\mathcal{M}_i) = \int L(\mathbf{y}|\theta_i, \mathcal{M}_i)\, p(\theta_i|\mathcal{M}_i)\, d\theta_i \tag{2}$$

is the marginal likelihood of model $\mathcal{M}_i$. The combined forecast is obtained as

$$E(y_{T+h}|\mathbf{y}) = \sum_{j=1}^{M} E(y_{T+h}|\mathbf{y}, \mathcal{M}_j)\, p(\mathcal{M}_j|\mathbf{y})$$

by weighting the forecasts from each model by the posterior model probabilities. It is easily seen that the Bayesian forecast combination is a special case of the general result that the marginal (over all models) posterior distribution for some function $\phi$ of the parameters is

$$p(\phi|\mathbf{y}) = \sum_{j=1}^{M} p(\phi|\mathbf{y}, \mathcal{M}_j)\, p(\mathcal{M}_j|\mathbf{y}). \tag{3}$$

The crucial feature of the marginal distribution (3) is that it takes account of both parameter and model uncertainty. It is thus relatively easy to produce prediction intervals that incorporates model uncertainty.

The marginal likelihood (2) is the basic Bayesian measure of fit of a model and is a joint assessment of how well the likelihood and parameter prior agrees with the data. It is the key quantity for determining the posterior model probabilities and hence the weights assigned to the forecasts from the different models.

## 2.1 Predictive Likelihood

The marginal likelihood is well suited for combination of univariate forecasting models but, unfortunately, problematic when it comes to the combination of forecasts from multivariate forecasting models. Multivariate forecasting models, e.g. VAR-models, are typically built with the express purpose of forecasting a single variable and the remaining dependent variables in the model are only included if they are deemed to improve the forecasting performance for the variable of interest. As the marginal likelihood measures the fit of the whole model it is easy to see that the forecast performance can remain unaffected by a change in the model that either increases or decreases the marginal likelihood. This can happen when a dependent is exchanged for another variable or the dimension of the model changes as variables are added or dropped from the model.

To overcome these problems with the marginal likelihood we propose to base the forecast combination on the predictive likelihood as suggested by Eklund and Karlsson (2007) in the context of univariate forecasting models. Our primary motivation for using the predictive likelihood is that it is meaningful to marginalize this over the non-forecasted variables to obtain a measure that is focused on the variable of interest. An added benefit of the predictive likelihood is that it is a true out of sample measure of fit whereas the marginal likelihood depends on the predictive content of the parameter prior. When combining the forecasts from a large set of models it is often to time consuming to provide well thought out parameter priors for all the models. Instead uninformative default priors such as the ones suggested by Fernández, Ley and Steel (2001) are used and with this type of prior the marginal likelihood essentially reduces to an in-sample measure of fit.

Our use of the predictive likelihood is based on a split of the data, $\mathbf{Y} = (\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_T')'$, into two parts, the training sample, $\mathbf{Y}_n^* = (\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_n')'$ of size $n$, and an evaluation or hold out sample, $\widetilde{\mathbf{Y}}_n = \left(\mathbf{y}_{n+1}', \mathbf{y}_{n+2}', \ldots, \mathbf{y}_T'\right)'$ of size $m = T - n$, where $\mathbf{y}_t = (y_{1t}, \ldots, y_{qt})$ is the vector of modelled variables. The training sample is used to convert the prior into a posterior and the predictive likelihood is obtained by marginalizing out the parameters from the joint distribution of data and parameters,

$$p\left(\widetilde{\mathbf{Y}}_n \middle| \mathbf{Y}_n^*, \mathcal{M}_i\right) = \int L\left(\widetilde{\mathbf{Y}}_n \middle| \theta_i, \mathbf{Y}_n^*, \mathcal{M}_i\right) p\left(\theta_i \middle| \mathbf{Y}_n^*, \mathcal{M}_i\right) d\theta_i. \qquad (4)$$

Technically this is the predictive distribution of an unknown $\widetilde{\mathbf{Y}}_n$ conditional on the training sample, $\mathbf{Y}_n^*$. When evaluated at the observed $\widetilde{\mathbf{Y}}_n$ (4) provides a measure of the out of sample predictive performance and we refer to this as the predictive likelihood. Since our primary interest is to forecast a subset of the $q$ modelled variables the multivariate predictive likelihood (4) suffers from the same drawback as the marginal likelihood in that it is not directly informative about the forecasting performance for the variable of interest. To overcome this we marginalize the predictive distribution of $\widetilde{\mathbf{Y}}_n$ with respect to the auxiliary variables, with $y_1$ the variable of interest we have

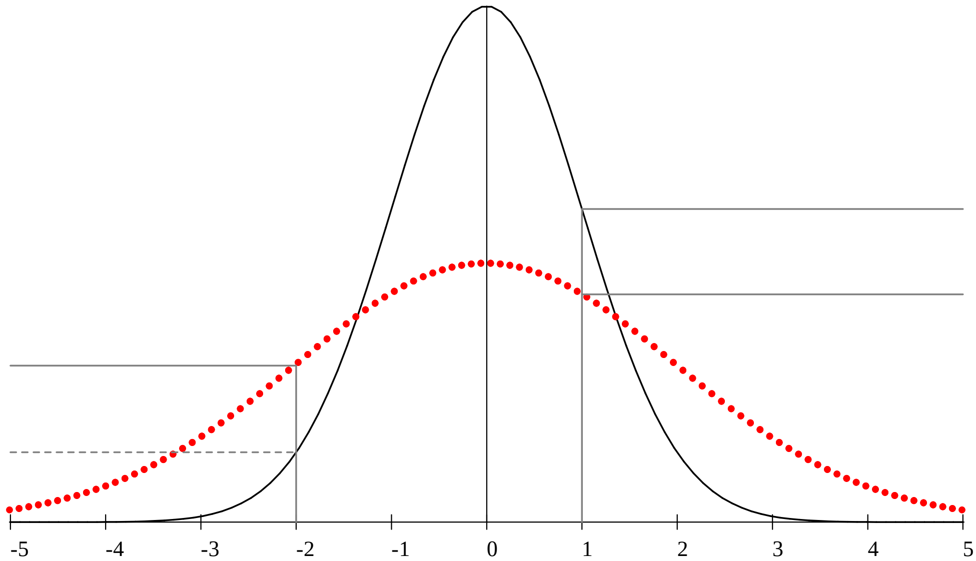
$$p\left(\widetilde{\mathbf{y}}_{1,n} \middle| \mathbf{Y}_n^*, \mathcal{M}_i\right) = \int p\left(\widetilde{\mathbf{Y}}_n \middle| \mathbf{Y}_n^*, \mathcal{M}_i\right) d\widetilde{\mathbf{y}}_{2,n} \ldots d\widetilde{\mathbf{y}}_{q,n} \qquad (5)$$

the marginal predictive likelihood for the hold of sample of $y_1$ as a measure of the average predictive performance for the variable of interest.

Replacing the marginal likelihood with the marginal predictive likelihood in (1) yields the predictive weights

$$w\left(\mathcal{M}_i \middle| \widetilde{\mathbf{y}}_{1,n}, \mathbf{Y}_n^*\right) = \frac{p\left(\widetilde{\mathbf{y}}_{1,n} \middle| \mathbf{Y}_n^*, \mathcal{M}_i\right) p\left(\mathcal{M}_i\right)}{\sum_{j=1}^{M} p\left(\widetilde{\mathbf{y}}_{1,n} \middle| \mathbf{Y}_n^*, \mathcal{M}_j\right) p\left(\mathcal{M}_j\right)} \qquad (6)$$

**Figure 1** Predictive likelihood for a "good" and a "bad" model




and the combined forecast

$$\hat{y}_{T+h} = \sum_{j=1}^{M} E\left(y_{T+h}|\, \mathbf{Y},\mathcal{M}_j\right) w\left(\mathcal{M}_i|\, \tilde{\mathbf{y}}_{1,n}, \mathbf{Y}_n^*\right).$$

While the predictive weights (6) strictly speaking can not be interpreted as posterior probabilities they have several appealing properties in addition to providing a basis for meaningful marginalization with respect to the auxiliary variables in the model.

- Proper prior distributions are not required for the parameters. The predictive likelihood is, in contrast to the marginal likelihood, well defined as long as the posterior distribution of the parameters conditioned on the training sample is proper.

- The predictive likelihood is not an absolute measure of forecasting performance. Instead it is relative to the precision of forecasts implied by the model and models with a good in-sample fit are penalized when a "good" and "bad" model forecast both forecasts poorly. This is illustrated in Figure 1. If the forecast error is small (1) as can be expected from a model with good in-sample fit, the predictive likelihood prefers the "good" model but the "bad" model is favoured if the forecast error (-2) is larger than what can be expected from the "good" model. The predictive weights will thus be small for models that overfit the data or models with structural breaks.

## 2.2 Dynamic Models

The predictive densities (4) and (5) are joint predictive distributions for lead times $h = 1$ through $h = m = T - n$. For dynamic models where the forecast precision typically deteriorates as the lead time increases these will not be appropriate measures of forecast performance if the focus is on producing forecasts for a few select lead times. One solution is to set $m$ to the largest lead time, $H$, considered but this will typically be small (say 8 quarters) and the Monte Carlo experiments in Eklund and Karlsson (2007) indicates that the hold out sample should be large, on the order of 70% of the data. To combine these two requirements we suggest using a series of short horizon predictive likelihoods,

$$g(\mathbf{Y}, n, H|\mathcal{M}_i) = \prod_{t=n}^{T-h_k} p\left(y_{1,t+h_1}, \ldots, y_{1,t+h_k} | \mathbf{Y}_t^*, \mathcal{M}_i\right) \tag{7}$$

where $h_1, \ldots, h_k$ represents the lead times at which we whish to evaluate the forecast performance.

The use of the predictive likelihood in dynamic models is complicated by the fact that the predictive likelihood is not available in closed form for lead times $h > 1$. Instead the predictive distribution must be simulated and the predictive likelihood estimated from the simulation output. Standard density estimation techniques can be used for this purpose and works quite well if the predictive likelihood is evaluated at a single lead time. Evaluating the predictive likelihood at multiple horizons leads to more complex multivariate density estimation.

To facilitate the use of multiple horizon predictive likelihoods we take advantage of the model structure and use the idea of Rao-Blackwellization to estimate the predictive likelihood. Consider the task of evaluating the unknown density $f_u$ at $u = x$ when we have draws from the joint distribution of $(u, v)$ or only the marginal distribution of $v$ and the conditional density $f_{u|v}$ is known. We want $f_u(x) = \int f_{u,v}(x, v)\, dv = \int f_{u|v}(x, v)\, f_v(v)\, dv = E_v\left[f_{u|v}(x, v)\right]$ where we make the dependence of $f_{u|v}$ on $v$ explicit by including it as an argument to the function. A simple Monte Carlo estimate is then given by $\widehat{f_u}(x) = \frac{1}{R}\sum_{i=1}^{R} f_{u|v}(x, v_i^*)$ where $v_i^*$ are the draws from the marginal distribution of $v$. The Rao-Blackwellized estimate will in general be quite precise even for moderate sample sizes and preserves any smoothness properties of the underlying density.

For the VAR-model

$$\begin{aligned}\mathbf{y}_t &= \sum_{i=1}^{p} \mathbf{y}_{t-i}\mathbf{A}_i + \mathbf{x}_t\mathbf{C} + \mathbf{u}_t \\ &= \mathbf{z}_t\boldsymbol{\Gamma} + \mathbf{u}_t\end{aligned} \tag{8}$$

or $\mathbf{Y} = \mathbf{Z}\boldsymbol{\Gamma} + \mathbf{U}$ with $\mathbf{z}_t = (\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}, \mathbf{x}_t)$ and $\mathbf{u}_t \sim N(0, \boldsymbol{\Psi})$ it is easiest to evaluate the predictive likelihood by way of the forecast errors. Define the forecast error at horizon $h$ for a particular set of parameters as

$$\mathbf{e}_{T+h} = \mathbf{y}_{T+h} - E(\mathbf{y}_{T+h}|\mathbf{Y}_T, \boldsymbol{\Gamma}, \boldsymbol{\Psi}) = \sum_{i=0}^{h-1} \mathbf{u}_{T+h-i}\mathbf{B}_i$$

where $\mathbf{B}_i$ are the parameter matrices in the MA-representation $\mathbf{y}_t = \sum_{i=0}^{\infty} \mathbf{x}_{t-i}\mathbf{C}\mathbf{B}_i + \sum_{i=0}^{\infty} \mathbf{u}_{t-i}\mathbf{B}_i$. The distribution of $\mathbf{e}_{T+h}$ conditional on the data and the parameters is normal, $\mathbf{e}_{T+h}|\mathbf{Y}_T, \boldsymbol{\Gamma}, \boldsymbol{\Psi} \sim N\left(\mathbf{0}, \sum_{i=0}^{h-1} \mathbf{B}_i'\boldsymbol{\Psi}\mathbf{B}_i\right)$. Further define $\widetilde{\mathbf{e}}_{T+h} = (\mathbf{e}_{T+1}, \ldots, \mathbf{e}_{T+h})$, the joint distribution of the lead time 1 through $h$ is normal with mean zero and variance covariance matrix $\mathbf{\Omega} = \widetilde{\mathbf{B}}' (\mathbf{I}_h \otimes \mathbf{\Psi}) \widetilde{\mathbf{B}}$ with

$$\widetilde{\mathbf{B}} = \begin{pmatrix} \mathbf{B}_0 & \mathbf{B}_1 & \cdots & \mathbf{B}_{h-1} \\ \mathbf{0} & \mathbf{B}_0 & \cdots & \mathbf{B}_{h-2} \\ \vdots & & \ddots & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_0 \end{pmatrix}.$$

and block $i, j$ of $\mathbf{\Omega}$ is given by $\sum_{m=i-j}^{j} \mathbf{B}'_{m+i-j} \mathbf{\Psi} \mathbf{B}_m$ for $i \geq j$. The matrices in the MA-polynomial are easily obtained through the recursion

$$\mathbf{B}_0 = \mathbf{I}$$
$$\mathbf{B}_i = \sum_{m=1}^{q} \mathbf{A}_m \mathbf{B}_{i-m},\ i > 0$$

for $q = \min(i, p)$. Note that the recursion is well defined for finite $i$ even if the model is nonstationary.

It is thus trivial to obtain the marginal distribution of any subset of the variables and forecast horizons conditional on the parameters as a multivariate normal distribution. The Rao-Blackwellized estimate of $p(y_{1,t+h_1}, \ldots, y_{1,t+h_k} | \mathbf{Y}_t^*, \mathcal{M}_i)$ is then obtained by letting the parameters play the role of $v$ and the forecast errors play the role of $u$ above. The draws from the posterior distribution of the parameters are, in our case, obtained from a standard Gibbs sampler.

