Economies of Scale in Banking, Indeterminacy, and Business Cycles*

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Abstract

This paper investigates an environment featuring inside money, nominal rigidities and indeterminacy due to increasing returns to scale (IRS) in the financial intermediary. Although the size of the financial intermediary sector is calibrated to match the observed (1-2 percent) value added to US output: (i) indeterminacy arises for small degrees of IRS and standard parameter assumptions; (ii) sunspot shocks have significant effects due to the endogenous money multiplier; (iii) idiosyncratic sunspot shocks qualitatively resemble exogenous monetary shocks; and (iv) although endogenous monetary policy can increase the degree of IRS needed for indeterminacy, policy can only stabilize sunspot shocks under complete information.

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1. Introduction

Business cycle environments focusing on the beliefs of agents (or *animal spirits*) have become popular tools for analyzing economic fluctuations. Following Farmer (1993), the literature has focused on environments featuring increasing returns to scale (IRS) in production which deliver indeterminate equilibria and allow an analysis of the economic impact of resulting sunspot shocks. While providing a rich environment to analyze the extent through which agents' beliefs influence business cycle fluctuations, some features of IRS in production are at odds with the empirical facts. For example, the degree of IRS needed to deliver indeterminacy in environments with one sector of production [i.e. Farmer and Guo (1994) and Gali (1994)] far exceeds the constant returns estimated by Basu and Fernald (1997).[1]

In contrast to the above literature, this paper considers a business cycle environment where indeterminacy of equilibria arises through a decidedly different sector: financial intermediation. Following Corbae and Dressler (2007) and their extension of the endogenous inside money mechanism of Freeman and Kydland (2000) and Dressler (2007), a standard cash-in-advance environment is extended to feature financial intermediaries (*banks*) and nominal wage rigidity. Households purchase consumption goods with either previously chosen currency balances or physical capital deposited with a bank. Banks are assumed to exhibit IRS by having decreasing marginal costs associated with managing household deposits. This model feature builds on empirical evidence reported by Hughes and Mester (1998) that banks of all sizes exhibit significant scale economies.[2] IRS in the banking sector delivers indeterminacy, and the resulting sunspot shocks have a nominal economic impact because they influence the households' decision of holding deposit (i.e. inside money) balances which in turn affect broad monetary aggregates and the price level. When combined with nominal wage rigidity, shocks to households' beliefs concerning the banking sector impact the real

[1] Benhabib and Farmer (1996) show, however, that including multiple productive sectors reduces the required degree of IRS. This result is more in line with the empirical evidence that modest IRS can be found in the production of consumer durables (Basu and Fernald, 1997) and investment goods (Harrison, 1996). Weder (2000) shows that modest IRS in the investment goods producing sector alone can deliver indeterminacy in a business cycle environment which performs as well as standard environments driven by shocks to fundamentals.

[2] This empirical evidence is not without controversy. See Berger and Mester (1999) for references both in support and in contrast to this result.

economy through changes in real wages and the demand for labor.

The full set of dynamic equilibria is characterized for the environment under alternative specifications of monetary policy. The results of the analysis are as follows. First, although the size of the intermediary sector is in line with U.S. data (a value added of 1-2 percent of output), indeterminacy easily arises in the model for small degrees of IRS in the financial intermediary without the need for multiple productive sectors or unusual parameter values.[3] Second, since the sunspot shock influences inside money balances, the response of the real economy to idiosyncratic sunspot shocks look qualitatively similar to persistent, exogenous monetary shocks. Finally, although endogenous monetary policy can increase the amount of IRS required to deliver indeterminacy when targeting real variables (the output gap), idiosyncratic sunspot shocks can only be stabilized when the monetary authority has complete information. In other words, a monetary authority can neutralize the real impact of sunspot shocks only when the shock can be fully observed. An underlying feature of these results is that although the sunspot shocks arising from indeterminacy resemble *nominal* shocks, indeterminacy arises due to IRS in a small, *real* cost to intermediation.

Although the analysis focuses on business-cycle fluctuations, it is closely related to the rich literature of banking crises where a strategic complementarity in intermediation delivers multiple equilibria based on shocks to confidence.[4] In contrast to models like Cooper and Corbae (2002) and Corbae and Dressler (2007) where shocks to confidence coordinates agents to multiple *steady-state* equilibria, this paper considers small belief-induced shocks which fluctuate the economy around a unique steady state. In doing so, similar intuition behind belief-induced banking crises can be applied on a smaller scale and treated as a commonly occurring event which influences equilibrium business cycles. This paper can therefore be considered a link between the literature on banking crises and the literature considering sunspot shocks as a source of business-cycle fluctuations.

The paper is organized as follows. Section 2 presents the model and equilibrium. Section 3 presents the conditions which deliver indeterminacy, and analyzes the dynamic predictions

[3]For example, it has been shown that indeterminacy arises in cash-in-advance economies only for very weak degrees of intertemporal substitution (see Farmer, 1993).

[4]Some of the first analyses along these lines are Bryant (1987) and Weil (1989). In addition, Boyd et al. (2000) provide empirical evidence in favor of a 'sunspot's view' of banking crises.

of the model under alternative treatments of the shocks and alternative monetary policy assumptions. Section 4 concludes.

2. The Model and Equilibrium

2.1. The Model

The model is in discrete time with an infinite horizon. There exists a continuum of households indexed by $i \in (0,1)$ which supply differentiated labor, a large number of intermediate firms which aggregate differentiated labor into homogeneous labor units, a large number of final good-producing firms, a large number of financial intermediaries, and a monetary authority. The environment is described by outlining the production sector, the household's problem, the financial intermediary sector, and the monetary authority.

Production

Following Erceg et al. (2000), there exists two competitive production sectors: an intermediate goods sector and a final goods sector.

Intermediate firms (employment agencies) hire differentiated labor from type-i households (h_{it}) and aggregate them according to the CES technology

$$h_t = \left(\int_0^1 h_{it}^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}},$$

where $\frac{\xi}{\xi-1}$ is the static gross markup of consumer i in the type-i labor market. The homogeneous labor good (h_t) is sold to final good producing firms. Nominal profits of the representative intermediate firm are given by

$$W_t h_t - \int_0^1 W_{it} h_{it} di,$$

where W_t denotes the aggregate nominal wage rate and W_{it} is the nominal wage rate paid to type-i labor. Profit maximization yields a standard demand function for the ith household's labor,

$$h_{it} = \left(\frac{W_t}{W_{it}} \right)^{\xi} h_t, \tag{1}$$

where ξ measures the elasticity of substitution between different labor types. As is common in the literature featuring nominal-wage rigidity, only symmetric labor market equilibria will be considered (i.e. $W_{it} = W_t$ and $h_{it} = h_t, \forall i$).

Output of the final goods at time t follows a constant returns to scale (CRS) function of capital (k_t) and homogeneous labor: $y_t = f(z_t, k_t, h_t)$, where z_t denotes the level of total factor productivity which evolves according to $z_t = \kappa_z + \rho_z z_{t-1} + \varepsilon_t^z$ with $\varepsilon_t^z \sim N(0, \sigma_z^2)$. The capital stock evolves according to $k_{t+1} = (1-\delta) k_t + i_t$, where δ denotes the depreciation rate and i_t denotes capital investment in period t.

Final good firms choose k_t and h_t in order to maximize profits,

$$f(z_t, k_t, h_t) + (1-\delta) k_t - r_t k_t - \frac{W_t}{P_t} h_t,$$

where r_t is the gross real rental rate of capital and P_t is the aggregate price level. A profit-maximizing firm equates the marginal product of each input with its marginal cost.

$$f_k(z_t, k_t, h_t) = r_t - 1 + \delta$$
$$f_h(z_t, k_t, h_t) = \frac{W_t}{P_t}$$

Households

Infinitely-lived households choose their nominal wage (W_{it}) and then supply differentiated labor h_{it} according to (1). It is well documented in the literature featuring nominal-wage rigidity that households make all subsequent decisions identically, so allocations and asset holdings need not be indexed.

Each household is endowed with an initial capital stock and one unit of time in every period. There exists a continuum of consumption goods of measure c_t^*, indexed by $j \in (0,1)$. Households rank the consumption of each good type (c_{jt}) and leisure in each period. The utility of a representative household is expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left[\min \left(\frac{c_{jt}}{2j} \right), h_{it} \right], \tag{2}$$

where $u[.,.]$ is assumed to be increasing (decreasing) in its first (second) argument, quasi-concave, twice continuously differentiable, and to satisfy the Inada conditions. E_0 is the expectation operator conditional on information available at time 0 and $\beta \in (0,1)$ is the discount rate.

The leontief-type argument in (2) follows Freeman and Kydland (2000) and delivers an endogenous separation of 'cash' goods and 'deposit' goods. For a given amount of total consumption c_t^*, households distribute consumption across the j good types according to the optimal rule: $c_{jt} = 2jc_t^*$. This rule results in the continuum of consumption goods to be given an ordinal rank by j; the smaller (larger) the value of j, the smaller (larger) the contribution of good c_{jt} to the utility of total consumption c_t^*.

A representative household begins each period with amounts of physical capital and nominal balances of currency (m_t), and then receives a lump-sum transfer of currency from the monetary authority (T_t). The household then divides its capital into a direct investment in the firm (a_t) and a deposit in the financial intermediary (d_t) at rates r_t and $\tilde{r}_t$, respectively. This implies the constraint $k_t = a_t + d_t$.

It is assumed that currency and deposits can both be used to purchase consumption. Currency balances are chosen in the previous period, similar to the assumption of standard cash-in-advance models. Deposits are chosen at the beginning of the period, pay interest $\tilde{r}_t$, and circumvent the opportunity cost of holding return-dominated assets across periods. While currency is costless to use, there is a real cost (γ) for each consumption good (c_{jt}) purchased with deposits. This cost can be interpreted as a per-check identity verification or check-clearing cost and is independent of the amount of the purchase. This fixed cost to using deposits implies that the net return on deposits is decreasing in the size of the purchase. While leaving the details to the following section, the net return on deposits approaches negative infinity as the purchase size j approaches zero. This implies that there is a critical good type j^*, below (above) which currency (deposits) is the preferred means of payment because it offers the higher return. These features deliver the following money balance conditions,

$$\frac{m_t + T_t}{P_t} \geq \int_0^{j^*} c_{jt} d = j_t^{*2} c_t^*, \tag{3}$$

$$d_t \geq \int_{j^*}^1 c_{jt} dj = \left(1 - j_t^{*2}\right) c_t^*, \tag{4}$$

where j_t^* illustrates the endogenous separation of 'cash' goods and 'deposit' goods.

The representative household maximizes (2) subject to (1) , (3), (4), and a flow budget constraint.

$$\frac{W_{it}}{P_t} h_{it} + r_t a_t + \tilde{r}_t d_t + \frac{m_t + T_t}{P_t} \geq \tag{5}$$
$$c_t^* + \frac{m_{t+1}}{P_t} + k_{t+1} + \frac{\phi}{2} \left[\frac{W_{it}}{\pi W_{it-1}} - 1 \right]^2 + \gamma \left(1 - j_t^*\right)$$

There are two costs worth noting on the right-hand side of (5). The fourth term measures the real Rotemberg (1982)-style cost of adjusting the nominal wage which is indexed by the long-run inflation rate π. The fifth term measures the aggregate cost to using deposits to make consumption purchases.

Financial Intermediaries

At the beginning of a period, intermediaries pool a portion of household capital (deposits) and transforms it into interest-bearing checking accounts. An intermediary then rents the capital to firms. An intermediary provides no additional services in the model such as screening or monitoring of the capital loans; it's sole purpose lies in providing check-writing services to depositors. From the perspective of a firm, bank loans and direct capital loans from households are perfect substitutes and therefore share the same rental rate (r_t). This prevents the financial intermediary from having monopoly power over loan supply and direct investment will always occur.

The profit function of an intermediary can be expressed as

$$r_t D_t - \left[\tilde{r}_t D_t + C\left(D_t\right)\right],$$

where D_t denotes total real deposits, and $C(D_t)$ denotes real operating costs. Profit maximization results in

$$\tilde{r}_t = r_t - \tau_t, \tag{6}$$

where $\tau_t = \partial C(D_t)/\partial D_t$ denotes marginal operating costs. If the return to deposits $(r_t - \tau_t)$ is marginally increasing in the aggregate amount of total deposits (all else equal), the financial intermediary will exhibit IRS and potentially be a source of indeterminacy. For example, by considering $C(D_t) = \frac{\Gamma}{1+\theta} D_t^{1+\theta}$, the marginal cost to managing deposits is given by

$$\tau_t = \Gamma D_t^{\theta}, \tag{7}$$

and IRS in financial intermediation would arise for any $-1 < \theta < 0$.[5]

The Monetary Authority

The budget constraint of the monetary authority is given by $T_t = M_{t+1} - M_t$, where M_{t+1} denotes the aggregate stock of currency (the monetary base) available at the end of period t. The currency base evolves according to $M_{t+1} = \mu_t M_t$ where μ_t denotes the gross growth rate.

The analysis considers several cases of how the monetary authority chooses μ_t. First, the benchmark case considers exogenous monetary policy where money growth evolves according to

$$\mu_t = \kappa_\mu + \rho_\mu \mu_{t-1} + \varepsilon_t^\mu, \tag{8}$$

where $\varepsilon_t^\mu \sim N(0, \sigma_\mu^2)$. Three additional cases consider variations of a standard, Taylor

[5] This intermediation technology is a slight abstraction from more formal analyses of IRS in the intermediary sector. For example, if the intermediary faces a fixed start-up cost Γ which must be paid at the beginning of every period (e.g. Boyd and Prescott, 1986), then an intermediary could be considered a coalition of depositors which equally distribute the cost across its members $(\tau_t = \Gamma/D_t)$. This case is nested in the present model with $\theta = -1$ in the limit.

(1993)-style rule,

$$E\left[\log\left(R_t/R\right) - \varpi_\pi \log\left(\pi_t/\pi\right) - \varpi_y \log\left(y_t/y\right)|\Omega_t\right] = 0, \tag{9}$$

where R_t denotes the nominal interest rate, $\pi_t = P_t/P_{t-1}$, and variables without a time subscript denote their long-run (steady state) values. By considering alternative values for ϖ_π and ϖ_y, as well as changing the set of information available to the monetary authority (Ω_t), the full interaction of monetary policy and indeterminacy in the model can be assessed.

2.2. Equilibrium

The household's problem in the previous section was simplified following Freeman and Kydland (2000). Substitution of the optimal rule $c_{jt} = 2jc_t^*$ into (2) yields a standard objective function.

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^*, h_{it}\right) \tag{10}$$

For each type of good purchased, a household must decide if it is more attractive to use its present balances of currency or to acquire deposits. Recall that deposits are physical capital lent to the intermediary after all relevant information is observed. Therefore, if a household decides to purchase a consumption good c_{jt} with deposits, the end-of-period cost of the purchase is given by

$$\left[\left(1+\tau_t\right)c_{jt} + \gamma\right]/r_t$$

which takes into account that the household is paying all costs after the firm repays the capital loans (i.e. the cost is deflated by market interest rates). On the other hand, if the household decides to purchase c_{jt} with previously chosen currency at given prices, the real cost is given by

$$P_t c_{jt}/P_{t-1}$$

which takes into account that the currency obtained to make the purchase was acquired last period. A direct comparison of these costs results in

$$\left[1+\tau_t+\frac{\gamma}{2jc_t^*}\right]r_t^{-1} \gtreqless \frac{P_t}{P_{t-1}},\ \forall j\in(0,1)$$

where the left-hand side is a decreasing function of the good index j. Therefore, the critical good type (j^*) mentioned previously results in the household being indifferent to purchasing c_{j^*t} with either currency or deposits because they share the same cost,

$$\left[1+\tau_t+\frac{\gamma}{2j^*c_t^*}\right]r_t^{-1} = \frac{P_t}{P_{t-1}}, \tag{11}$$

and all goods j above (below) j^* will be purchased with deposits (currency). The remainder of the analysis concentrates on the case in which $j^*\in(0,1)$.[6]

Taking all prices and the state of the economy as given, each household chooses c_t^*, j_t^*, d_t, W_{it}, m_{t+1} and k_{t+1} in order to maximize (10) subject to (3), (4), and

$$\frac{W_t}{P_t}h_{it}+r_tk_t-\tau_td_t+\frac{m_t+T_t}{P_t}\geq c_t^*+\frac{m_{t+1}}{P_t}+k_{t+1}+\frac{\phi}{2}\left[\frac{W_{it}}{\pi W_{it-1}}-1\right]^2+\gamma\left(1-j_t^*\right) \tag{12}$$

where $a_t=k_t-d_t$ and (6) have been substituted into (12).[7] Given optimal choices for these sequences, the household's labor supply (h_{it}) is given by (1).