The estimates of the predictive weights are then formed as

$$\widehat{w}(\mathcal{M}_i | \widetilde{\mathbf{y}}_{1,n}, \mathbf{Y}_n^*) = \frac{\widehat{g}(\mathbf{Y}, n, h | \mathcal{M}_i) p(\mathcal{M}_i)}{\sum_{j=1}^{M} \widehat{g}(\mathbf{Y}, n, h | \mathcal{M}_j) p(\mathcal{M}_j)} \tag{9}$$

with

$$\widehat{g}(\mathbf{Y}, n, h | \mathcal{M}_i) = \prod_{t=n}^{T-h_k} \widehat{p}(y_{1,t+h_1}, \ldots, y_{1,t+h_k} | \mathbf{Y}_t^*, \mathcal{M}_i). \tag{10}$$

# 3 Prior Specification

We use a Normal-Diffuse prior on the parameters in the VAR-model (8), i.e. $\text{vec}(\mathbf{\Gamma}) \sim N(\gamma_0, \mathbf{\Sigma}_0)$ and $\pi(\mathbf{\Psi}) \propto |\mathbf{\Psi}|^{-(q+1)/2}$, see Kadiyala and Karlsson (1997) for details and the Gibbs sampler for simulating from the posterior distribution of $\mathbf{\Gamma}$ and $\mathbf{\Psi}$. The prior for $\mathbf{\Gamma}$ is a Litterman type prior. That is, $\gamma_0$ is zero except for elements corresponding to the first own lag of variables. These are set to unity for variables believed to be non-stationary and to 0.9 for stationary variables. $\mathbf{\Sigma}_0$ is a diagonal matrix and the prior standard deviations are given by

$$\frac{\pi_1}{k^{\pi_3}},\ \text{own lags},\ k = 1, \ldots, p$$
$$\frac{s_i \pi_1 \pi_2}{s_j k^{\pi_3}},\ \text{lags of variable } j \text{ in equation } i,\ k = 1, \ldots, p$$
$$\pi_4,\ \text{deterministic variables}$$

where $s_i$ is the residual standard deviation for equation $i$ from the OLS fit of the VAR-model.

The model prior is given by

$$\pi\left(\mathcal{M}_j\right) \propto \prod_{k=1}^{K} \delta_k^{d_k}\left(1-\delta_k\right)^{d_k}$$

where $d_k = 1$ if variable $k$ is included in the model and $\delta_k$ is the prior inclusion probability of variable $k$.

# 4 Monte Carlo Experiment

We use three small Monte Carlo experiments to evaluate the forecasting performance of forecast combinations based on the predictive weights (9). The data generating processes are a bivariate VAR(1),

$$\text{DGP 1: } \mathbf{y}_t = \mathbf{y}_{t-1}\begin{pmatrix} 0.5 & 0.2 \\ 0.5 & 0.5 \end{pmatrix} + \mathbf{u}_t, \tag{11}$$

a bivariate VAR(2),

$$\text{DGP 2: } \mathbf{y}_t = \mathbf{y}_{t-1}\begin{pmatrix} 0.5 & 0.2 \\ 0.5 & 0.5 \end{pmatrix} + \mathbf{y}_{t-2}\begin{pmatrix} 0.1 & 0.1 \\ 0.2 & -0.3 \end{pmatrix} + \mathbf{u}_t, \tag{12}$$

and a trivariate VAR(1),

$$\text{DGP 3: } \mathbf{y}_t = \mathbf{y}_{t-1}\begin{pmatrix} 0.5 & 0.2 & 0.1 \\ 0.5 & 0.5 & 0.1 \\ 0.5 & 0.3 & 0.2 \end{pmatrix} + \mathbf{u}_t. \tag{13}$$

In addition we generate a set of 5 extraneous variables as

$$\begin{aligned} z_{1,t} &= 0.5y_{1,t-1} + 0.5z_{1,t-1} + e_{1,t} \\ z_{2,t} &= 0.5y_{2,t-1} + 0.5z_{2,t-1} + e_{2,t} \\ z_{3,t} &= 0.7z_{3,t-1} + e_{3,t} \\ z_{4,t} &= 0.2z_{4,t-1} + e_{4,t} \\ z_{5,t} &= e_{5,t}. \end{aligned}$$

with $u_{i,t}$, and $e_{i,t}$ iid standard normal random variables. The last, white noise, extraneous variable is dropped with the trivariate VAR-model so that the generated data sets in each Monte Carlo experiment consists of seven variables. For each experiment we generate 100 data sets of length 112 with the last 12 observations set aside for forecast evaluation.

The variable to be forecasted is $y_{1,t}$. For the bivariate DGPs we consider the 42 models arising from modelling $y_{1,t}$ alone or together with combinations of $y_{2,t}$ and $z_{1,t}, \ldots, z_{5,t}$ with a maximum of four variables in the model, for the trivariate DGP we consider the 57 possible models when allowing a maximum of five variables in the model. We use two settings for the lag length of the VAR-models, $p = 2$ and $p = 4$.

We are particularly concerned about the number of observations needed for the hold out sample, for this we consider three cases, $m = 30$, $m = 50$ and $m = 70$, ($m = 70$ is not used in combination with lag length 4 in the estimated models since this would reduce the number of available observations too much) and the effect of the lead time used for the calculation of the predictive weights, here we consider eight alternatives, the single lead times $h = 1$, 2, 3, 4 and 8 and the multiple lead times $h = (1, 2, 3, 4)$, $h = (1, 2, 3, 4, 5, 6, 7, 8)$ and $h = (1, 4, 8)$. We also experiment with two specifications of the model prior, setting $\delta_k = 0.2$ implying a prior expected model size of 2.15 when we allow for four variables in the model and 2.19 when we allow five variables. The other settings $\delta_k = 0.5$, with all models equally likely and prior expected model sizes 3.29 and 3.74. The prior for $\mathbf{\Gamma}$ is specified with $\pi_1 = 0.5$, $\pi_2 = 0.5$, $\pi_3 = 1$ and $\pi_4 = 5.0$.

When conducting the Monte Carlo exercise we simplify the estimation of the predictive likelihoods by not updating the posterior distribution of the parameters as $t$ increases in the product (10), this allow us to perform all the calculations for the predictive weights within a single Gibbs sampler run instead of running one Gibbs sampler for each value of $t$. [1] The predictive likelihoods are estimated based on 5000 draws from Markov chain and the final forecast, $E\left(y_{T+h}\right| \mathbf{Y}, \mathcal{M}_j)$, is estimated from 5000 draws from the Markov chain based on the full sample. To increase the precision of the estimate we use antithetic variates where an antithetic draw of $\mathbf{\Gamma}$, conditional on $\mathbf{\Psi}$, is obtained in each step of the Markov chain.

## 4.1 Results

We will focus on DGP 1, a bivariate VAR(1), when the models are estimated with lag length $p = 2$ when reporting the results. The qualitative results are similar for the other DGPs as well as models estimated with $p = 4$. A comprehensive set of results are available in Appendix B.

Table 1 reports on the posterior variable inclusion probabilities, or more precisely the sum of the predictive weights for the set of models containing the variable. It is clear that the procedure is able to discriminate between the variable $y_2$ which is in the true model and the extraneous variables. The strongest discrimination is achieved when the predictive likelihood is evaluated at $h = 1$. This is not too surprising given that prediction intervals rapidly becomes very wide as the forecast horizon increases with a correspondingly diminishing discriminatory power. Longer lead times might, however, be important for seasonal or cyclical data. This is to some extent indicated by the results for DGP 2 which contains a cycle. Evaluating the predictive likelihood at multiple horizons discriminates almost as well as the single $h = 1$ and can be a useful alternative. Increasing the size of the hold out sample is beneficial for discriminating between the variables although the estimation sample can obviously not be made too small (in particular when the posterior is not updated with new observations and always based on the first $T - m$ observations). As can be

[1] We do a limited check on the effect of not updating the prior by rerunning a few experiments for the first DGP with the prior updated as new observations are added. The results are slightly better when the prior is updated, particularly for $m = 70$, but overall the differences are small.

**Table 1** Posterior variable inclusion probabilites, DGP 1, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 70$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.79 | 0.17 | 4.71 | 0.92 | 0.15 | 6.11 |
| 4 | 0.42 | 0.19 | 2.26 | 0.49 | 0.20 | 2.47 |
| 8 | 0.31 | 0.19 | 1.57 | 0.28 | 0.20 | 1.40 |
| $1-4$ | 0.76 | 0.17 | 4.38 | 0.79 | 0.19 | 4.10 |
| $1-8$ | 0.70 | 0.18 | 3.81 | 0.66 | 0.18 | 3.76 |
| $1,4,8$ | 0.76 | 0.17 | 4.49 | 0.76 | 0.16 | 4.68 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 70$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.88 | 0.31 | 2.79 | 0.96 | 0.28 | 3.48 |
| 4 | 0.60 | 0.36 | 1.67 | 0.63 | 0.32 | 1.96 |
| 8 | 0.49 | 0.37 | 1.32 | 0.40 | 0.32 | 1.27 |
| $1-4$ | 0.85 | 0.30 | 2.89 | 0.84 | 0.26 | 3.25 |
| $1-8$ | 0.78 | 0.28 | 2.80 | 0.71 | 0.22 | 3.27 |
| $1,4,8$ | 0.85 | 0.29 | 2.88 | 0.82 | 0.23 | 3.64 |

expected we also achieve better discrimination with the $\delta_k = 0.2$ model prior which favours small models.

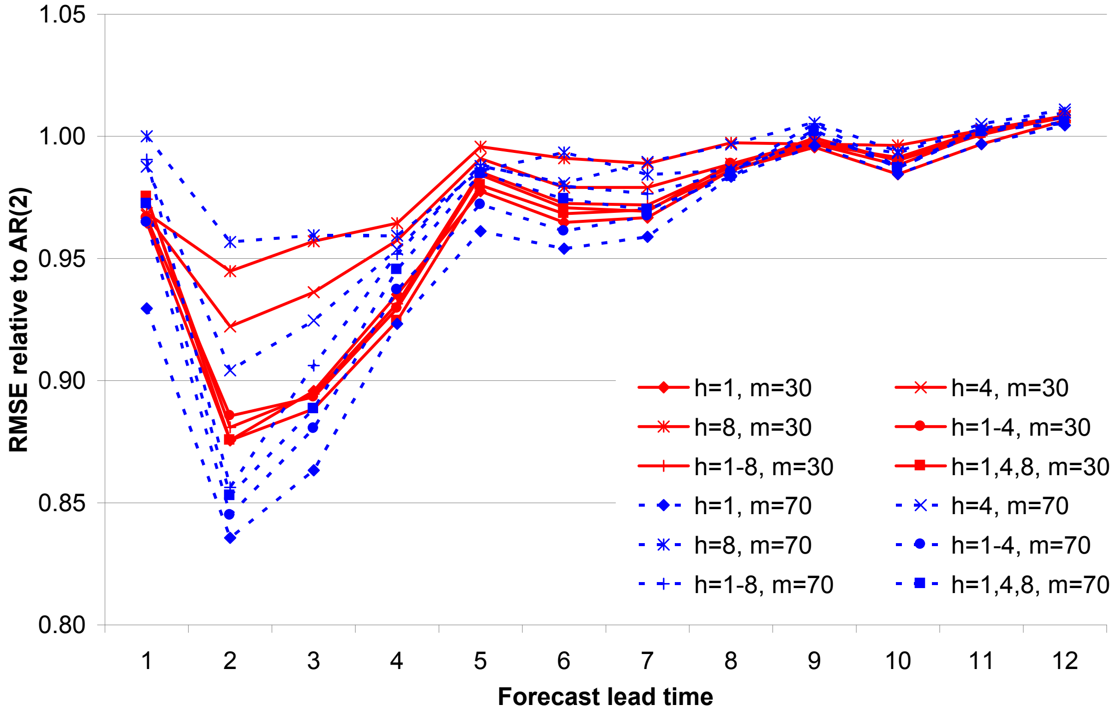
Table 2 summarizes the model selection properties of the predictive likelihood. The posterior "probabilities" for the true model are not particularly large but the performance is reasonable in terms of model selection. With the $\delta_k = 0.2$ model prior the correct model is selected in between 70% and 87% of the Monte Carlo replicates when the predictive likelihood is evaluated at $h = 1$. Performance is, on the other hand, quite poor with the uninformative model prior which favours large models.

Figure 2 summarizes the forecast performance for DGP 1 and models estimated with lag length $p = 2$. The figure compare the root mean square forecast error

**Table 2** Model selection, DGP 1, models estimated with lag length $p = 2$. Average posterior probability and proportion selected for true model.

| | Model prior, $\delta_k = 0.2$ | | | | Model prior, $\delta_k = 0.5$ | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | hold out, $m = 30$ | | hold out, $m = 70$ | | hold out, $m = 30$ | | hold out, $m = 70$ | |
| | Prob | Selected | Prob | Selected | Prob | Selected | Prob | Selected |
| 1 | 0.31 | 0.87 | 0.44 | 0.70 | 0.08 | 0.20 | 0.18 | 0.39 |
| 4 | 0.16 | 0.29 | 0.23 | 0.34 | 0.05 | 0.18 | 0.15 | 0.26 |
| 8 | 0.12 | 0.19 | 0.12 | 0.13 | 0.05 | 0.25 | 0.10 | 0.15 |
| $1-4$ | 0.33 | 0.61 | 0.42 | 0.46 | 0.13 | 0.28 | 0.33 | 0.37 |
| $1-8$ | 0.33 | 0.50 | 0.31 | 0.34 | 0.19 | 0.30 | 0.28 | 0.28 |
| $1,4,8$ | 0.34 | 0.66 | 0.41 | 0.45 | 0.14 | 0.31 | 0.34 | 0.40 |

**Figure 2** RMSE for forecast combination relative to AR(2), DGP 1, $\delta_k = 0.2$




(RMSE) for the forecast combination to that of the forecasts from the model with only $y_{1,t}$, i.e. an AR(2). There is clearly a substantial gain for shorter forecast lead times. The larger hold out sample, $m = 70$, provides the best forecasts together with predictive criteria that puts weight on lead time 1. The difference between the $\delta_k = 0.2$ and $\delta_k = 0.5$ model priors is small for this DGP and models estimated with lag length $p = 4$ gives slightly worse forecasts.

The results for DGP 2 shown in Figure 3 show a larger improvement from the forecast combination at lead time 1 than for DGP 1 but the results are slightly worse than an AR(2) at the longer lead times. Again, the forecasts combinations based on the predictive likelihood evaluated at $h = 4$ and 8 provides the least improvement on an AR(2). Performance is slightly better for the $\delta_k = 0.5$ model prior with smaller differences between combinations based on predictive likelihoods evaluated at different horizons.

With DGP 3 (Figure 4) the forecast combination improves on an AR(2) at all but the longest lead times. The difference between the different forecast combinations is small except for when the predictive likelihood is evaluated at $h = 8$ which performs worse than the other combinations. The difference between model priors is very small, the $\delta_k = 0.5$ prior does slightly better at longer lead times and the $\delta_k = 0.2$ prior does slightly better at short lead times.

Overall it is clear that forecast combination based on the predictive likelihood can improve substantially on the common benchmark of a univariate AR-model. The improvement is larger for short lead times and is also larger for more complex DGPs. The performance is in general better when the predictive likelihood is evaluated at a single short horizon although the use of multiple horizons may be more robust. With a single horizon the use of standard density estimation techniques is uncomplicated

**Figure 3** RMSE for forecast combination relative to AR(2), DGP 2, $\delta_k = 0.2$

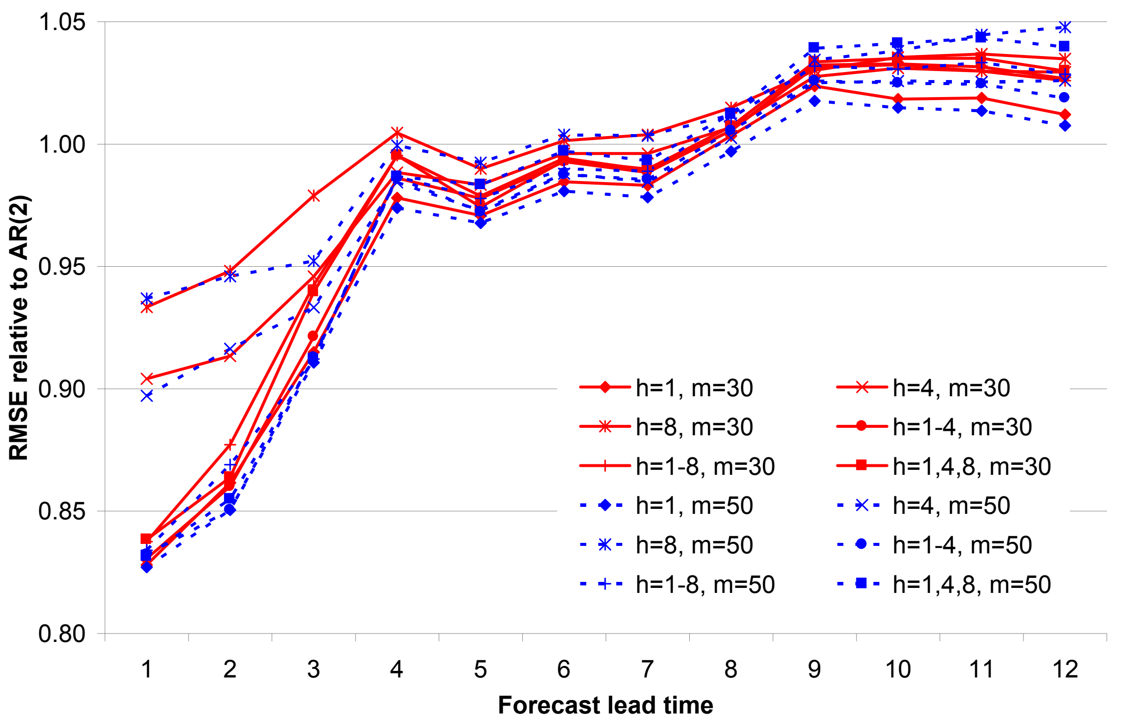



**Figure 4** RMSE for forecast combination relative to AR(2), DGP 3, $\delta_k = 0.2$

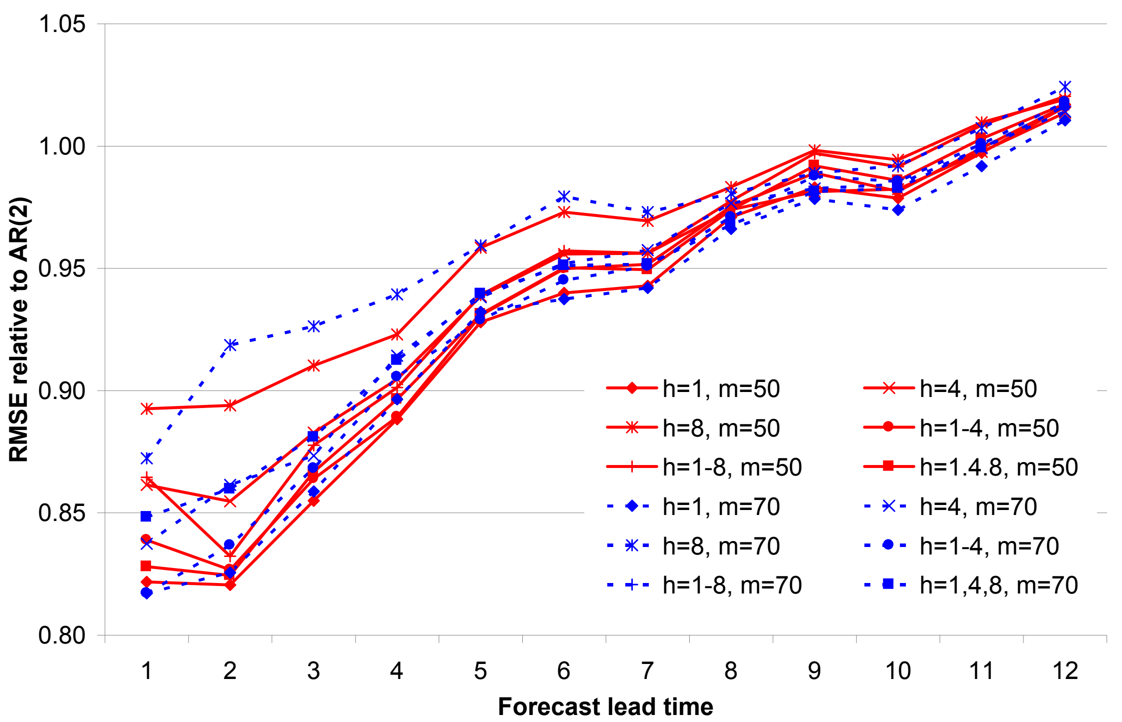
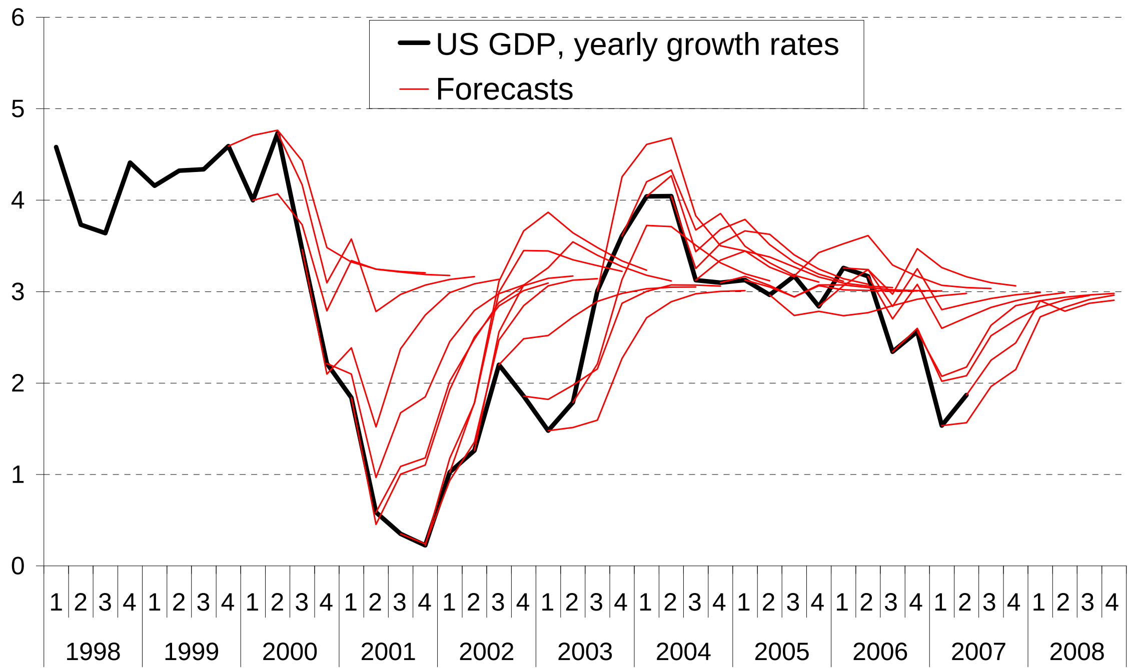


**Figure 5** Sequential forecasts from 1999:1 to 2008:3.




Note: The figure presents the median of the predictive distribution.

and the procedure generalizes readily to situations where the model structure does not allow the use of the Rao-Blackwellization device.

# 5 Forecasting US GDP

This section illustrates the predictive likelihood forecast combination procedure at work. The forecast variable is U.S. gross domestic product (GDP). The VAR models are of dimensions one to four and we use a data set of 20 series (GDP included) ranging from second quarter 1971 to the second quarter 2007. The full list variables can be found in Appendix A. This implies estimation of 1,160 (unique) model combinations. The series are modelled in their first differences or in the levels, but in the presentation the forecasts, as well as the data, are in the fourth log-differences (as an approximation to yearly growth rates).

The prior variable probabilities, $\delta_k$, are all set to 0.2, but we have also tried a value of 0.5 (which is equivalent to a uniform prior over the model space). The final results do not change much when the prior distribution is changed. However, the procedure puts a larger posterior model probability on larger systems when the prior 0.5 is used. The predictive likelihood is computed through 5000 Gibbs samples and 50 evaluation points in time. The final forecasts arises as the mean forecast from 1000 Gibbs samples. The prior specification for the parameters is of the same Litterman type as in the Monte Carlo experiment; we set the first (own) lag mean to zero for difference stationary variables and the first lag mean to 0.9 for stationary series. The overall tightness ($\pi_1$) is 0.2, the cross-equation tightness ($\pi_2$) is 0.5 the lag decay ($\pi_3$) is 1 and the thightness on the constant term ($\pi_4$) is 5.

**Table 3** Forecast accuracy

| Lead | For. comb. | Top mod. | AR(2) | R. Walk | Rec. mean | No Fcsts |
|---|---|---|---|---|---|---|
| 1 | 0.43 | 1.07 | 1.09 | 1.30 | 2.06 | 30 |
| 2 | 0.63 | 1.06 | 1.14 | 1.34 | 1.56 | 29 |
| 3 | 0.93 | 1.03 | 1.14 | 1.24 | 1.30 | 28 |
| 4 | 1.20 | 1.01 | 1.06 | 1.28 | 1.17 | 27 |
| 5 | 1.30 | 0.98 | 1.04 | 1.35 | 1.13 | 26 |
| 6 | 1.32 | 0.98 | 1.04 | 1.42 | 1.09 | 25 |
| 7 | 1.25 | 0.98 | 1.04 | 1.54 | 1.05 | 24 |
| 8 | 1.14 | 0.98 | 1.04 | 1.71 | 0.98 | 23 |
| 9 | 0.99 | 0.99 | 1.03 | 1.97 | 0.91 | 22 |
| 10 | 0.91 | 1.00 | 1.02 | 2.21 | 0.84 | 21 |
| 11 | 0.83 | 1.01 | 1.01 | 2.47 | 0.77 | 20 |
| 12 | 0.83 | 1.01 | 1.00 | 2.52 | 0.72 | 19 |
| | $Stdev\,(GDP)$ | 1.15 | | | | |

RMSE for forecast combination. Ratio of RMSE to RMSE for forecast combination for other procedures.

In order to compare the general forecasting performance of our procedure, we compute (pseudo out-of-sample) root of the mean squared errors (RMSE) for the combination estimator and compare it to the model with the highest model posterior probability (which may be a different model for different forecast occasions). Furthermore, the performance is also compared to a Bayesian second-order autoregressive model, a random walk forecasts and a recent mean construct (based on the last eight quarters of data). The RMSE's, for horizons 1-12, are calculated for forecasts ranging from first quarter 2000 to second quarter 2007. The reported results concern average performance of the procedure (in terms of RMSE:s) but we also present a current situation analysis (in terms of forecasts and posterior probabilities).

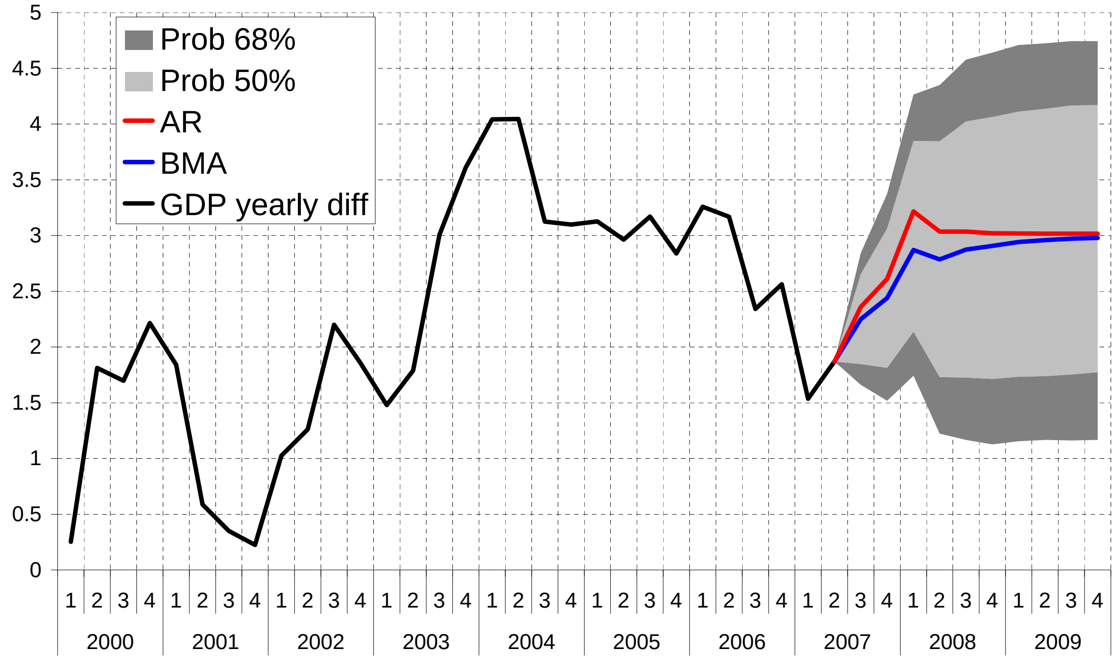
## 5.1 Average Forecasting Performance

Figure 5 presents a cascade plot of forecasts (one to eight steps ahead) from different points in time. This picture reveals how well the forecasts track the development of GDP growth (if we neglect data revisions which may be sizeable). For example, the first BMA forecast is constructed with data up to the last quarter 1999. From the forecast cascade it is demonstrated that the BMA procedure underestimated the weakness of the economy during 2001, but predicts the period 2002 to 2005 reasonably well. The forecasts did not quite catch the down turn in the recent past and GDP growth is somewhat overpredicted, but not to the same degree as in 2001.

Turning to a more formal evaluation of the forecasts, Table 3 shows that the forecast combination improves on the top model and especially the AR(2) for shorter lead times but does slightly worse than the top model for lead times 5 and higher.[2]

[2]The ability of autoregressions to compare well with more sophisticated approaches is a familiar phenomenon. See, for example, Stock and Watson (2002*a*) and Stock and Watson (2004).

**Figure 6** Forecast from 2007:2. Posterior mean and probability intervals for forecast combination and mean forecast from a Bayesian AR(2).




Due to the small evaluation sample no formal testing is performed. This improvement is somewhat more articulated when we use the uniform prior for the models, $\delta_k = 0.5$. The two simplest alternative forecasts, namely the random walk and the recent mean forecasts, perform notably worse in the short run than the other forecasts. However, for the longest horizons, where the model forecast capacity is consumed, the recent mean construct produces the overall best forecasts. This indicate that the sample mean (or process steady state) may be a preferred forecast in the long run (i.e. when the dynamics of the model is used up).

The size of the RMSE of the forecast combination for lead times $h = 4$ and higher is approximately the same as the standard deviation of the GDP series. Our procedure can thus be regarded as a complement to traditional forecasts for short horizons. This is in line with previous studies, see for instance Galbraith and Tkacz (2006).