Goods market clearing requires

$$y_t=c_t^*+i_t+\tau_td_t+\gamma\left(1-j_t^*\right)+\frac{\phi}{2}\left[\frac{W_{it}}{\pi W_{it-1}}-1\right]^2, \tag{13}$$

which states that output produced in period t must be distributed amongst consumption, investment, and total financial and wage adjustment costs.

The condition for currency market clearing is $m_t=M_t$, which states that the monetary authority only controls the amount of outside money available in the economy. Broader

[6] Given the parameterized model, it has been numerically verified that $[1+\tau_t+\gamma/2j^*c_t^*]\,r_t^{-1}-P_t/P_{t-1}$ is monotonically decreasing in j_t and equals zero for a single value $j^*\in(0,1)$.

[7] Substituting $(r_t-\tau_t)$ for $\tilde{r}_t$ does not alter the decentralization of the household's problem due to the household taking both r_t and τ_t as given.

monetary aggregates such as M1 are defined as the nominal sum of currency and deposits,

$$
\begin{aligned}
M1_t &= M_t + P_t D_t \\
M1_t &= M_t \left(1 + \frac{P_t D_t}{M_t}\right),
\end{aligned}
\tag{14}
$$

where the second expression defines M1 as the product of the currency base and the endogenously determined money multiplier.

The decision rules of the households, firms, and pricing functions can now be defined as functions of k_t, W_{t-1}, μ_t (exogenous or endogenous), the fundamental shock z_t, and the nonfundamental sunspot shock denoted by ζ_t. Therefore, for all $\{k_t, W_{t-1}, \mu_t, z_t, \zeta_t\}$, an equilibrium is defined as a list of prices $\{P_t, r_t, \tilde{r}_t, W_t\}$ and allocations $\{k_{t+1}, h_t, c_t^*, j_t^*, d_t, m_{t+1}\}$ such that: (i) households maximize (10) subject to (3), (4), and (12), (ii) firms maximize profits, (iii) labor demand is determined by (1), (iv) the markets for goods (13), currency, and deposits ($D_t = d_t$) clear, and (v) $\tau_t = \Gamma d_t^{\theta}$.

2.3. Intuition behind Indeterminacy

For intuition on how indeterminacy arises in the model, consider the depositing decision of a household when the financial intermediary exhibits IRS. Given that (6) is anticipated, a household decides how much consumption to purchase with their present currency holdings and how much to purchase with deposits. A strategic complementarity emerges from this decision: the more households choose to deposit, the higher the returns to using deposits (all else equal). Therefore, the size of the deposit market is determined in a non-cooperative fashion by the simultaneous choices of the (identical) households. For example, if the representative household believes the size of the deposit market will be large and the net returns on deposits high, the household will choose to hold more deposits resulting in smaller costs to managing deposits and effectively higher deposit returns. The expectation is supported by having decreasing marginal costs for the intermediary in managing deposits.

These *self-fulfilling* beliefs described above are the classic (banking crises) interpretation of sunspot shocks to the financial intermediary. In this environment, a belief-induced change in deposit holdings influences the price level due to the household's desire to purchase a different amount of consumption with currency. This change in the price level will impact the real economy through its interaction with nominal wage rigidity.

3. Quantitative Analysis

The model is log-linearized around its steady-state values and solved following Lubik and Schorfheide (2003). While leaving the details to an appendix, the system is cast in the canonical form of linear rational expectations models following Sims (2001),

$$\Xi_0 (\Theta) s_t = \Xi_1 (\Theta) s_{t-1} + \Upsilon (\Theta) \varepsilon_t + \Pi (\Theta) \vartheta_t,$$

where s_t is a vector of model variables, ε_t is a vector of exogenous shocks, and ϑ_t is a vector of expectational forecast errors [e.g. $k_{t+1} - E_{t-1} (k_{t+1})$]. $\Xi_0 (\Theta)$, $\Xi_1 (\Theta)$, $\Upsilon (\Theta)$, and $\Pi (\Theta)$ are coefficient matracies dependent upon a vector of model parameters Θ. The general form of the solution is given by

$$s_t = \Xi_1^* (\Theta) s_{t-1} + [\Upsilon^* (\Theta) + \Pi^* (\Theta) C] \varepsilon_t + \Pi^* (\Theta) D\zeta_t,$$

where $\Xi_1^* (\Theta)$, $\Upsilon^* (\Theta) + \Pi^* (\Theta) C$, and $\Pi^* (\Theta) D$ are matrices which govern the response of the choice variables to changes in the endogenous states (s_{t-1}), the fundamental shocks, and the reduced-form sunspot shock (ζ_t), respectively. Lubik and Schorfheide (2003) show that the complete set of solutions to the model is characterized by decomposing the expectational forecast errors into that attributable to fundamental and sunspot shocks: $\vartheta_t = \Upsilon^* (\Theta) \varepsilon_t + \Pi^* (\Theta) (C\varepsilon_t + D\zeta_t)$. If the model exhibits a unique equilibrium, then $C = D = 0$ and the methodology reduces to Sims (2001).[8] If the model exhibits indeterminacy, D comprises the influence of sunspots on the model dynamics and C is then used to consider the assumption of whether or not the influence of the fundamental and sunspot shocks on the endogenous forecast errors are orthogonal. Assuming *orthogonality* implies that the influence of fundamental and sunspot shocks on the expectational forecast errors are independent

[8] In other words, a stable solution exists if one can choose the expectation errors ϑ_t as a function of the exogenous ε_t to eliminate explosive components of s_t. The solution is unique if the mapping from ϑ_t to ε_t is one-to-one.

($C = 0$). Otherwise, sunspot shocks are assumed to alter the influence of fundamental shocks, and C is determined in order to impose *continuity* between the model's impulse responses to fundamental shocks at the boundary of the parameter space between determinacy and indeterminacy.[9] Imposing continuity ensures that the impact of fundamental shocks do not abruptly change when the economy transitions from determinacy to indeterminacy.

This section sets out by stating the functional form assumptions, model calibration, and alternative specifications of monetary policy to be used throughout the analysis. Given the parameterized model, a search is conducted over a small subset of the parameter space for zones where the model dynamics are either unique or indeterminate. The dynamic properties of the model under indeterminacy are then assessed by examining the impulse responses from fundamental and nonfundamental shocks under alternative monetary policy assumptions. The section concludes with a sensitivity analysis to show that the results presented here are insensitive to the parameter assumptions.

3.1. Functional Forms and Calibration

The functional forms and parameter values are determined according to the business-cycle literature (e.g. Cooley and Hansen, 1989) and so the resulting steady state of the model matches particular long-run properties of the US economy. All calibrated parameter values detailed below are summarized in Table 1.

The annual money growth rate ($\mu - 1$) is set to 3 percent, and the discount parameter β is calibrated so the annual real interest rate is 4 percent.

Investment is one quarter of steady state output. With a 10 percent depreciation rate, the capital stock to annual output ratio is 2.5. The production function is assumed to be $y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$, and α is calibrated so labor's share of national income is roughly two-thirds. The parameters governing the evolution of technology shocks ($\rho_z,\ \sigma_z$) are respectively set to 0.95 and 0.0076 as in Prescott (1986).

The utility function is assumed to be $\left[c_t^{*\eta} \left(1 - h_t\right)^{1-\eta}\right]^{1-V} / \left(1 - V\right)$. The parameter η is

[9] Since $\Upsilon^* \left(\Theta\right)$ is dependent on the parameterized model, C is calculated in order to minimize the distance between the impulse responces of the model to fundamental shocks in corresponding determinate and indeterminate equilibria. See Lubik and Schorfheide (2003) or the appendix for further details.

calibrated so a household's average allocation of time to market activity (net of sleep and personal care) is 0.3 which is in line with estimates of Ghez and Becker (1975). V is set to 2 which is within the range of results reported by Hansen and Singleton (1983) and Neely et al. (2001).

The parameter ξ is calibrated so the average mark-up of household labor type-i is five percent, as in post-war U.S. data (see Christiano et al., 2005). The cost parameter governing nominal wage changes (ϕ) is set to 6 which roughly corresponds to an average sticky wage duration of 3 quarters.[10]

The benchmark model assumes purely exogenous monetary policy which evolves according to (8) with ρ_μ and σ_μ respectively set to 0.32 and 0.0038 as in Christiano (1991) and Fuerst (1992). Three additional cases with variations of the endogenous monetary policy rule (9) are considered. For the first two cases, it is assumed that the monetary authority follows pure inflation targeting and sets $\varpi_\pi = 1.5$ and $\varpi_y = 0$. These two cases differ on the information available to the monetary authority. The first case is referred to as the partial information (PI) model and assumes the sunspot shock is not observable ($\zeta_t \notin \Omega_t$). The second case is referred to as the full information (FI) model and assumes $\zeta_t \in \Omega_t$. The final case extends the PI model by considering output targeting (the YT model) and follows Taylor's (1993) original parameterization with $\varpi_\pi = 1.5$ and $\varpi_y = 0.5$.

The three remaining parameters to be determined are associated with costs to financial intermediation: the check-writing cost (γ), and the parameters defining τ_t (Γ and θ). Since θ is central to delivering indeterminacy, it is treated as a free parameter and analyzed separately below. The remaining two parameters are pinned down so the model's steady state matches the US deposit-currency ratio and the value added of the financial intermediation sector. In the context of the model, the deposit-currency ratio is defined as dP/m and set to 9 following Dressler (2007).[11] Value added is defined in the model as total banking costs per unit of output $([\tau d + \gamma(1-j^*)]/y)$, and is used as a proxy for the size of the intermediary

[10] See Chugh (2006) for a mapping from Rotemberg-style costs to Calvo-style rigidity.

[11] This value is chosen by considering that two-thirds to three-quarters of the total U.S. currency base is held abroad (see Porter and Judson, 1996). This model differs from Dressler (2007) by abstracting away from (ten percent) fractional reserves, therefore the value used in that paper is ten percent of the value used here (i.e. 0.9 was considered a reserves - currency ratio). A similar measure was also considered by Freeman and Kydland (2000).

Table 1: Calibrated Parameter Values

Symbol	Description	Value
α	capital's share	0.3397
β	discount factor	0.9902
δ	depreciation rate	0.0241
ς	consumption's share	0.3773
V	risk aversion	2
ξ	labor elasticity	20
ϕ	wage cost parameter	6
γ	check-clearing cost	$9.47e^{-5}$
ρ_z	AR coefficient (z_t)	0.95
σ_z	standard deviation (z_t)	0.0076
ρ_μ	exog. AR coefficient (μ_t)	0.32^a
σ_μ	exog. standard deviation (μ_t)	0.0038^a
ϖ_π	endog. policy parameter	1.5^b
ϖ_y	endog. policy parameter	$0, 0.5^c$
θ	banking cost (curvature)	$-0.05, 0, -0.25^d$
Γ	banking cost (level)	$1.92e^{-2}, 1.96e^{-2}, 1.77e^{-2d}$

Notes: [a]Values under benchmark model
[b]Value for PI, FI, and YT model
[c]First (second) value for PI and FI (YT) models
[d]First value for benchmark, PI, and FI models,
second value for determinacy, third value for YT model.

sector. Diaz-Gimenez et al. (1992) compute a range of value added for subsectors of the financial intermediation sector as percentages of GNP (Table 3a). For the years 1970 to 1989, the value added from 'banking and credit agencies other than banks' was 1.8 to 2.7 percent. While one can easily argue that the value added of this subsector has probably increased over the last 18 years, it is not clear how much of this measure is represented by the simple structure of financial intermediaries in the model. Still, this information delivers a plausible size range of the financial intermediary which will be explored further in the following section.

3.2. Increasing Returns and Indeterminacy

In the decentralized economy, clearing of the deposit market results in the marginal costs of managing deposits be equal to

$$\frac{\partial C\left(d_t\right)}{\partial d_t} = \Gamma d_t^{\theta}. \tag{15}$$

Standard textbook models of banking (e.g. Freixas and Rochet, 1997) claim that for banks to exhibit economies of scale, the cost function must be concave and marginal costs must be decreasing. While $-1 < \theta < 0$ is sufficient for delivering IRS, it may not be sufficient for delivering indeterminacy in the quantitative environment. This is the case in the present framework because the bank is a small portion of the aggregate economy (due to value added).

Results from a numerical analysis of the equilibrium properties of the model over values of θ and the value added of the financial intermediary sector are illustrated in Figure 1. For the benchmark model with exogenous monetary policy, points in the shaded region (Zone 1) correspond to parameter values which deliver a unique (saddle-path stable) equilibria, while points in the non-shaded region (Zone 2) correspond to indeterminate equilibria. While keeping all other parameters consistent with the calibration section, the figure illustrates that slight changes to the size of the intermediary have a large impact on the minimum (absolute value) of θ required to deliver indeterminacy. For amounts of value added below 1.203 percent, indeterminacy requires $\theta < -1$ which would imply that the original cost function $\left[\Gamma d_t^{1+\theta} / \left(1+\theta\right)\right]$ was negative. For value added roughly between 1.203 and 1.328 percent, indeterminate equilibria arise for values of θ where total costs are positive and marginal costs are decreasing.[12]

Similar searches for indeterminacy were conducted for versions of the model with endogenous monetary policy. For the PI and FI models, the indeterminacy zone is identical to the benchmark model. When monetary policy is also targeting the real output gap (the YT model), the parameter space which delivers indeterminacy shrinks and is denoted in Figure

[12] For value added greater than 1.329 percent, the calibration methodology described above delivers negative values for either γ or Γ. Since the benefits to banking are small, there exists a negative relationship between the size of the bank and the parameters delivering value added.

1 by the area under the dashed line. The reason for this result stems from the response of real output to a sunspot shock, and the ability of the monetary authority to stabilize it. A more detailed explanation for this result will be given when comparing the model's impulse responses to the sunspot shock under alternative monetary policies.

The numerical analysis concludes that indeterminacy arises for a banking sector well within the size range determined by Diaz-Gimenez et al. (1992), and for many values of θ. Therefore, the quantitative analysis proceeds with a very conservative degree of IRS for the benchmark, PI and FI models and sets $\theta = -0.05$. The minimum value added which delivers indeterminacy under this degree of IRS is approximately 1.322 percent. In the case of the YT model, the same value added requires θ be roughly -0.25. Given these values, the resulting value of Γ is given at the bottom of Table 1. While these parameter values are used throughout the quantitative analysis, a sensitivity analysis below considers several points within the indeterminacy zone depicted in Figure 1 to confirm the robustness of the model predictions.

3.3. Model Results

Benchmark Model: Exogenous Monetary Policy

The analysis first considers the dynamic properties of the benchmark model with exogenous monetary policy. This version of the model is solved under the two alternative assumptions considered by Lubik and Schorfheide (2003): *orthogonality* which refers to the independent influence of the fundamental and sunspot shocks, and *continuity* which allows the shocks to interact when influencing the expectational forecast errors. Under continuity, Lubik and Schorfheide (2003) identify the joint impact of the shocks by minimizing the distance between the impulse responses in the model due to fundamental shocks under both determinacy and indeterminacy.[13] Since the choice of θ is close to the border between determinacy and indeterminacy, the result of this assumption is that the model's impulse responses to the fundamental shocks are equivalent to those under determinacy. Therefore, these impulse responses can conveniently be used as a comparison to assess not only the

[13] Given the parameter values discussed above, the corresponding determinacy result needed to compute the joint impact was chosen by setting $\theta = 0$. The resulting value for Γ is given in Table 1.

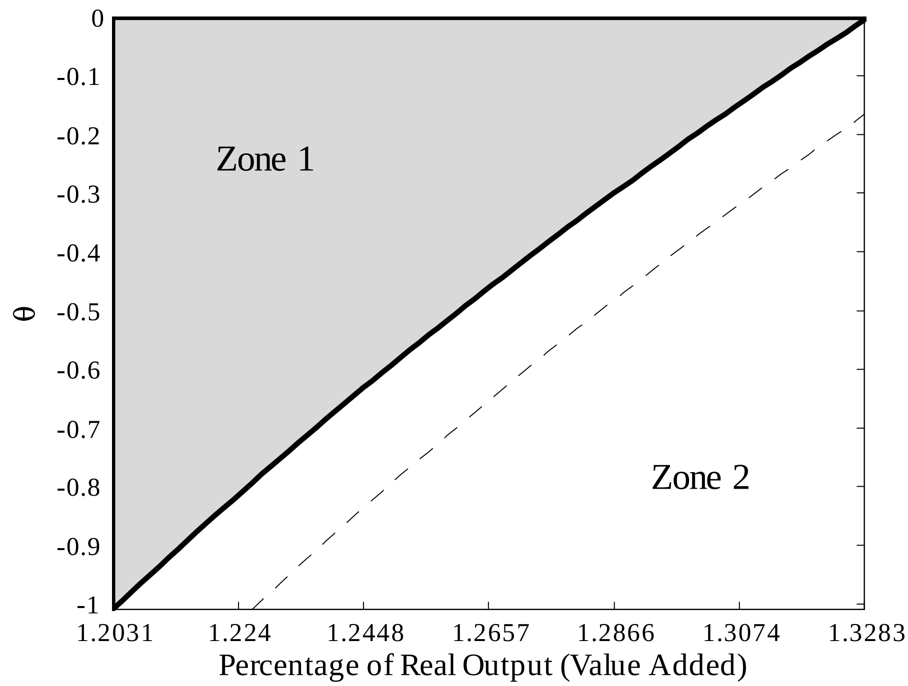

Figure 1: Zone 2 (1) corresponds to pairs of the value added to financial intermediation and θ that deliver indeterminate (saddle-path stable) equilibria for the benchmark, PI and FI models. The dashed line indicates the reduced indeterminacy region under the ouput-targeting monetary policy (YT) model.

effect of the sunspot shock on the economy, but the effect of IRS in banking on the impact of fundamental shocks.