## 5.2 Contemporaneous Forecasts from the Procedure

Figure 6 presents the posterior mean of the combination forecasts given data up to second quarter 2007. The forecast cover the period 2007:3 to 2009:4. The figure also presents the associated 50 and 68 per cent probability intervals for the forecast combination and the forecasts from a Bayesian autoregression. The intervals demonstrate that there is considerable forecast uncertainty. The combination forecast suggests that the US economy will slowly approach the potential growth rate. The autoregressive forecast only considers the dynamics contained in GDP itself, whereas, the combination procedure also takes the other nineteen variables into account. Figure 6 demonstrate that the information contained in the indicator variables leads to a lower forecast for the whole forecast period compared to not

**Figure 7** Posterior Variable Inclusion Probabilities.

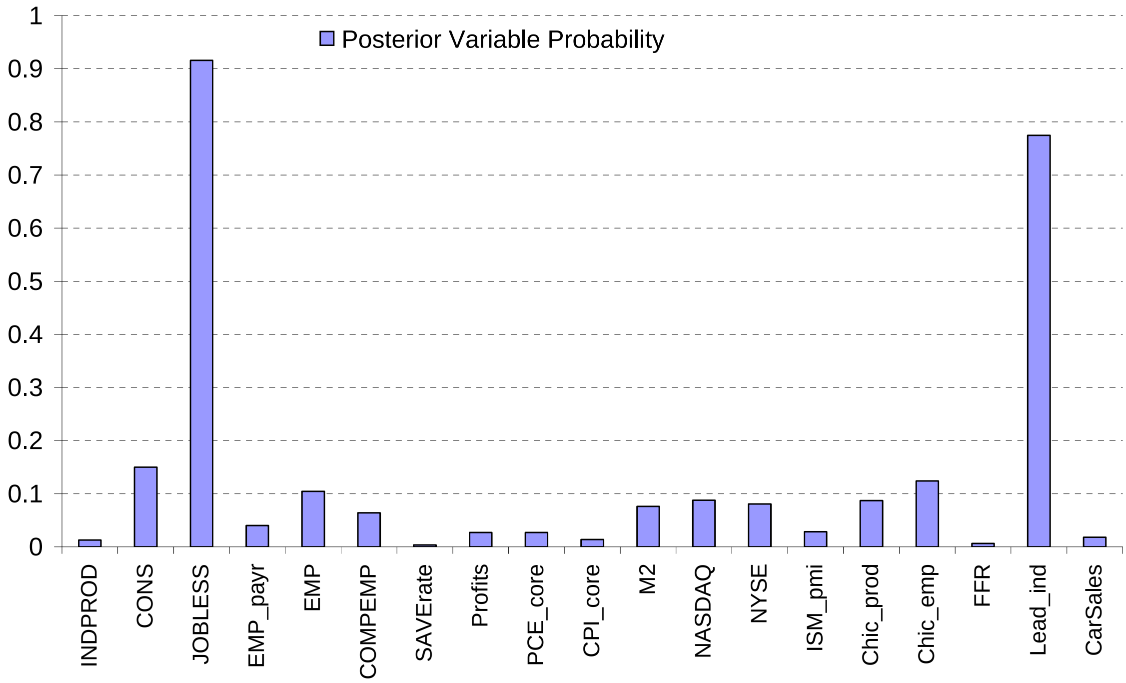



**Table 4** Top 10 Models

| Rank | Variables | | | | Post.prob. |
|---|---|---|---|---|---|
| 1 | GDP | JOBLESS | Lead_ind | | 0.173 |
| 2 | GDP | JOBLESS | Chic_emp | Lead_ind | 0.098 |
| 3 | GDP | CONS | JOBLESS | Lead_ind | 0.074 |
| 4 | GDP | JOBLESS | EMP | Lead_ind | 0.068 |
| 5 | GDP | JOBLESS | Chic_prod | Lead_ind | 0.047 |
| 6 | GDP | JOBLESS | COMPEMP | Lead_ind | 0.042 |
| 7 | GDP | JOBLESS | NYSE | Lead_ind | 0.035 |
| 8 | GDP | JOBLESS | M2 | Lead_ind | 0.035 |
| 9 | GDP | JOBLESS | NASDAQ | Lead_ind | 0.034 |
| 10 | GDP | JOBLESS | EMP_payr | Lead_ind | 0.028 |
| Equal weights for all specifications/models | | | | | 0.0009 |

The table presents the top ten models based on data from 1971:2 to 2007:2. The column Post. prob. reports the posterior probability of each model.

using the indicator information. Thus, the indicators contain a signal of a weaker growth than the GDP series by itself.

Figure 7 presents the posterior probabilities for each variable, based on the present full data-set. This information may be useful by itself, e.g., this information may be incorporated in judgementally based forecasting schemes. The highest variable inclusion probabilities are found for the jobless claims (JOBLESS) and the Conference Board leading indicator index (Lead_ind). The other real variables exhibit notably lower posterior probabilities, and the nominal variables even lower probabilities.

Table 4 presents posterior analysis for the top ten models, using the current data set. As a point of reference the table also gives the "posterior probability", 1/1160, for an equal weighting scheme. Given the variable posterior probabilities it is not a surprise that the top ranked model consists of GDP, jobless claims and the Conference Board leading indicator. Furthermore, the jobless claims and Conference Board leading indicator variables are found in all top ten specifications and appears to be important GDP predictors for the moment.

# 6 Conclusions

This paper proposes to use weights based on the predictive likelihood for combining forecasts from dynamic multivariate forecasting models such as VAR-models. Our approach overcomes a basic difficulty with standard Bayesian forecast combination based on the marginal with multivariate forecasting models, that the marginal likelihood can change with the dimension of the model in ways that are unrelated to the forecasting performance for the variable of interest. This is achieved by considering the marginal predictive likelihood for the variable of interest rather than the joint predictive likelihood which suffers from the same problem.

The predictive likelihood is not available in closed form for forecasts at lead times greater than 1 and we propose simulation strategies for estimating the predictive likelihood. Our approach is completely general and does not rely on natural conjugate priors or the availability of closed form solutions for the posterior quantities. All that is required is the ability to simulate from the posterior distribution of the parameters and to simulate one step ahead forecasts. The approach is thus also well suited for non-linear forecasting models.

We evaluate the performance of the forecast combination procedure in a small Monte Carlo study and in an application to forecasting US GDP growth. Overall the forecast combinations perform very well. In the Monte Carlo study the forecast combination outperforms our benchmark autoregression by as much as 15% and improves on the AR model for all but the longest lead time in the application to US GDP.

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# Appendices

## A Data used for the US GDP forecasts

The data set consists of real, nominal and indicator type variables:

- GDP: National Income Account, Overall, Total, Constant Prices, SA (US Dept. of Commerce)
- INDPROD: Production, Overall, Total, SA (Federal Reserve)
- CONS: Personal Outlays, Overall, Total, Constant Prices, SA (US Dept. of Commerce)
- JOBLESS: Jobless claims, SA (US Dept. of Labor)
- EMP_payr: Employment, Overall, Nonfarm Payroll, Total, SA (Bureau of Labor Statistics)
- EMP: Civilian Employment, Business Cycles Indicators, SA (The Conference Board)
- COMPEMP: National Income Account, Compensation of Employees, Total, SA (The US Dept. of Commerce)
- SAVErate: Personal Savings, Rate, SA (Federal Reserve)
- Profits: National Income Account, Corporate Profits, with IVA and CCAdj, Total, SA (The US Dept. of Commerce)
- PCE_core: Price Index, PCE, Overall, Personal Consumption Expenditures less Food and Energy, SA (Bureau of Economic Analysis)
- CPI_core: Consumer Prices, All Items less Food and Energy, SA (Bureau of Labor Statistics)
- M2: Money Supply M2, SA (Federal Board of Governors)
- NASDAQ: Composite Index, Close (NASDAQ)
- NYSE: Composite Index, Close (NYSE)
- ISM_pmi: Business Surveys, ISM Manufacturing, PMI Total, SA (Institute for Supply Management)
- Chic_prod: Business Surveys, Chicago PMI, Production, SA (PMAC)
- Chic_emp: Business Surveys, Chicago PMI, Employment, SA (PMAC)
- FFR: Policy Rates, Fed Funds Effective Rate (Federal Reserve)
- Lead_ind: Leading Index, Total, SA (The Conference Board)
- CarSales:

- Car Sales, Domestic, SA (The US Dept. of Commerce)
- Car Sales, Imported, SA (The US Dept. of Commerce)
- Truck Sales, Domestic Light, SA (The US Dept. of Commerce)
- Truck Sales, Imported Light, SA (The US Dept. of Commerce)

# B Monte Carlo Experiments

## B.1 DGP 1

The DGP is

$$\mathbf{y}_t = \mathbf{y}_{t-1} \begin{pmatrix} 0.5 & 0.2 \\ 0.5 & 0.5 \end{pmatrix} + \mathbf{u}_t,$$

and the irrelevant variables are generated as

$$\begin{aligned} z_{1,t} &= 0.5y_{1,t-1} + 0.5z_{1,t-1} + e_{1,t} \\ z_{2,t} &= 0.5y_{2,t-1} + 0.5z_{2,t-1} + e_{2,t} \\ z_{3,t} &= 0.7z_{3,t-1} + e_{3,t} \\ z_{4,t} &= 0.2z_{4,t-1} + e_{4,t} \\ z_{5,t} &= e_t \end{aligned}$$

with $u_{i,t}$ and $e_{i,t}$ iid $N(0,1)$. $T = 100$ (not accounting for lag lengths) and an additional 12 observations are set aside for forecast evaluation. Model averaging and model selection over the 42 possible models with up to four variables. $y_1$ is always included in the model. The results are based on 100 Monte Carlo replicates.

**Table B1** Posterior variable inclusion probabilities, DGP 1, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | | hold out sample, $m = 70$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.79 | 0.17 | 4.71 | 0.90 | 0.17 | 5.41 | 0.92 | 0.15 | 6.11 |
| 2 | 0.67 | 0.18 | 3.68 | 0.75 | 0.19 | 4.06 | 0.83 | 0.19 | 4.28 |
| 3 | 0.51 | 0.18 | 2.78 | 0.60 | 0.19 | 3.13 | 0.65 | 0.20 | 3.30 |
| 4 | 0.42 | 0.19 | 2.26 | 0.48 | 0.20 | 2.40 | 0.49 | 0.20 | 2.47 |
| 8 | 0.31 | 0.19 | 1.57 | 0.32 | 0.18 | 1.77 | 0.28 | 0.20 | 1.40 |
| $1-4$ | 0.76 | 0.17 | 4.38 | 0.83 | 0.19 | 4.27 | 0.79 | 0.19 | 4.10 |
| $1-8$ | 0.70 | 0.18 | 3.81 | 0.76 | 0.19 | 4.00 | 0.66 | 0.18 | 3.76 |
| $1,4,8$ | 0.76 | 0.17 | 4.49 | 0.83 | 0.16 | 5.25 | 0.76 | 0.16 | 4.68 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | | hold out sample, $m = 70$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.88 | 0.31 | 2.79 | 0.96 | 0.31 | 3.09 | 0.96 | 0.28 | 3.48 |
| 2 | 0.80 | 0.33 | 2.43 | 0.86 | 0.32 | 2.64 | 0.89 | 0.32 | 2.74 |
| 3 | 0.68 | 0.35 | 1.96 | 0.74 | 0.34 | 2.17 | 0.76 | 0.31 | 2.45 |
| 4 | 0.60 | 0.36 | 1.67 | 0.66 | 0.35 | 1.88 | 0.63 | 0.32 | 1.96 |
| 8 | 0.49 | 0.37 | 1.32 | 0.49 | 0.35 | 1.38 | 0.40 | 0.32 | 1.27 |
| $1-4$ | 0.85 | 0.30 | 2.89 | 0.88 | 0.29 | 3.03 | 0.84 | 0.26 | 3.25 |
| $1-8$ | 0.78 | 0.28 | 2.80 | 0.82 | 0.26 | 3.15 | 0.71 | 0.22 | 3.27 |
| $1,4,8$ | 0.85 | 0.29 | 2.88 | 0.89 | 0.27 | 3.33 | 0.82 | 0.23 | 3.64 |

**Table B2** Posterior variable inclusion probabilities, DGP 1, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.77 | 0.17 | 4.49 | 0.88 | 0.17 | 5.31 |
| 2 | 0.65 | 0.19 | 3.41 | 0.73 | 0.19 | 3.93 |
| 3 | 0.47 | 0.19 | 2.44 | 0.55 | 0.19 | 2.86 |
| 4 | 0.38 | 0.20 | 1.91 | 0.41 | 0.19 | 2.15 |
| 8 | 0.28 | 0.19 | 1.48 | 0.27 | 0.18 | 1.51 |
| $1-4$ | 0.75 | 0.19 | 3.90 | 0.79 | 0.20 | 3.92 |
| $1-8$ | 0.66 | 0.19 | 3.51 | 0.68 | 0.19 | 3.57 |
| $1,4,8$ | 0.72 | 0.17 | 4.23 | 0.77 | 0.15 | 5.00 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.88 | 0.32 | 2.71 | 0.94 | 0.31 | 3.07 |
| 2 | 0.79 | 0.35 | 2.26 | 0.83 | 0.35 | 2.40 |
| 3 | 0.65 | 0.37 | 1.76 | 0.71 | 0.36 | 1.97 |
| 4 | 0.57 | 0.37 | 1.52 | 0.59 | 0.36 | 1.67 |
| 8 | 0.48 | 0.37 | 1.30 | 0.45 | 0.34 | 1.33 |
| $1-4$ | 0.85 | 0.31 | 2.77 | 0.86 | 0.29 | 2.92 |
| $1-8$ | 0.76 | 0.28 | 2.70 | 0.75 | 0.26 | 2.92 |
| $1,4,8$ | 0.83 | 0.31 | 2.72 | 0.83 | 0.26 | 3.21 |

**Table B3** Posterior variable inclusion probabilities, DGP 1, models estimated with lag length $p = 2$ and updated posterior distributions

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 70$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.78 | 0.17 | 4.60 | 0.97 | 0.14 | 6.84 |
| 4 | 0.39 | 0.18 | 2.16 | 0.56 | 0.19 | 3.00 |
| $1-4$ | 0.77 | 0.17 | 4.39 | 0.92 | 0.15 | 6.35 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 70$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.88 | 0.32 | 2.76 | 0.99 | 0.28 | 3.54 |
| 4 | 0.58 | 0.36 | 1.60 | 0.72 | 0.36 | 2.00 |
| $1-4$ | 0.86 | 0.31 | 2.80 | 0.96 | 0.25 | 3.78 |

**Table B4** Model selection, DGP 1, models estimated with lag length $p = 2$. Average posterior probability and proportion selected for true model.