The impulse responses of the benchmark model to positive (one-percent) monetary and sunspot shocks are illustrated in Figure 2. Consider first the series of events following an injection of currency (a positive monetary shock) to the household under the continuity assumption. Recall that the continuity assumption results in impulse responses that are equivalent to a model exhibiting determinacy. The increase of currency immediately increases the price level. According to (11), the increase in inflation makes deposits more attractive than currency (i.e. j_t^* decreases), and this shift towards deposit holdings further amplifies prices because more currency is used to purchase less consumption. Nominal wage rigidity implies that nominal wages increase less than prices, and real wages decline. The decline in real wages delivers an increase in the demand for labor and an increase in all other real aggregates. In the period following the shock, prices remain above steady state, along with the portion of consumption purchased with deposits (i.e. j_t^* remains below steady state). Real wages remain below steady state, so real aggregates remain above. Eventually, an increase in the demand for currency results in all nominal variables to return to their steady state values. Once the paths of prices and nominal wages align, the real wage rate returns to its steady state value along with all other real aggregates.

Under the orthogonality assumption, the initial impact to a monetary shock is qualitatively similar to the impact under the continuity assumption. Prices, M1, and j_t^* all illustrate that deposits are becoming more attractive to make consumption purchases. IRS in the intermediary implies that the shift towards increased deposits influences the net deposit rate $(r_t - \tau_t)$, and the initial impact of a monetary shock is diminished. In the following period, prices decline below steady state resulting in currency becoming more attractive. As households choose to hold less deposits, the net return to holding deposits declines. This results in a persistent shift away from deposits, illustrated by the persistent increase in j_t^* and the persistent decrease in M1. This persistence is only exhibited in nominal variables. Once nominal wages and prices align, the real economy returns to its steady state levels.

The final set of impulse responses in Figure 2 illustrates the impact of a positive one-percent sunspot shock. The sunspot shock has a significant real impact on the economy and qualitatively mirrors the real impact of monetary shocks. Quantitatively speaking, the real impact of a sunspot shock is approximately one-half the size of a monetary shock calculated under continuity and three-quarters the size under orthogonality. The reason for these similar predictions stems from the fact that the monetary and sunspot shocks both impact the economy through the households' portfolio choice of cash and deposit holdings. A sunspot shock induces agents to increase deposit holdings because of a perceived decrease in the cost to intermediating assets, resulting in deposits dominating currency for a larger portion of total consumption purchases (illustrated by the decline in j_t^*). The increase in deposit holdings results in an immediate increase in M1 and prices, while the nominal interest rate declines. The decline in real wages again increases the demand for labor and real aggregates. In the following period, the increase in deposit holdings keeps the net return high. This explains the persistence in Prices, M1, and j_t^*. Although nominal aggregates continue to remain far from steady state for several periods, nominal wages and prices eventually align so the real economy converges to its pre-shock state.

Figure 2 illustrates that the real impact of monetary and sunspot shocks are qualitatively similar, and exogenous monetary shocks differ under determinacy and indeterminacy. These results are particularly interesting because the decreasing marginal costs which deliver indeterminacy are *real* costs. The significance of a sunspot shock is felt through the presence of nominal rigidity and the inside money mechanism, just like monetary shocks. Since the size of the financial sector is small relative to the rest of the real economy, one might expect that IRS in the banking sector would not add much to real shocks. Figure 3 illustrates the impulse responses of the economy to a positive, one-percent productivity shock under both the continuity and orthogonality assumptions. As is common in business-cycle analyses, a positive technology shock increases real output as well as real and nominal interest rates. This make deposits more attractive than currency, so there is an endogenous increase in broad monetary aggregates. When comparing the responses of the real variables under the orthogonality and continuity assumptions, the top six panels of Figure 3 show that the model's responses are roughly identical. This supports the fact that although indeterminacy is introduced through a real cost, the impact of this mechanism lies in the nominal side of the economy. For nominal variables (the bottom six panels), the continuity and orthogonality

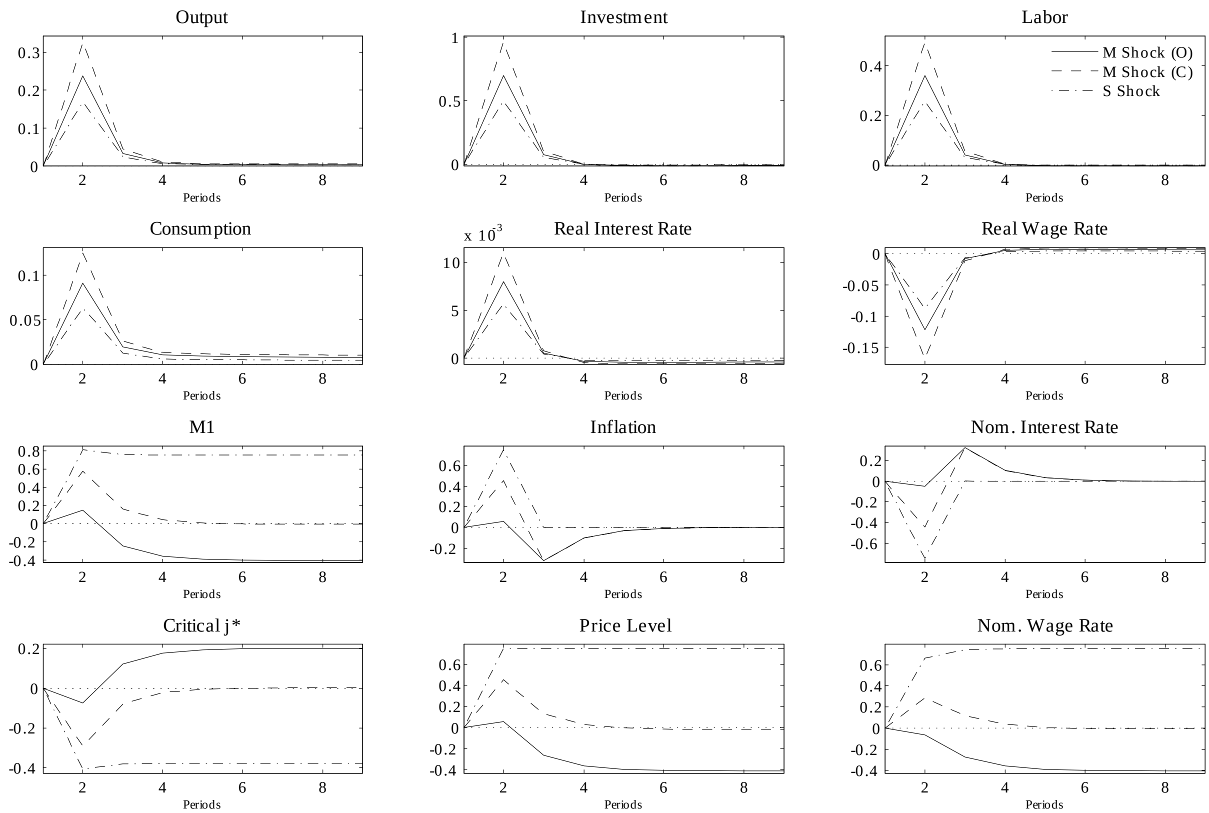

Figure 2: Impulse responses to a one percent increase in the monetary base (M Shock) and the reduced-form sunspot shock (S Shock). The Y-axes denote percentage changes from steady state. Impulse responses calculated under the orthogonality (continuity) assumption are denoted with O (C).

assumptions result in different impulse responses. Under orthogonality, IRS in banking implies a larger increase in deposits due to decreasing marginal costs. A larger portion of total consumption is purchased with these deposits, resulting in a smaller decline in prices because the household's previously held currency balance is used to purchase a smaller portion of total consumption. Nonetheless, the nominal movements have no noticeable impact on the real aggregates under either assumption.

Endogenous Monetary Policy

Since the model's response to sunspot shocks resemble the response to exogenous monetary shocks, what does the model predict for the impact of sunspot shocks under endogenous monetary policy? The impulse responses to a sunspot shock for the three remaining models

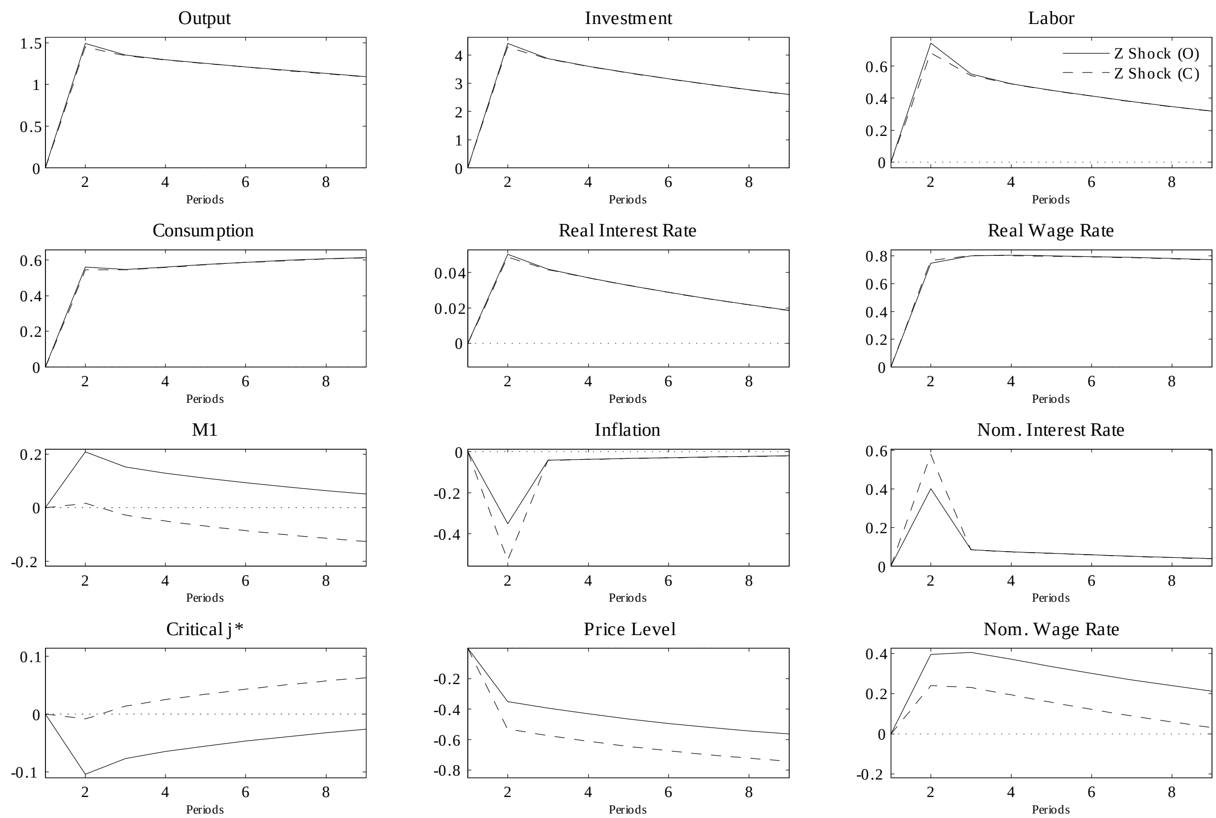

Figure 3: Impulse responses to a one percent increase in technology (Z Shock). The Y-axes denote percentage changes from steady state. Impulse responses calculated under the orthogonality (continuity) assumption are denoted with O (C).

(PI, FI, and YT) are compared with the benchmark model in Figure 4.[14]

Recall that the partial information (PI) model considers a pure inflation-targeting policy where the monetary authority is unable to observe the initial sunspot shock. In this scenario, the model's impulse responses are identical to the benchmark model. This is because the sunspot shock is idiosyncratic and $E_{t-1}\zeta_t = 0$. The immediate increase in inflation due to the sunspot shock makes the monetary authority completely unable to offset it in the impact period. Since this increase in inflation has no persistence, inflation returns to trend on its own before the monetary authority can act.

The full information (FI) model considers the same policy as the PI model, only now the monetary authority is able to observe the initial sunspot shock. In this case, the monetary authority is able to almost completely offset the real effects of the shock. Upon observing a sunspot shock, the monetary authority dampens the initial increase in prices, and therefore increases the persistence of inflation. The increased persistence in inflation results in a persistent shift towards deposits, and more importantly steers the path of prices to mirror the path of nominal wages. This results in the real wage remaining close to its steady state value, and the real impact of the sunspot shock through the nominal channel has been neutralized. The small departures of the real variables from their steady state values are due to the fact that the sunspot shock influences a real (albeit small) cost to managing deposits.

The final model considered (the YT model) is an extension of the PI model which includes output-gap targeting. The inability to immediately observe the sunspot shock results in the real impulse responses of the model to look qualitatively identical to the PI model. However, the monetary authority is now able to slightly diminish the initial impact of the shock on real variables by slightly diminishing the increase in prices (which in turn slightly diminishes the decline in real wages). The persistence in output resulting from the idiosyncratic sunspot shock gives the monetary authority information relevant for subsequent periods, which explains the slight increased persistence in inflation. While this result suggests that the monetary authority can partially offset a sunspot shock by targeting the output gap, it should be noted that Taylor's (1993) original value for ϖ_y used here (0.5) is much larger

[14] Restricting attention to sunspot shocks implies no distinction between the orthogonality or continuity assumptions - the assumption only impacts fundamental shocks.

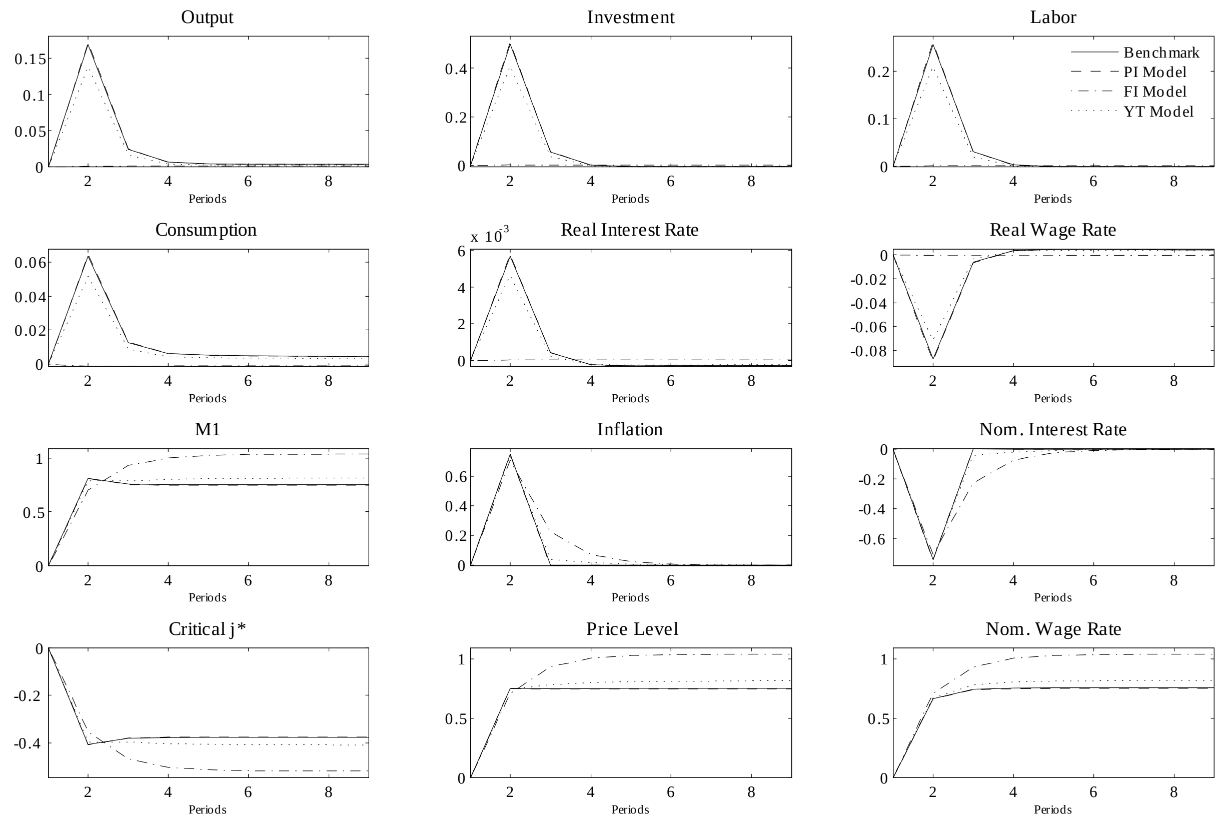

Figure 4: Impulse responses to a one percent increase in the reduced-form sunspot shock under different specifications of monetary policy. The Y-axes denote percentage changes from steady state. Benchmark refers to the model with exogenous monetary policy. The PI (FI) model refers to a pure inflation-targeting policy where the monetary authority is unable (able) to observe the sunspot shock. The YT model refers to an extension of the PI model by including output-gap targeting.