Model prior, $\delta_k = 0.2$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | | hold out, $m = 70$ | |
|---|---|---|---|---|---|---|
| | Prob | Selected | Prob | Selected | Prob | Selected |
| 1 | 0.31 | 0.87 | 0.37 | 0.78 | 0.44 | 0.70 |
| 2 | 0.26 | 0.69 | 0.31 | 0.68 | 0.38 | 0.61 |
| 3 | 0.19 | 0.46 | 0.24 | 0.51 | 0.30 | 0.45 |
| 4 | 0.16 | 0.29 | 0.19 | 0.34 | 0.23 | 0.34 |
| 8 | 0.12 | 0.19 | 0.15 | 0.19 | 0.12 | 0.13 |
| $1-4$ | 0.33 | 0.61 | 0.40 | 0.59 | 0.42 | 0.46 |
| $1-8$ | 0.33 | 0.50 | 0.38 | 0.46 | 0.31 | 0.34 |
| $1,4,8$ | 0.34 | 0.66 | 0.42 | 0.60 | 0.41 | 0.45 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | | hold out, $m = 70$ | |
|---|---|---|---|---|---|---|
| | Prob | Selected | Prob | Selected | Prob | Selected |
| 1 | 0.08 | 0.20 | 0.10 | 0.20 | 0.18 | 0.39 |
| 2 | 0.07 | 0.18 | 0.09 | 0.25 | 0.17 | 0.34 |
| 3 | 0.06 | 0.17 | 0.08 | 0.21 | 0.15 | 0.32 |
| 4 | 0.05 | 0.18 | 0.08 | 0.18 | 0.15 | 0.26 |
| 8 | 0.05 | 0.25 | 0.08 | 0.19 | 0.10 | 0.15 |
| $1-4$ | 0.13 | 0.28 | 0.22 | 0.32 | 0.33 | 0.37 |
| $1-8$ | 0.19 | 0.30 | 0.29 | 0.35 | 0.28 | 0.28 |
| $1,4,8$ | 0.14 | 0.31 | 0.24 | 0.38 | 0.34 | 0.40 |

Model prior, $\delta_k = 0.2$, updated posterior distributions

| $h$ | hold out, $m = 30$ | | | | hold out, $m = 70$ | |
|---|---|---|---|---|---|---|
| | Prob | Selected | | | Prob | Selected |
| 1 | 0.31 | 0.86 | | | 0.47 | 0.90 |
| 4 | 0.15 | 0.26 | | | 0.24 | 0.48 |
| $1-4$ | 0.33 | 0.61 | | | 0.49 | 0.72 |

Model prior, $\delta_k = 0.5$, updated posterior distributions

| $h$ | hold out, $m = 30$ | | | | hold out, $m = 70$ | |
|---|---|---|---|---|---|---|
| | Prob | Selected | | | Prob | Selected |
| 1 | 0.07 | 0.15 | | | 0.14 | 0.38 |
| 4 | 0.05 | 0.23 | | | 0.09 | 0.24 |
| $1-4$ | 0.11 | 0.29 | | | 0.26 | 0.41 |

**Table B5** Model selection, DGP 1, models estimated with lag length $p = 4$. Average posterior probability and proportion selected for true model.

Model prior, $\delta_k = 0.2$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.31 | 0.82 | 0.37 | 0.77 |
| 2 | 0.24 | 0.60 | 0.30 | 0.61 |
| 3 | 0.18 | 0.39 | 0.23 | 0.42 |
| 4 | 0.15 | 0.22 | 0.17 | 0.28 |
| 8 | 0.11 | 0.13 | 0.14 | 0.15 |
| $1-4$ | 0.30 | 0.53 | 0.38 | 0.50 |
| $1-8$ | 0.31 | 0.47 | 0.34 | 0.37 |
| $1,4,8$ | 0.32 | 0.65 | 0.39 | 0.57 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.08 | 0.17 | 0.11 | 0.26 |
| 2 | 0.07 | 0.12 | 0.09 | 0.22 |
| 3 | 0.06 | 0.11 | 0.08 | 0.18 |
| 4 | 0.05 | 0.14 | 0.07 | 0.21 |
| 8 | 0.05 | 0.22 | 0.08 | 0.24 |
| $1-4$ | 0.12 | 0.22 | 0.23 | 0.36 |
| $1-8$ | 0.19 | 0.28 | 0.29 | 0.36 |
| $1,4,8$ | 0.13 | 0.27 | 0.24 | 0.33 |

**Table B6** Forecast performance, RMSE relative to univariate AR(2), hold out sample $m = 30$, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Posterior distribution not updated | | | | | | | | Updated posterior | | |
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ | 1 | 4 | $1-4$ |
| 1 | 0.965 | 0.970 | 0.966 | 0.968 | 0.969 | 0.967 | 0.974 | 0.975 | 0.960 | 0.971 | 0.966 |
| 2 | 0.876 | 0.890 | 0.902 | 0.922 | 0.945 | 0.886 | 0.881 | 0.876 | 0.871 | 0.925 | 0.874 |
| 3 | 0.896 | 0.907 | 0.929 | 0.936 | 0.957 | 0.893 | 0.895 | 0.888 | 0.894 | 0.941 | 0.892 |
| 4 | 0.935 | 0.941 | 0.952 | 0.957 | 0.964 | 0.930 | 0.931 | 0.924 | 0.935 | 0.962 | 0.930 |
| 5 | 0.978 | 0.983 | 0.988 | 0.991 | 0.996 | 0.984 | 0.985 | 0.980 | 0.977 | 0.989 | 0.979 |
| 6 | 0.965 | 0.972 | 0.976 | 0.979 | 0.991 | 0.971 | 0.973 | 0.968 | 0.964 | 0.978 | 0.968 |
| 7 | 0.967 | 0.970 | 0.973 | 0.979 | 0.989 | 0.969 | 0.972 | 0.970 | 0.965 | 0.978 | 0.966 |
| 8 | 0.986 | 0.987 | 0.988 | 0.989 | 0.997 | 0.989 | 0.987 | 0.986 | 0.986 | 0.987 | 0.988 |
| 9 | 0.996 | 0.996 | 0.996 | 0.997 | 0.997 | 0.998 | 0.999 | 0.997 | 0.996 | 0.996 | 0.999 |
| 10 | 0.985 | 0.986 | 0.986 | 0.991 | 0.996 | 0.990 | 0.990 | 0.988 | 0.984 | 0.989 | 0.987 |
| 11 | 0.997 | 0.998 | 0.999 | 1.003 | 1.003 | 1.001 | 1.002 | 1.001 | 0.996 | 1.001 | 0.999 |
| 12 | 1.006 | 1.008 | 1.008 | 1.008 | 1.008 | 1.008 | 1.008 | 1.008 | 1.006 | 1.007 | 1.009 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Posterior distribution not updated | | | | | | | | Updated posterior | | |
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ | 1 | 4 | $1-4$ |
| 1 | 0.961 | 0.962 | 0.959 | 0.957 | 0.953 | 0.961 | 0.969 | 0.967 | 0.956 | 0.959 | 0.959 |
| 2 | 0.869 | 0.873 | 0.882 | 0.897 | 0.911 | 0.873 | 0.870 | 0.864 | 0.861 | 0.896 | 0.858 |
| 3 | 0.887 | 0.893 | 0.907 | 0.912 | 0.931 | 0.884 | 0.887 | 0.879 | 0.883 | 0.915 | 0.880 |
| 4 | 0.926 | 0.929 | 0.936 | 0.940 | 0.947 | 0.921 | 0.925 | 0.917 | 0.924 | 0.945 | 0.920 |
| 5 | 0.973 | 0.978 | 0.982 | 0.985 | 0.988 | 0.976 | 0.980 | 0.975 | 0.970 | 0.985 | 0.973 |
| 6 | 0.959 | 0.965 | 0.969 | 0.971 | 0.981 | 0.963 | 0.967 | 0.962 | 0.958 | 0.970 | 0.961 |
| 7 | 0.963 | 0.965 | 0.967 | 0.971 | 0.980 | 0.964 | 0.967 | 0.963 | 0.961 | 0.970 | 0.961 |
| 8 | 0.985 | 0.986 | 0.986 | 0.987 | 0.995 | 0.987 | 0.985 | 0.984 | 0.985 | 0.987 | 0.988 |
| 9 | 0.996 | 0.997 | 0.997 | 0.998 | 0.998 | 0.999 | 0.999 | 0.997 | 0.997 | 0.999 | 1.001 |
| 10 | 0.985 | 0.986 | 0.985 | 0.991 | 0.994 | 0.987 | 0.987 | 0.986 | 0.985 | 0.990 | 0.986 |
| 11 | 0.998 | 0.999 | 0.999 | 1.003 | 1.003 | 0.999 | 0.999 | 0.999 | 0.998 | 1.003 | 0.999 |
| 12 | 1.008 | 1.009 | 1.008 | 1.010 | 1.010 | 1.009 | 1.007 | 1.007 | 1.007 | 1.009 | 1.009 |

**Table B7** Forecast performance, RMSE relative to univariate AR(2), hold out sample $m = 50$, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.936 | 0.954 | 0.969 | 0.978 | 0.982 | 0.961 | 0.993 | 0.987 |
| 2 | 0.840 | 0.860 | 0.886 | 0.912 | 0.954 | 0.864 | 0.881 | 0.865 |
| 3 | 0.867 | 0.884 | 0.908 | 0.926 | 0.958 | 0.888 | 0.902 | 0.887 |
| 4 | 0.920 | 0.926 | 0.940 | 0.956 | 0.973 | 0.923 | 0.940 | 0.935 |
| 5 | 0.965 | 0.971 | 0.980 | 0.992 | 0.992 | 0.975 | 0.984 | 0.977 |
| 6 | 0.953 | 0.962 | 0.971 | 0.984 | 0.992 | 0.973 | 0.980 | 0.973 |
| 7 | 0.959 | 0.964 | 0.971 | 0.986 | 0.993 | 0.978 | 0.987 | 0.979 |
| 8 | 0.983 | 0.983 | 0.987 | 0.995 | 0.996 | 0.991 | 0.992 | 0.989 |
| 9 | 0.994 | 0.996 | 0.997 | 1.003 | 1.002 | 1.002 | 1.001 | 1.004 |
| 10 | 0.982 | 0.983 | 0.984 | 0.992 | 0.996 | 0.985 | 0.989 | 0.990 |
| 11 | 0.996 | 0.997 | 0.997 | 1.003 | 1.005 | 1.001 | 1.001 | 1.005 |
| 12 | 1.005 | 1.005 | 1.003 | 1.006 | 1.008 | 1.004 | 1.004 | 1.006 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.941 | 0.949 | 0.957 | 0.968 | 0.971 | 0.960 | 0.989 | 0.980 |
| 2 | 0.844 | 0.856 | 0.870 | 0.888 | 0.923 | 0.864 | 0.875 | 0.863 |
| 3 | 0.865 | 0.874 | 0.893 | 0.908 | 0.937 | 0.882 | 0.895 | 0.879 |
| 4 | 0.913 | 0.916 | 0.929 | 0.942 | 0.958 | 0.913 | 0.931 | 0.925 |
| 5 | 0.965 | 0.968 | 0.974 | 0.987 | 0.985 | 0.970 | 0.979 | 0.975 |
| 6 | 0.952 | 0.958 | 0.963 | 0.977 | 0.983 | 0.968 | 0.974 | 0.969 |
| 7 | 0.957 | 0.961 | 0.966 | 0.980 | 0.987 | 0.974 | 0.980 | 0.975 |
| 8 | 0.982 | 0.982 | 0.986 | 0.994 | 0.992 | 0.989 | 0.987 | 0.987 |
| 9 | 0.993 | 0.996 | 0.997 | 1.003 | 1.002 | 1.001 | 0.998 | 1.001 |
| 10 | 0.981 | 0.983 | 0.985 | 0.991 | 0.993 | 0.984 | 0.986 | 0.989 |
| 11 | 0.995 | 0.997 | 0.997 | 1.003 | 1.005 | 1.000 | 0.998 | 1.003 |
| 12 | 1.005 | 1.005 | 1.005 | 1.007 | 1.009 | 1.005 | 1.003 | 1.006 |

**Table B8** Forecast performance, RMSE relative to univariate AR(2), hold out sample $m = 70$, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Posterior distribution not updated | | | | | | | | Updated posterior | | |
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ | 1 | 4 | $1-4$ |
| 1 | 0.930 | 0.966 | 0.963 | 0.988 | 1.000 | 0.965 | 0.990 | 0.972 | 0.943 | 0.987 | 0.955 |
| 2 | 0.836 | 0.845 | 0.860 | 0.904 | 0.957 | 0.845 | 0.856 | 0.853 | 0.836 | 0.913 | 0.836 |
| 3 | 0.863 | 0.874 | 0.900 | 0.925 | 0.960 | 0.880 | 0.906 | 0.888 | 0.866 | 0.923 | 0.866 |
| 4 | 0.923 | 0.923 | 0.939 | 0.954 | 0.959 | 0.937 | 0.952 | 0.945 | 0.920 | 0.949 | 0.919 |
| 5 | 0.961 | 0.965 | 0.972 | 0.988 | 0.986 | 0.972 | 0.988 | 0.985 | 0.965 | 0.982 | 0.964 |
| 6 | 0.954 | 0.953 | 0.965 | 0.981 | 0.993 | 0.961 | 0.980 | 0.974 | 0.953 | 0.975 | 0.952 |
| 7 | 0.959 | 0.956 | 0.966 | 0.990 | 0.984 | 0.967 | 0.977 | 0.970 | 0.958 | 0.982 | 0.961 |
| 8 | 0.983 | 0.983 | 0.990 | 0.997 | 0.986 | 0.985 | 0.985 | 0.983 | 0.982 | 0.989 | 0.982 |
| 9 | 0.996 | 0.999 | 1.004 | 1.006 | 0.998 | 1.003 | 1.005 | 1.002 | 0.994 | 1.000 | 0.999 |
| 10 | 0.985 | 0.986 | 0.990 | 0.994 | 0.993 | 0.987 | 0.988 | 0.987 | 0.981 | 0.989 | 0.984 |
| 11 | 0.997 | 1.001 | 1.004 | 1.005 | 1.003 | 1.003 | 1.003 | 1.002 | 0.994 | 1.002 | 0.997 |
| 12 | 1.005 | 1.005 | 1.008 | 1.011 | 1.010 | 1.005 | 1.008 | 1.006 | 1.003 | 1.008 | 1.004 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Posterior distribution not updated | | | | | | | | Updated posterior | | |
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ | 1 | 4 | $1-4$ |
| 1 | 0.935 | 0.970 | 0.964 | 0.980 | 0.985 | 0.961 | 0.986 | 0.960 | 0.951 | 0.971 | 0.960 |
| 2 | 0.839 | 0.846 | 0.852 | 0.878 | 0.929 | 0.836 | 0.848 | 0.843 | 0.841 | 0.882 | 0.837 |
| 3 | 0.865 | 0.871 | 0.887 | 0.905 | 0.937 | 0.870 | 0.899 | 0.880 | 0.866 | 0.898 | 0.862 |
| 4 | 0.920 | 0.920 | 0.932 | 0.942 | 0.946 | 0.930 | 0.947 | 0.937 | 0.915 | 0.937 | 0.911 |
| 5 | 0.962 | 0.964 | 0.971 | 0.985 | 0.983 | 0.967 | 0.984 | 0.979 | 0.962 | 0.979 | 0.961 |
| 6 | 0.955 | 0.951 | 0.960 | 0.975 | 0.986 | 0.955 | 0.973 | 0.968 | 0.952 | 0.968 | 0.950 |
| 7 | 0.960 | 0.957 | 0.965 | 0.985 | 0.976 | 0.961 | 0.973 | 0.967 | 0.957 | 0.977 | 0.959 |
| 8 | 0.985 | 0.984 | 0.989 | 0.995 | 0.982 | 0.983 | 0.984 | 0.983 | 0.982 | 0.991 | 0.983 |
| 9 | 0.997 | 1.002 | 1.005 | 1.007 | 0.997 | 1.003 | 1.004 | 1.003 | 0.994 | 1.003 | 0.999 |
| 10 | 0.985 | 0.988 | 0.993 | 0.996 | 0.992 | 0.988 | 0.987 | 0.987 | 0.981 | 0.991 | 0.982 |
| 11 | 0.997 | 1.003 | 1.006 | 1.005 | 1.004 | 1.003 | 1.001 | 1.001 | 0.995 | 1.004 | 0.997 |
| 12 | 1.006 | 1.006 | 1.009 | 1.011 | 1.011 | 1.006 | 1.006 | 1.005 | 1.003 | 1.010 | 1.005 |

**Table B9** Forecast performance, RMSE relative to univariate AR(2), hold out sample $m = 30$, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.978 | 0.980 | 0.979 | 0.981 | 0.970 | 0.976 | 0.978 | 0.986 |
| 2 | 0.886 | 0.908 | 0.920 | 0.939 | 0.947 | 0.904 | 0.912 | 0.900 |
| 3 | 0.914 | 0.925 | 0.941 | 0.948 | 0.968 | 0.916 | 0.913 | 0.908 |
| 4 | 0.938 | 0.945 | 0.956 | 0.959 | 0.970 | 0.940 | 0.942 | 0.930 |
| 5 | 0.979 | 0.986 | 0.991 | 0.993 | 0.996 | 0.990 | 0.985 | 0.981 |
| 6 | 0.962 | 0.970 | 0.974 | 0.978 | 0.989 | 0.971 | 0.966 | 0.962 |
| 7 | 0.977 | 0.979 | 0.981 | 0.986 | 0.992 | 0.981 | 0.974 | 0.973 |
| 8 | 0.995 | 0.993 | 0.995 | 0.995 | 1.003 | 0.999 | 0.993 | 0.992 |
| 9 | 1.019 | 1.015 | 1.014 | 1.012 | 1.008 | 1.017 | 1.016 | 1.018 |
| 10 | 0.994 | 0.996 | 0.998 | 1.001 | 1.008 | 1.002 | 1.003 | 0.999 |
| 11 | 1.009 | 1.009 | 1.012 | 1.016 | 1.015 | 1.010 | 1.011 | 1.013 |
| 12 | 1.021 | 1.021 | 1.021 | 1.024 | 1.021 | 1.019 | 1.016 | 1.020 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.972 | 0.971 | 0.975 | 0.970 | 0.959 | 0.974 | 0.975 | 0.981 |
| 2 | 0.886 | 0.896 | 0.910 | 0.919 | 0.920 | 0.897 | 0.904 | 0.894 |
| 3 | 0.907 | 0.914 | 0.925 | 0.926 | 0.945 | 0.907 | 0.903 | 0.900 |
| 4 | 0.931 | 0.935 | 0.942 | 0.945 | 0.952 | 0.932 | 0.933 | 0.922 |
| 5 | 0.975 | 0.979 | 0.984 | 0.985 | 0.985 | 0.981 | 0.978 | 0.974 |
| 6 | 0.959 | 0.964 | 0.967 | 0.969 | 0.977 | 0.964 | 0.961 | 0.958 |
| 7 | 0.975 | 0.975 | 0.975 | 0.978 | 0.984 | 0.976 | 0.971 | 0.967 |
| 8 | 0.997 | 0.997 | 0.997 | 0.995 | 1.003 | 0.999 | 0.995 | 0.993 |
| 9 | 1.022 | 1.020 | 1.019 | 1.017 | 1.015 | 1.022 | 1.021 | 1.021 |
| 10 | 0.996 | 0.996 | 0.996 | 1.001 | 1.003 | 0.998 | 0.999 | 0.996 |
| 11 | 1.011 | 1.010 | 1.011 | 1.015 | 1.015 | 1.010 | 1.012 | 1.012 |
| 12 | 1.023 | 1.023 | 1.023 | 1.025 | 1.024 | 1.022 | 1.019 | 1.021 |

**Table B10** Forecast performance, RMSE relative to univariate AR(2), hold out sample $m = 50$, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.950 | 0.963 | 0.984 | 0.985 | 0.980 | 0.959 | 1.001 | 0.986 |
| 2 | 0.853 | 0.865 | 0.900 | 0.921 | 0.955 | 0.876 | 0.913 | 0.880 |
| 3 | 0.890 | 0.903 | 0.930 | 0.945 | 0.974 | 0.904 | 0.928 | 0.906 |
| 4 | 0.924 | 0.929 | 0.948 | 0.962 | 0.974 | 0.927 | 0.957 | 0.935 |
| 5 | 0.968 | 0.975 | 0.984 | 0.993 | 0.998 | 0.977 | 0.995 | 0.982 |
| 6 | 0.953 | 0.962 | 0.971 | 0.982 | 0.991 | 0.969 | 0.979 | 0.964 |
| 7 | 0.973 | 0.979 | 0.986 | 0.996 | 0.999 | 0.990 | 1.002 | 0.986 |
| 8 | 0.994 | 0.995 | 0.999 | 0.998 | 1.003 | 1.005 | 1.009 | 0.999 |
| 9 | 1.022 | 1.023 | 1.019 | 1.016 | 1.010 | 1.028 | 1.029 | 1.026 |
| 10 | 0.996 | 0.999 | 0.998 | 1.007 | 1.011 | 1.001 | 1.005 | 1.002 |
| 11 | 1.010 | 1.014 | 1.014 | 1.017 | 1.018 | 1.016 | 1.021 | 1.020 |
| 12 | 1.021 | 1.023 | 1.022 | 1.022 | 1.019 | 1.023 | 1.025 | 1.025 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.952 | 0.960 | 0.971 | 0.970 | 0.970 | 0.952 | 0.985 | 0.980 |
| 2 | 0.861 | 0.870 | 0.885 | 0.900 | 0.925 | 0.873 | 0.904 | 0.882 |
| 3 | 0.889 | 0.897 | 0.913 | 0.925 | 0.952 | 0.895 | 0.917 | 0.899 |
| 4 | 0.918 | 0.923 | 0.935 | 0.945 | 0.959 | 0.923 | 0.952 | 0.931 |
| 5 | 0.967 | 0.974 | 0.976 | 0.985 | 0.993 | 0.975 | 0.992 | 0.978 |
| 6 | 0.952 | 0.960 | 0.962 | 0.973 | 0.983 | 0.964 | 0.973 | 0.957 |
| 7 | 0.969 | 0.975 | 0.978 | 0.990 | 0.995 | 0.982 | 0.992 | 0.979 |
| 8 | 0.995 | 0.997 | 0.999 | 0.999 | 1.005 | 1.003 | 1.005 | 0.998 |
| 9 | 1.021 | 1.025 | 1.022 | 1.023 | 1.020 | 1.026 | 1.029 | 1.026 |
| 10 | 0.997 | 1.000 | 1.000 | 1.006 | 1.009 | 1.001 | 1.004 | 1.002 |
| 11 | 1.010 | 1.015 | 1.014 | 1.018 | 1.021 | 1.015 | 1.021 | 1.019 |
| 12 | 1.022 | 1.025 | 1.025 | 1.025 | 1.024 | 1.023 | 1.024 | 1.024 |

## B.2 DGP 2

The DGP is

$$\mathbf{y}_t = \mathbf{y}_{t-1}\begin{pmatrix} 0.5 & 0.2 \\ 0.5 & 0.5 \end{pmatrix} + \mathbf{y}_{t-2}\begin{pmatrix} 0.1 & 0.1 \\ 0.2 & -0.3 \end{pmatrix} + \mathbf{u}_t,$$

and the irrelevant variables are generated as

$$\begin{aligned} z_{1,t} &= 0.5y_{1,t-1} + 0.5z_{1,t-1} + e_{1,t} \\ z_{2,t} &= 0.5y_{2,t-1} + 0.5z_{2,t-1} + e_{2,t} \\ z_{3,t} &= 0.7z_{3,t-1} + e_{3,t} \\ z_{4,t} &= 0.2z_{4,t-1} + e_{4,t} \\ z_{5,t} &= e_t \end{aligned}$$

with $u_{i,t}$ and $e_{i,t}$ iid $N(0,1)$. $T = 100$ (not accounting for lag lengths) and an additional 12 observations are set aside for forecast evaluation. Model averaging and model selection over the 42 possible models with up to four variables. $y_1$ is always included in the model. The results are based on 100 Monte Carlo replicates.

**Table B11** Posterior variable inclusion probabilities, DGP 2, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.86 | 0.18 | 4.67 | 0.95 | 0.16 | 5.80 |
| 2 | 0.83 | 0.19 | 4.26 | 0.93 | 0.19 | 4.82 |
| 3 | 0.62 | 0.21 | 2.95 | 0.79 | 0.23 | 3.42 |
| 4 | 0.45 | 0.24 | 1.87 | 0.58 | 0.27 | 2.13 |
| 8 | 0.35 | 0.25 | 1.40 | 0.37 | 0.29 | 1.28 |
| $1-4$ | 0.89 | 0.20 | 4.38 | 0.94 | 0.20 | 4.60 |
| $1-8$ | 0.84 | 0.24 | 3.46 | 0.86 | 0.24 | 3.56 |
| $1,4,8$ | 0.84 | 0.23 | 3.62 | 0.91 | 0.25 | 3.68 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.92 | 0.33 | 2.78 | 0.98 | 0.30 | 3.31 |
| 2 | 0.90 | 0.33 | 2.71 | 0.96 | 0.30 | 3.17 |
| 3 | 0.76 | 0.35 | 2.19 | 0.87 | 0.33 | 2.61 |
| 4 | 0.63 | 0.38 | 1.67 | 0.72 | 0.37 | 1.96 |
| 8 | 0.52 | 0.38 | 1.35 | 0.52 | 0.39 | 1.33 |
| $1-4$ | 0.93 | 0.31 | 2.98 | 0.96 | 0.28 | 3.44 |
| $1-8$ | 0.88 | 0.32 | 2.72 | 0.89 | 0.29 | 3.04 |
| $1,4,8$ | 0.89 | 0.34 | 2.63 | 0.95 | 0.32 | 2.98 |

**Table B12** Posterior variable inclusion probabilities, DGP 2, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.85 | 0.18 | 4.76 | 0.94 | 0.17 | 5.69 |
| 2 | 0.81 | 0.19 | 4.19 | 0.91 | 0.19 | 4.69 |
| 3 | 0.53 | 0.21 | 2.51 | 0.67 | 0.21 | 3.15 |
| 4 | 0.34 | 0.23 | 1.50 | 0.42 | 0.23 | 1.83 |
| 8 | 0.28 | 0.23 | 1.20 | 0.33 | 0.24 | 1.38 |
| $1-4$ | 0.84 | 0.21 | 4.04 | 0.91 | 0.21 | 4.42 |
| $1-8$ | 0.79 | 0.22 | 3.56 | 0.83 | 0.19 | 4.26 |
| $1, 4, 8$ | 0.77 | 0.21 | 3.63 | 0.88 | 0.19 | 4.73 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 30$ | | | hold out sample, $m = 50$ | | |
|---|---|---|---|---|---|---|
| | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $p(y_2)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ |
| 1 | 0.91 | 0.32 | 2.82 | 0.97 | 0.31 | 3.17 |
| 2 | 0.89 | 0.33 | 2.72 | 0.95 | 0.31 | 3.08 |
| 3 | 0.68 | 0.36 | 1.92 | 0.79 | 0.33 | 2.37 |
| 4 | 0.54 | 0.37 | 1.44 | 0.60 | 0.37 | 1.62 |
| 8 | 0.45 | 0.37 | 1.23 | 0.49 | 0.37 | 1.33 |
| $1-4$ | 0.89 | 0.31 | 2.83 | 0.94 | 0.28 | 3.34 |
| $1-8$ | 0.85 | 0.30 | 2.86 | 0.88 | 0.24 | 3.62 |
| $1, 4, 8$ | 0.84 | 0.31 | 2.68 | 0.93 | 0.27 | 3.43 |

**Table B13** Model selection, DGP 2, models estimated with lag length $p = 2$. Average posterior probability and proportion selected for true model.

Model prior, $\delta_k = 0.2$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.32 | 0.85 | 0.39 | 0.77 |
| 2 | 0.29 | 0.73 | 0.34 | 0.63 |
| 3 | 0.20 | 0.51 | 0.27 | 0.52 |
| 4 | 0.14 | 0.24 | 0.18 | 0.31 |
| 8 | 0.10 | 0.14 | 0.12 | 0.19 |
| $1-4$ | 0.33 | 0.55 | 0.37 | 0.49 |
| $1-8$ | 0.30 | 0.44 | 0.35 | 0.41 |
| $1,4,8$ | 0.30 | 0.60 | 0.35 | 0.47 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.07 | 0.15 | 0.11 | 0.28 |
| 2 | 0.07 | 0.27 | 0.10 | 0.23 |
| 3 | 0.06 | 0.17 | 0.09 | 0.26 |
| 4 | 0.04 | 0.12 | 0.07 | 0.21 |
| 8 | 0.04 | 0.13 | 0.07 | 0.19 |
| $1-4$ | 0.11 | 0.27 | 0.19 | 0.27 |
| $1-8$ | 0.15 | 0.24 | 0.26 | 0.31 |
| $1,4,8$ | 0.10 | 0.19 | 0.19 | 0.29 |

**Table B14** Model selection, DGP 2, models estimated with lag length $p = 4$. Average posterior probability and proportion selected for true model.