than estimated values from the literature.[15]

3.4. Sensitivity Analysis

The quantitative results presented above are dependent upon several model assumptions. The most important of these are the degree of IRS in the intermediary (θ), and the size of the intermediation sector (quantified by value added).

To get some sense of where the value of θ lies in the data, equations (6) and (7) from the

[15] For example, different empirical exercises conducted by Clarida et al. (1999) and Dressler (2007) both deliver estimates of ϖ_y which are not significantly different from zero.

Table 2: Increasing Return Estimates

Data	θ	R^2
1959:1-2006:4	-0.8666 (0.3139)	0.42
1959:1-1979:1	-5.6641 (2.5143)	0.57
1986:1-2006:4	-0.3024 (0.1574)	0.36

Notes: Standard errors are in parentheses.

financial intermediary's problem can be combined to deliver a simple expression identifying θ as

$$\log(\tilde{r}_t - r_t) = \log(\Gamma) + \theta \log(D_t). \tag{16}$$

The left-hand side of (16) is the logged spread between real lending and deposit rates, while the right-hand side is the log-linearized version of (7). The results for estimating (16) over US post-war data are presented in Table 2.[16] The table considers the full post-war sample, as well as subsamples of the data which excludes several years after 1979 when the documented change in Federal Reserve policy procedure and the most volatile interest rate movements are observed. For all cases, considering up to two lagged dependent variables was sufficient to render white noise residuals. For the full data sample, θ is estimated to be around -0.87 and is significantly less than zero. This estimate is much lower in the earlier subsample (-5.66), but not significantly different than the full-sample estimate at the 95 percent confidence level. The estimate of θ in the later subsample is significantly higher than the full-sample estimate (-0.30), but still significantly less than zero at the 90 percent confidence level. While this simple exercise is far from concrete evidence confirming IRS in the financial intermediary sector, it does suggest that assuming marginally decreasing cost functions for financial intermediaries is not completely inconsistent with the data.

[16] The spread between lending and deposit rates was taken to be the spread between the prime lending rate (series name: MPRIME) and the 3 month Tbill rate (series name TB3MS), while real deposits were defined as the sum of M1: demand deposits and M1: other checkable deposits (series names: DD.US and OCD.US) deflated by the GDP deflator (series name: GDPDEF). The annualized interest rate data was transformed into gross, monthly rates to make them consistent with the model, and a non-linear trend was removed from both variables using the HP filter. All monthly data was transformed to quarterly by taking three-month averages. The data sample is from 1959:1 to 2006:4, and is available from the Board of Governors of the Federal Reserve System.

To assess the stability of the model results, the model was analyzed under several points within Zone 2 of Figure 1: (Case 1) an extremely small degree of IRS ($\theta = -0.01$) and value added equal to 1.328 percent, (Case 2) the degree of IRS estimated in (16) using the full post-war data sample ($\theta = -0.8666$) and value added equal to 1.21 percent, and (Case 3) the same high degree of IRS as in Case 2 ($\theta = -0.8666$) with value added equal to 1.322 percent as in the benchmark calibration. These three cases roughly span the entire indeterminacy zone where $\theta \in (-1, 0)$.

Figure 5 compares the (orthogonal) impulse responses of several variables to a technology shock (left column), monetary shock (middle column), and the sunspot shock (right column). The nominal variables (M1, aggregate prices, and nominal wages) were chosen because these variables change the most across the three shocks, while output was chosen to illustrate the stability of the real economy. There is a large degree of (short-run) stability in the impulse response functions of the model. Across the four cases (including the benchmark case), there are no noticeable differences in the real or nominal responses to a monetary or sunspot shock. When looking at the model's response to a technology shock, there are no changes in either the real variables or the nominal variables in the initial periods. However, for cases which consider a larger degree of IRS in financial intermediation, the reduced cost to intermediation results in a continued rise in deposit holdings, and a continued decline in aggregate prices. The decline in prices is larger than the rise in deposits, which results in the decline in M1. The real effect of the persistent decline in prices is offset by an equivalent decline in nominal wages, which explains the equivalence in the real responses.

4. Conclusion

The goal of this paper was to analyze the economic effects of indeterminacy resulting from IRS in the financial intermediation sector. Empirical evidence of increasing returns in banks of all sizes are provided by Hughes and Mester (1998), and have not been previously considered as a source for commonly occurring, business-cycle fluctuations. The environment is a cash-in-advance framework extended to feature financial intermediaries which have the potential to exhibit increasing returns through decreasing marginal costs to

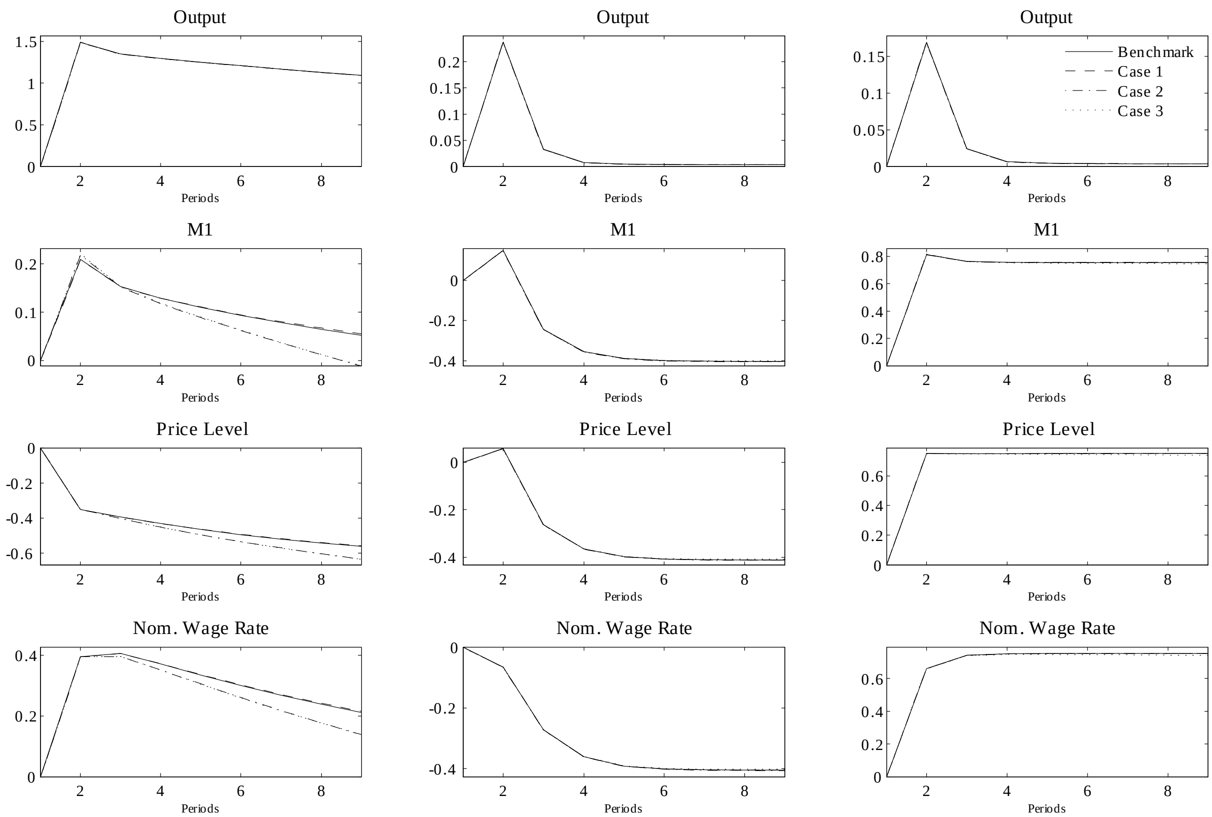

Figure 5: Impulse responses to a one percent increase in technology (left column), the monetary base (middle column), and the reduced-form sunspot shock (right column). The Y-axes denote percentage changes from steady state. The benchmark model uses $\theta = -0.05$ and value added (VA) = 1.322 percent. Case 1 sets $\theta = -0.01$ and VA = 1.328 percent. Case 2 sets $\theta = -0.8622$ and VA = 1.21 percent. Case 3 sets $\theta = -0.8622$ and VA = 1.322 percent.

managing deposits. Although the size of the financial intermediary sector is calibrated to be approximately 1-2 percent of value added to output, the model illustrates: (i) indeterminacy arises for small degrees of IRS and standard parameter assumptions; (ii) sunspot shocks have significant effects due to the money multiplier; (iii) idiosyncratic sunspot shocks qualitatively resemble exogenous monetary shocks; and (iv) although endogenous monetary policy can increase the degree of IRS needed for indeterminacy when targeting real output, policy can only stabilize the real effects of sunspot shocks under complete information.