Model prior, $\delta_k = 0.2$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.33 | 0.83 | 0.39 | 0.75 |
| 2 | 0.30 | 0.78 | 0.35 | 0.60 |
| 3 | 0.18 | 0.46 | 0.24 | 0.45 |
| 4 | 0.11 | 0.13 | 0.14 | 0.20 |
| 8 | 0.09 | 0.09 | 0.11 | 0.12 |
| $1-4$ | 0.33 | 0.56 | 0.38 | 0.45 |
| $1-8$ | 0.31 | 0.46 | 0.36 | 0.39 |
| $1,4,8$ | 0.30 | 0.52 | 0.35 | 0.45 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out, $m = 30$ | | hold out, $m = 50$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.08 | 0.16 | 0.12 | 0.27 |
| 2 | 0.07 | 0.17 | 0.11 | 0.26 |
| 3 | 0.05 | 0.12 | 0.09 | 0.23 |
| 4 | 0.04 | 0.07 | 0.06 | 0.18 |
| 8 | 0.04 | 0.11 | 0.07 | 0.12 |
| $1-4$ | 0.11 | 0.21 | 0.22 | 0.34 |
| $1-8$ | 0.15 | 0.20 | 0.28 | 0.35 |
| $1,4,8$ | 0.11 | 0.20 | 0.21 | 0.31 |

**Table B15** Forecast performance RMSE relative to univariate AR(2), DGP 2, hold out sample $m = 30$, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.828 | 0.838 | 0.871 | 0.904 | 0.933 | 0.831 | 0.838 | 0.838 |
| 2 | 0.862 | 0.861 | 0.887 | 0.913 | 0.948 | 0.860 | 0.877 | 0.864 |
| 3 | 0.915 | 0.918 | 0.924 | 0.946 | 0.979 | 0.921 | 0.942 | 0.940 |
| 4 | 0.978 | 0.987 | 0.987 | 0.988 | 1.005 | 0.986 | 0.995 | 0.995 |
| 5 | 0.971 | 0.976 | 0.979 | 0.983 | 0.990 | 0.978 | 0.979 | 0.974 |
| 6 | 0.985 | 0.989 | 0.993 | 0.996 | 1.001 | 0.993 | 0.994 | 0.993 |
| 7 | 0.983 | 0.987 | 0.992 | 0.996 | 1.004 | 0.988 | 0.989 | 0.990 |
| 8 | 1.003 | 1.008 | 1.009 | 1.007 | 1.015 | 1.006 | 1.006 | 1.007 |
| 9 | 1.024 | 1.031 | 1.035 | 1.028 | 1.030 | 1.032 | 1.031 | 1.034 |
| 10 | 1.018 | 1.027 | 1.033 | 1.031 | 1.035 | 1.033 | 1.032 | 1.035 |
| 11 | 1.019 | 1.027 | 1.034 | 1.030 | 1.037 | 1.032 | 1.030 | 1.035 |
| 12 | 1.012 | 1.021 | 1.031 | 1.029 | 1.035 | 1.027 | 1.026 | 1.030 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.831 | 0.839 | 0.857 | 0.881 | 0.902 | 0.840 | 0.831 | 0.838 |
| 2 | 0.849 | 0.845 | 0.862 | 0.882 | 0.916 | 0.852 | 0.868 | 0.856 |
| 3 | 0.911 | 0.913 | 0.918 | 0.935 | 0.966 | 0.915 | 0.940 | 0.936 |
| 4 | 0.977 | 0.983 | 0.983 | 0.985 | 1.001 | 0.983 | 0.993 | 0.994 |
| 5 | 0.970 | 0.973 | 0.974 | 0.978 | 0.984 | 0.976 | 0.978 | 0.974 |
| 6 | 0.981 | 0.984 | 0.986 | 0.989 | 0.996 | 0.987 | 0.992 | 0.990 |
| 7 | 0.983 | 0.984 | 0.989 | 0.991 | 0.998 | 0.985 | 0.988 | 0.988 |
| 8 | 1.003 | 1.006 | 1.006 | 1.005 | 1.013 | 1.003 | 1.004 | 1.006 |
| 9 | 1.026 | 1.031 | 1.034 | 1.028 | 1.031 | 1.031 | 1.031 | 1.033 |
| 10 | 1.021 | 1.027 | 1.032 | 1.031 | 1.035 | 1.031 | 1.032 | 1.034 |
| 11 | 1.023 | 1.028 | 1.034 | 1.029 | 1.037 | 1.030 | 1.030 | 1.035 |
| 12 | 1.016 | 1.022 | 1.029 | 1.027 | 1.032 | 1.025 | 1.025 | 1.029 |

**Table B16** Forecast performance RMSE relative to univariate AR(2), DGP 2, hold out sample $m = 50$, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.827 | 0.846 | 0.872 | 0.897 | 0.937 | 0.832 | 0.834 | 0.831 |
| 2 | 0.850 | 0.850 | 0.882 | 0.916 | 0.946 | 0.850 | 0.869 | 0.855 |
| 3 | 0.911 | 0.909 | 0.913 | 0.933 | 0.952 | 0.913 | 0.912 | 0.912 |
| 4 | 0.974 | 0.978 | 0.979 | 0.984 | 0.999 | 0.986 | 0.987 | 0.987 |
| 5 | 0.968 | 0.971 | 0.973 | 0.973 | 0.992 | 0.972 | 0.978 | 0.983 |
| 6 | 0.981 | 0.984 | 0.988 | 0.988 | 1.004 | 0.987 | 0.990 | 0.997 |
| 7 | 0.978 | 0.980 | 0.985 | 0.985 | 1.003 | 0.986 | 0.989 | 0.993 |
| 8 | 0.997 | 1.000 | 1.002 | 1.003 | 1.012 | 1.006 | 1.011 | 1.012 |
| 9 | 1.018 | 1.024 | 1.030 | 1.025 | 1.034 | 1.026 | 1.032 | 1.039 |
| 10 | 1.015 | 1.021 | 1.027 | 1.026 | 1.038 | 1.025 | 1.031 | 1.041 |
| 11 | 1.014 | 1.020 | 1.027 | 1.025 | 1.045 | 1.025 | 1.033 | 1.043 |
| 12 | 1.008 | 1.016 | 1.025 | 1.026 | 1.048 | 1.019 | 1.029 | 1.040 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.832 | 0.847 | 0.866 | 0.885 | 0.914 | 0.833 | 0.829 | 0.831 |
| 2 | 0.847 | 0.850 | 0.866 | 0.891 | 0.919 | 0.850 | 0.858 | 0.847 |
| 3 | 0.911 | 0.912 | 0.914 | 0.926 | 0.940 | 0.916 | 0.914 | 0.914 |
| 4 | 0.975 | 0.979 | 0.982 | 0.983 | 0.994 | 0.986 | 0.989 | 0.990 |
| 5 | 0.969 | 0.973 | 0.975 | 0.973 | 0.989 | 0.975 | 0.980 | 0.984 |
| 6 | 0.981 | 0.984 | 0.987 | 0.986 | 0.999 | 0.988 | 0.991 | 0.998 |
| 7 | 0.979 | 0.982 | 0.985 | 0.983 | 0.999 | 0.987 | 0.991 | 0.995 |
| 8 | 0.998 | 1.001 | 1.004 | 1.002 | 1.010 | 1.006 | 1.013 | 1.014 |
| 9 | 1.021 | 1.028 | 1.032 | 1.027 | 1.034 | 1.028 | 1.035 | 1.040 |
| 10 | 1.018 | 1.025 | 1.029 | 1.027 | 1.039 | 1.026 | 1.034 | 1.042 |
| 11 | 1.018 | 1.025 | 1.029 | 1.026 | 1.045 | 1.027 | 1.036 | 1.045 |
| 12 | 1.012 | 1.020 | 1.025 | 1.024 | 1.046 | 1.021 | 1.031 | 1.040 |

**Table B17** Forecast performance RMSE relative to univariate AR(2), DGP 2, hold out sample $m = 30$, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.818 | 0.836 | 0.919 | 0.945 | 0.935 | 0.846 | 0.857 | 0.872 |
| 2 | 0.885 | 0.888 | 0.943 | 0.964 | 0.988 | 0.902 | 0.917 | 0.921 |
| 3 | 0.935 | 0.937 | 0.962 | 0.975 | 1.004 | 0.954 | 0.973 | 0.988 |
| 4 | 0.997 | 1.005 | 1.011 | 1.009 | 1.025 | 1.010 | 1.019 | 1.030 |
| 5 | 0.993 | 0.999 | 1.005 | 1.005 | 1.011 | 1.006 | 1.009 | 1.014 |
| 6 | 1.002 | 1.006 | 1.014 | 1.016 | 1.026 | 1.015 | 1.024 | 1.030 |
| 7 | 1.004 | 1.006 | 1.016 | 1.017 | 1.028 | 1.015 | 1.024 | 1.028 |
| 8 | 1.019 | 1.024 | 1.028 | 1.025 | 1.036 | 1.028 | 1.035 | 1.040 |
| 9 | 1.044 | 1.050 | 1.055 | 1.047 | 1.053 | 1.056 | 1.060 | 1.065 |
| 10 | 1.044 | 1.049 | 1.053 | 1.049 | 1.059 | 1.060 | 1.064 | 1.070 |
| 11 | 1.051 | 1.053 | 1.058 | 1.053 | 1.065 | 1.066 | 1.074 | 1.078 |
| 12 | 1.045 | 1.048 | 1.054 | 1.053 | 1.063 | 1.059 | 1.067 | 1.071 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.816 | 0.825 | 0.891 | 0.920 | 0.905 | 0.837 | 0.844 | 0.865 |
| 2 | 0.872 | 0.864 | 0.909 | 0.924 | 0.948 | 0.887 | 0.899 | 0.897 |
| 3 | 0.933 | 0.934 | 0.961 | 0.966 | 0.997 | 0.951 | 0.961 | 0.982 |
| 4 | 1.001 | 1.005 | 1.012 | 1.007 | 1.025 | 1.014 | 1.016 | 1.030 |
| 5 | 0.995 | 0.999 | 1.004 | 1.002 | 1.010 | 1.009 | 1.007 | 1.014 |
| 6 | 1.003 | 1.005 | 1.011 | 1.011 | 1.022 | 1.016 | 1.021 | 1.026 |
| 7 | 1.006 | 1.007 | 1.014 | 1.012 | 1.025 | 1.016 | 1.020 | 1.025 |
| 8 | 1.023 | 1.025 | 1.028 | 1.024 | 1.037 | 1.028 | 1.034 | 1.039 |
| 9 | 1.054 | 1.056 | 1.058 | 1.051 | 1.059 | 1.062 | 1.064 | 1.067 |
| 10 | 1.053 | 1.055 | 1.056 | 1.051 | 1.063 | 1.064 | 1.066 | 1.071 |
| 11 | 1.062 | 1.062 | 1.062 | 1.057 | 1.070 | 1.071 | 1.077 | 1.079 |
| 12 | 1.054 | 1.055 | 1.056 | 1.053 | 1.065 | 1.064 | 1.068 | 1.071 |

**Table B18** Forecast performance RMSE relative to univariate AR(2), DGP 2, hold out sample $m = 50$, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.807 | 0.823 | 0.901 | 0.927 | 0.933 | 0.825 | 0.835 | 0.822 |
| 2 | 0.875 | 0.874 | 0.924 | 0.968 | 0.978 | 0.884 | 0.893 | 0.874 |
| 3 | 0.942 | 0.949 | 0.963 | 0.974 | 0.987 | 0.966 | 0.958 | 0.954 |
| 4 | 1.000 | 1.010 | 1.011 | 1.010 | 1.020 | 1.017 | 1.005 | 1.013 |
| 5 | 0.990 | 0.998 | 1.006 | 0.999 | 1.007 | 1.001 | 0.989 | 0.997 |
| 6 | 1.004 | 1.009 | 1.016 | 1.014 | 1.013 | 1.015 | 1.000 | 1.009 |
| 7 | 1.006 | 1.008 | 1.016 | 1.013 | 1.015 | 1.020 | 1.001 | 1.010 |
| 8 | 1.022 | 1.025 | 1.023 | 1.022 | 1.024 | 1.030 | 1.025 | 1.027 |
| 9 | 1.044 | 1.049 | 1.050 | 1.046 | 1.039 | 1.053 | 1.040 | 1.046 |
| 10 | 1.041 | 1.044 | 1.046 | 1.043 | 1.039 | 1.049 | 1.037 | 1.044 |
| 11 | 1.047 | 1.047 | 1.049 | 1.048 | 1.040 | 1.054 | 1.038 | 1.046 |
| 12 | 1.041 | 1.043 | 1.048 | 1.050 | 1.043 | 1.050 | 1.032 | 1.041 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.815 | 0.818 | 0.877 | 0.907 | 0.908 | 0.818 | 0.824 | 0.812 |
| 2 | 0.868 | 0.866 | 0.893 | 0.927 | 0.937 | 0.877 | 0.883 | 0.868 |
| 3 | 0.947 | 0.952 | 0.961 | 0.964 | 0.977 | 0.966 | 0.956 | 0.949 |
| 4 | 1.003 | 1.010 | 1.015 | 1.011 | 1.017 | 1.019 | 1.005 | 1.013 |
| 5 | 0.993 | 1.000 | 1.007 | 1.001 | 1.006 | 1.002 | 0.988 | 0.997 |
| 6 | 1.005 | 1.010 | 1.016 | 1.012 | 1.009 | 1.014 | 0.998 | 1.007 |
| 7 | 1.006 | 1.009 | 1.016 | 1.011 | 1.012 | 1.019 | 1.000 | 1.008 |
| 8 | 1.023 | 1.026 | 1.029 | 1.023 | 1.025 | 1.031 | 1.023 | 1.027 |
| 9 | 1.050 | 1.054 | 1.057 | 1.050 | 1.042 | 1.056 | 1.040 | 1.049 |
| 10 | 1.046 | 1.049 | 1.053 | 1.046 | 1.041 | 1.052 | 1.037 | 1.045 |
| 11 | 1.052 | 1.053 | 1.057 | 1.052 | 1.044 | 1.056 | 1.038 | 1.048 |
| 12 | 1.046 | 1.049 | 1.054 | 1.050 | 1.042 | 1.052 | 1.031 | 1.045 |

## B.3 DGP 3

The DGP is

$$\mathbf{y}_t = \mathbf{y}_{t-1} \begin{pmatrix} 0.5 & 0.2 & 0.1 \\ 0.5 & 0.5 & 0.1 \\ 0.5 & 0.3 & 0.2 \end{pmatrix} + \mathbf{u}_t,$$

and the irrelevant variables are generated as

$$\begin{aligned} z_{1,t} &= 0.5y_{1,t-1} + 0.5z_{1,t-1} + e_{1,t} \\ z_{2,t} &= 0.5y_{2,t-1} + 0.5z_{2,t-1} + e_{2,t} \\ z_{3,t} &= 0.7z_{3,t-1} + e_{3,t} \\ z_{4,t} &= 0.2z_{4,t-1} + e_{4,t} \end{aligned}$$

with $u_{i,t}$ and $e_{i,t}$ iid $N(0,1)$. $T = 100$ (not accounting for lag lengths) and an additional 12 observations are set aside for forecast evaluation. Model averaging and model selection over the 57 possible models with up to five variables. $y_1$ is always included in the model. The results are based on 100 Monte Carlo replicates.

**Table B19** Posterior variable inclusion probabilities, DGP 3, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 30$ | | | | | hold out sample, $m = 50$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $p(y_2)$ | $p(y_3)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $\frac{p(y_3)}{\max[p(z_i)]}$ | $p(y_2)$ | $p(y_3)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $\frac{p(y_3)}{\max[p(z_i)]}$ |
| 1 | 0.89 | 0.87 | 0.18 | 4.90 | 4.79 | 0.91 | 0.87 | 0.19 | 4.75 | 4.53 |
| 2 | 0.80 | 0.75 | 0.21 | 3.81 | 3.58 | 0.84 | 0.75 | 0.25 | 3.32 | 2.94 |
| 3 | 0.65 | 0.62 | 0.23 | 2.80 | 2.64 | 0.74 | 0.64 | 0.25 | 2.93 | 2.54 |
| 4 | 0.56 | 0.50 | 0.25 | 2.27 | 2.03 | 0.65 | 0.49 | 0.28 | 2.35 | 1.79 |
| 8 | 0.41 | 0.32 | 0.27 | 1.54 | 1.20 | 0.47 | 0.36 | 0.33 | 1.44 | 1.11 |
| $1-4$ | 0.81 | 0.85 | 0.21 | 3.95 | 4.14 | 0.83 | 0.74 | 0.23 | 3.71 | 3.30 |
| $1-8$ | 0.76 | 0.74 | 0.23 | 3.25 | 3.18 | | | | | |
| $1,4,8$ | 0.82 | 0.78 | 0.21 | 3.82 | 3.62 | 0.70 | 0.62 | 0.21 | 3.33 | 2.94 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 30$ | | | | | hold out sample, $m = 50$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $p(y_2)$ | $p(y_3)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $\frac{p(y_3)}{\max[p(z_i)]}$ | $p(y_2)$ | $p(y_3)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $\frac{p(y_3)}{\max[p(z_i)]}$ |
| 1 | 0.94 | 0.93 | 0.34 | 2.72 | 2.70 | 0.94 | 0.92 | 0.33 | 2.85 | 2.79 |
| 2 | 0.87 | 0.86 | 0.38 | 2.31 | 2.27 | 0.89 | 0.84 | 0.38 | 2.38 | 2.24 |
| 3 | 0.79 | 0.77 | 0.41 | 1.90 | 1.86 | 0.82 | 0.76 | 0.39 | 2.07 | 1.92 |
| 4 | 0.71 | 0.68 | 0.43 | 1.64 | 1.59 | 0.74 | 0.65 | 0.40 | 1.84 | 1.61 |
| 8 | 0.56 | 0.51 | 0.46 | 1.22 | 1.11 | 0.57 | 0.48 | 0.41 | 1.38 | 1.15 |
| $1-4$ | 0.87 | 0.90 | 0.33 | 2.64 | 2.73 | 0.86 | 0.82 | 0.30 | 2.88 | 2.75 |
| $1-8$ | 0.81 | 0.82 | 0.32 | 2.53 | 2.56 | | | | | |
| $1,4,8$ | 0.86 | 0.85 | 0.34 | 2.53 | 2.51 | 0.77 | 0.68 | 0.28 | 2.72 | 2.39 |

**Table B20** Posterior variable inclusion probabilities, DGP 3, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| $h$ | hold out sample, $m = 50$ | | | | |
|---|---|---|---|---|---|
| | $p(y_2)$ | $p(y_3)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $\frac{p(y_3)}{\max[p(z_i)]}$ |
| 1 | 0.89 | 0.86 | 0.16 | 5.59 | 5.43 |
| 2 | 0.77 | 0.71 | 0.19 | 4.06 | 3.74 |
| 3 | 0.62 | 0.57 | 0.20 | 3.04 | 2.78 |
| 4 | 0.52 | 0.46 | 0.23 | 2.29 | 2.01 |
| 8 | 0.32 | 0.30 | 0.26 | 1.24 | 1.16 |
| $1-4$ | | | | | |
| $1-8$ | | | | | |
| $1,4,8$ | 0.71 | 0.73 | 0.20 | 3.50 | 3.59 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out sample, $m = 50$ | | | | |
|---|---|---|---|---|---|
| | $p(y_2)$ | $p(y_3)$ | $\max[p(z_i)]$ | $\frac{p(y_2)}{\max[p(z_i)]}$ | $\frac{p(y_3)}{\max[p(z_i)]}$ |
| 1 | 0.94 | 0.93 | 0.32 | 2.92 | 2.89 |
| 2 | 0.86 | 0.84 | 0.37 | 2.33 | 2.28 |
| 3 | 0.76 | 0.75 | 0.39 | 1.96 | 1.91 |
| 4 | 0.68 | 0.66 | 0.40 | 1.69 | 1.62 |
| 8 | 0.48 | 0.49 | 0.45 | 1.07 | 1.09 |
| $1-4$ | | | | | |
| $1-8$ | | | | | |
| $1,4,8$ | 0.78 | 0.84 | 0.31 | 2.49 | 2.67 |

**Table B21** Model selection, DGP 3, models estimated with lag length $p = 2$. Average posterior probability and proportion selected for true model.

Model prior, $\delta_k = 0.2$

| $h$ | hold out, $m = 50$ | | hold out, $m = 70$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.37 | 0.72 | 0.40 | 0.61 |
| 2 | 0.26 | 0.54 | 0.27 | 0.44 |
| 3 | 0.15 | 0.32 | 0.18 | 0.26 |
| 4 | 0.10 | 0.17 | 0.11 | 0.14 |
| 8 | 0.04 | 0.04 | 0.05 | 0.06 |
| $1-4$ | 0.35 | 0.50 | 0.36 | 0.43 |
| $1-8$ | 0.28 | 0.33 | | |
| $1,4,8$ | 0.32 | 0.49 | 0.20 | 0.22 |

Model prior, $\delta_k = 0.5$

| $h$ | hold out, $m = 50$ | | hold out, $m = 70$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.37 | 0.72 | 0.40 | 0.61 |
| 2 | 0.26 | 0.54 | 0.27 | 0.44 |
| 3 | 0.15 | 0.32 | 0.18 | 0.26 |
| 4 | 0.10 | 0.17 | 0.11 | 0.14 |
| 8 | 0.04 | 0.04 | 0.05 | 0.06 |
| $1-4$ | 0.35 | 0.50 | 0.36 | 0.43 |
| $1-8$ | 0.28 | 0.33 | | |
| $1,4,8$ | 0.32 | 0.49 | 0.20 | 0.22 |

**Table B22** Model selection, DGP 3, models estimated with lag length $p = 4$. Average posterior probability and proportion selected for true model.

Hold out sample, $m = 50$

| $h$ | Model prior, $\delta_k = 0.2$ | | Model prior, $\delta_k = 0.5$ | |
|---|---|---|---|---|
| | Prob | Selected | Prob | Selected |
| 1 | 0.39 | 0.72 | 0.15 | 0.33 |
| 2 | 0.26 | 0.53 | 0.11 | 0.26 |
| 3 | 0.14 | 0.25 | 0.07 | 0.16 |
| 4 | 0.09 | 0.12 | 0.06 | 0.14 |
| 8 | 0.03 | 0.03 | 0.03 | 0.09 |
| $1-4$ | | | | |
| $1-8$ | | | | |
| $1,4,8$ | 0.27 | 0.39 | 0.19 | 0.26 |

**Table B23** Forecast performance RMSE relative to univariate AR(2), DGP 3, hold out sample $m = 50$, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.833 | 0.867 | 0.880 | 0.904 | 0.930 | 0.852 | 0.896 | 0.851 |
| 2 | 0.820 | 0.831 | 0.861 | 0.882 | 0.930 | 0.833 | 0.856 | 0.834 |
| 3 | 0.858 | 0.872 | 0.884 | 0.900 | 0.933 | 0.874 | 0.896 | 0.875 |
| 4 | 0.889 | 0.895 | 0.908 | 0.913 | 0.939 | 0.894 | 0.909 | 0.899 |
| 5 | 0.927 | 0.931 | 0.940 | 0.943 | 0.966 | 0.932 | 0.949 | 0.936 |
| 6 | 0.938 | 0.943 | 0.951 | 0.960 | 0.980 | 0.954 | 0.968 | 0.956 |
| 7 | 0.941 | 0.946 | 0.955 | 0.959 | 0.975 | 0.957 | 0.968 | 0.955 |
| 8 | 0.966 | 0.966 | 0.971 | 0.972 | 0.985 | 0.978 | 0.983 | 0.976 |
| 9 | 0.978 | 0.975 | 0.976 | 0.979 | 0.997 | 0.992 | 1.001 | 0.993 |
| 10 | 0.975 | 0.973 | 0.979 | 0.979 | 0.994 | 0.986 | 0.996 | 0.988 |
| 11 | 0.993 | 0.990 | 0.995 | 0.994 | 1.007 | 1.004 | 1.012 | 1.005 |
| 12 | 1.010 | 1.007 | 1.009 | 1.007 | 1.014 | 1.020 | 1.022 | 1.018 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.822 | 0.844 | 0.851 | 0.861 | 0.893 | 0.839 | 0.865 | 0.828 |
| 2 | 0.821 | 0.828 | 0.846 | 0.855 | 0.894 | 0.827 | 0.832 | 0.824 |
| 3 | 0.855 | 0.868 | 0.873 | 0.883 | 0.910 | 0.864 | 0.878 | 0.867 |
| 4 | 0.888 | 0.894 | 0.902 | 0.905 | 0.923 | 0.889 | 0.901 | 0.896 |
| 5 | 0.928 | 0.932 | 0.937 | 0.939 | 0.958 | 0.931 | 0.939 | 0.931 |
| 6 | 0.940 | 0.944 | 0.947 | 0.956 | 0.973 | 0.950 | 0.957 | 0.950 |
| 7 | 0.943 | 0.948 | 0.953 | 0.956 | 0.969 | 0.952 | 0.956 | 0.950 |
| 8 | 0.971 | 0.972 | 0.973 | 0.974 | 0.983 | 0.975 | 0.978 | 0.974 |
| 9 | 0.983 | 0.980 | 0.978 | 0.981 | 0.998 | 0.989 | 0.997 | 0.992 |
| 10 | 0.979 | 0.977 | 0.980 | 0.982 | 0.994 | 0.982 | 0.992 | 0.986 |
| 11 | 0.997 | 0.994 | 0.996 | 0.998 | 1.010 | 0.999 | 1.009 | 1.003 |
| 12 | 1.016 | 1.013 | 1.013 | 1.014 | 1.019 | 1.016 | 1.020 | 1.017 |

**Table B24** Forecast performance RMSE relative to univariate AR(2), DGP 3, hold out sample $m = 70$, models estimated with lag length $p = 2$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.830 | 0.833 | 0.855 | 0.866 | 0.920 | 0.830 | | 0.858 |
| 2 | 0.824 | 0.844 | 0.878 | 0.877 | 0.943 | 0.841 | | 0.867 |
| 3 | 0.858 | 0.865 | 0.876 | 0.882 | 0.943 | 0.868 | | 0.886 |
| 4 | 0.898 | 0.898 | 0.910 | 0.918 | 0.948 | 0.906 | | 0.918 |
| 5 | 0.932 | 0.925 | 0.935 | 0.943 | 0.961 | 0.929 | | 0.941 |
| 6 | 0.938 | 0.937 | 0.945 | 0.959 | 0.983 | 0.946 | | 0.955 |
| 7 | 0.942 | 0.938 | 0.945 | 0.958 | 0.973 | 0.952 | | 0.957 |
| 8 | 0.964 | 0.962 | 0.973 | 0.978 | 0.979 | 0.970 | | 0.967 |
| 9 | 0.977 | 0.974 | 0.978 | 0.980 | 0.986 | 0.987 | | 0.981 |
| 10 | 0.973 | 0.971 | 0.974 | 0.981 | 0.989 | 0.985 | | 0.982 |
| 11 | 0.990 | 0.990 | 0.995 | 0.998 | 1.003 | 1.001 | | 0.997 |
| 12 | 1.007 | 1.008 | 1.010 | 1.014 | 1.017 | 1.017 | | 1.014 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.817 | 0.813 | 0.819 | 0.837 | 0.872 | 0.817 | | 0.848 |
| 2 | 0.826 | 0.833 | 0.852 | 0.861 | 0.919 | 0.837 | | 0.860 |
| 3 | 0.859 | 0.863 | 0.864 | 0.873 | 0.926 | 0.868 | | 0.881 |
| 4 | 0.896 | 0.896 | 0.904 | 0.914 | 0.939 | 0.906 | | 0.912 |
| 5 | 0.932 | 0.925 | 0.931 | 0.938 | 0.959 | 0.929 | | 0.940 |
| 6 | 0.937 | 0.937 | 0.939 | 0.952 | 0.979 | 0.945 | | 0.951 |
| 7 | 0.942 | 0.941 | 0.944 | 0.958 | 0.973 | 0.951 | | 0.952 |
| 8 | 0.966 | 0.965 | 0.975 | 0.977 | 0.981 | 0.971 | | 0.968 |
| 9 | 0.978 | 0.977 | 0.983 | 0.983 | 0.989 | 0.988 | | 0.981 |
| 10 | 0.974 | 0.974 | 0.978 | 0.984 | 0.992 | 0.986 | | 0.983 |
| 11 | 0.992 | 0.993 | 0.998 | 0.999 | 1.007 | 1.001 | | 0.999 |
| 12 | 1.011 | 1.011 | 1.013 | 1.014 | 1.024 | 1.018 | | 1.016 |

**Table B25** Forecast performance RMSE relative to univariate AR(2), DGP 3, hold out sample $m = 50$, models estimated with lag length $p = 4$

Model prior, $\delta_k = 0.2$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.843 | 0.878 | 0.909 | 0.939 | 0.929 | 0.861 | | 0.896 |
| 2 | 0.831 | 0.840 | 0.868 | 0.906 | 0.933 | 0.843 | | 0.881 |
| 3 | 0.873 | 0.886 | 0.898 | 0.923 | 0.948 | 0.882 | | 0.906 |
| 4 | 0.897 | 0.905 | 0.918 | 0.935 | 0.962 | 0.896 | | 0.925 |
| 5 | 0.931 | 0.934 | 0.941 | 0.961 | 0.994 | 0.934 | | 0.960 |
| 6 | 0.950 | 0.953 | 0.961 | 0.981 | 1.006 | 0.962 | | 0.984 |
| 7 | 0.953 | 0.955 | 0.966 | 0.981 | 1.008 | 0.964 | | 0.990 |
| 8 | 0.974 | 0.971 | 0.983 | 0.994 | 1.019 | 0.984 | | 1.006 |
| 9 | 0.996 | 0.991 | 0.997 | 1.002 | 1.022 | 1.010 | | 1.023 |
| 10 | 0.981 | 0.978 | 0.990 | 0.994 | 1.024 | 0.989 | | 1.020 |
| 11 | 1.002 | 0.999 | 1.007 | 1.007 | 1.028 | 1.011 | | 1.029 |
| 12 | 1.023 | 1.019 | 1.022 | 1.021 | 1.037 | 1.031 | | 1.046 |

Model prior, $\delta_k = 0.5$

| | Predictive likelihood evaluated at horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 2 | 3 | 4 | 8 | $1-4$ | $1-8$ | $1,4,8$ |
| 1 | 0.829 | 0.849 | 0.870 | 0.891 | 0.901 | 0.841 | | 0.865 |
| 2 | 0.838 | 0.842 | 0.855 | 0.877 | 0.907 | 0.836 | | 0.862 |
| 3 | 0.876 | 0.882 | 0.886 | 0.905 | 0.929 | 0.873 | | 0.891 |
| 4 | 0.900 | 0.904 | 0.912 | 0.927 | 0.950 | 0.895 | | 0.916 |
| 5 | 0.935 | 0.937 | 0.940 | 0.957 | 0.984 | 0.932 | | 0.951 |
| 6 | 0.954 | 0.957 | 0.958 | 0.977 | 1.001 | 0.960 | | 0.974 |
| 7 | 0.955 | 0.958 | 0.961 | 0.974 | 1.002 | 0.960 | | 0.976 |
| 8 | 0.979 | 0.978 | 0.983 | 0.993 | 1.016 | 0.984 | | 1.000 |
| 9 | 1.003 | 1.000 | 1.002 | 1.007 | 1.027 | 1.011 | | 1.022 |
| 10 | 0.984 | 0.982 | 0.990 | 0.996 | 1.024 | 0.988 | | 1.013 |
| 11 | 1.006 | 1.003 | 1.008 | 1.009 | 1.028 | 1.010 | | 1.026 |
| 12 | 1.028 | 1.025 | 1.026 | 1.027 | 1.041 | 1.030 | | 1.044 |