One immediate question resulting from the above analysis is how do the fundamental and sunspot shocks interact and influence business-cycle moments? A formal answer to this question is beyond the scope of the present analysis because it would require a structural estimation of key model parameters such as those governing endogenous monetary policy, the laws of motion of the fundamental shocks, and of course the degree of IRS in financial intermediaries as well as the volatility of the resulting sunspot shocks. Since the model predictions presented above are robust to changes in key parameters that are all within plausible ranges, it is not immediately clear how the parameterization of the model will influence business-cycle predictions or how this parameterization will change over specific episodes of US data. Furthermore, since sunspot shocks qualitatively resemble monetary shocks, additional care must be taken to properly identify them in the data. The results presented here suggest that sunspot shocks stemming from the financial intermediary can have significant effects on the economy, and these remaining empirical questions are presently being explored.

Appendix: Model Solution

The solution methodology described in this appendix follows Lubik and Schorfheide (2003) and their extension of Sims (2001). After removing all multipliers from the household's first order conditions and imposing symmetry, the normalized system of equations comprising the dynamic solution are given by

$$u_{ct}\Psi_t - \beta E_t \frac{P_t}{P_{t+1}\mu_{t+1}} u_{ct+1}\left(1 + \frac{\gamma}{2j^*_{t+1}c^*_{t+1}} + \Gamma d^\theta_{t+1}\right)\Psi_{t+1} = 0$$

$$u_{ct}\Psi_t - \beta E_t r_{t+1} u_{ct+1}\Psi_{t+1} = 0$$

$$u_{ht}\xi h_t + u_{ct}\Psi_t\left[(1-\xi)\frac{W_t h_t}{P_t} - \phi\left(\frac{\mu_t W_t}{\pi W_{t-1}} - 1\right)\frac{\mu_t W_t}{\pi W_{t-1}}\right] - ...$$

$$\beta E_t \Psi_{t+1}\phi\left(\frac{\mu_{t+1}W_{t+1}}{\pi W_t} - 1\right)\frac{\mu_{t+1}W_{t+1}}{\pi W_t} = 0$$

$$z_t = \kappa_z + \rho_z z_{t-1} + \varepsilon_{zt}$$

$$\mu_t = \kappa_\mu + \rho_\mu \mu_{t-1} + \varepsilon_{\mu t}$$

$$z_t k_t^\alpha h_t^{1-\alpha} + (1-\delta)k_t = c_t + k_{t-1} + \phi\left(\frac{\mu_t W_t}{\pi W_{t-1}} - 1\right)^2 + \Gamma d_t^{1+\theta} + \gamma(1 - j^*_t)$$

$$\frac{1}{P_t} = j_t^{*2} c^*_t$$

$$d_t = \left(1 - j_t^{*2}\right)c^*_t$$

$$r_t = \alpha z_t \left(\frac{h_t}{k_t}\right)^{1-\alpha} + 1 - \delta$$

$$\frac{W_t}{P_t} = (1-\alpha) z_t \left(\frac{k_t}{h_t}\right)^\alpha$$

where $\Psi_t = \left[1 + \frac{\gamma j^*_t}{2c^*_t} + \Gamma d^\theta_t\right]^{-1}$. After the above system is log-linearized around the model's steady state, the dimension of the system is reduced by using the bottom five equations to remove $\{c^*_t, h_t, j^*_t, r_t, d_t\}$. The remaining five equations (and six identities) comprise the linear rational expectations model and can be represented in the canonical form:

$$\Xi_0 s_t = \Xi_1 s_{t-1} + \Upsilon\varepsilon_t + \Pi\vartheta_t \quad (17)$$

where

$$\begin{aligned} s_t &= \left[\begin{array}{c} k_{t+1}, k_t, W_t, W_{t-1}, P_t, P_{t-1}, z_t, \\ \mu_t, E_t(k_{t+2}), E_t(W_{t+1}), E_t(P_{t+1}) \end{array}\right]' \\ \varepsilon_t &= [\varepsilon_{zt}, \varepsilon_{\mu t}]' \\ \vartheta_t &= [k_{t+1} - E_{t-1}(k_{t+1}), W_t - E_{t-1}(W_t), P_t - E_{t-1}(P_t)]' \end{aligned}$$

Solving the model requires the use of the generalized Schur decomposition (QZ) of Ξ_0 and Ξ_1. This results in matrices Q, Z, Λ and Ω such that $QQ' = ZZ' = I_n$, Λ and Ω are upper triangular, and $\Xi_0 = Q'\Lambda Z$ and $\Xi_1 = Q'\Omega Z$. Defining $\varpi_t = Z's_t$, premultiplying (17) by Q results in

$$\left[\begin{array}{cc} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{array}\right]\left[\begin{array}{c} \varpi_{1t} \\ \varpi_{1t} \end{array}\right] = \left[\begin{array}{cc} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{array}\right]\left[\begin{array}{c} \varpi_{1t-1} \\ \varpi_{1t-1} \end{array}\right] + \left[\begin{array}{c} Q_{1\cdot} \\ Q_{2\cdot} \end{array}\right](\Upsilon\varepsilon_t + \Pi\vartheta_t)$$

where, without loss of generality, the system has been partitioned such that the lower blocks of Λ, Ω and Q correspond to the portion of the system delivering unstable eigenvalues. In other words, the lower block contains all equations in which the ratio between the diagonal elements of Ω and Λ are greater than unity.

This 'explosive' block is written as

$$\varpi_{2t} = \Lambda_{22}^{-1}\Omega_{22}\varpi_{2t-1} + \Lambda_{22}^{-1}Q_{2\cdot}\left(\Upsilon\varepsilon_t + \Pi\vartheta_t\right).$$

A non-explosive solution of the model requires $\varpi_{2t} = 0 \forall t \geq 0$. This is accomplished by choosing $\varpi_{20} = 0$ and for every vector ε_t the endogenous forecast error ϑ_t that satisfies

$$\Upsilon^*\varepsilon_t + \Pi^*\vartheta_t = 0 \tag{18}$$

where $\Upsilon^* = Q_{2\cdot}\Upsilon$ and $\Pi^* = Q_{2\cdot}\Pi$. If the number of endogenous forecast errors is equal to the number of unstable eigenvalues, then (18) uniquely determines ϑ_t. If the number of endogenous forecast errors exceeds the number of unstable eigenvalues, then the system is undetermined and sunspot fluctuations can arise.

Using the singular value decomposition $\Pi^* = UDV'$, a general solution for the endogenous forecast errors is given by

$$\vartheta_t = \left(-V_{\cdot 1}D_{11}^{-1}U_{\cdot 1}'\Upsilon^* + V_{\cdot 2}M_1\right)\varepsilon_t + V_{\cdot 2}M_2\zeta_t$$

where M_1 and M_2 govern the influence of the sunspot shock.

Assuming Ξ_0^{-1} exists, the solution of the model takes the form

$$s_t = \Xi_1^* s_{t-1} + \left[\Upsilon^* - \Pi^* V_{\cdot 1}D_{11}^{-1}U_{\cdot 1}'\Upsilon^*\right]\varepsilon_t + \Pi^* V_{\cdot 2}\left(M_1\varepsilon_t + M_2\zeta_t\right)\right]. \tag{19}$$

Setting $M_2 = 1$ results in the interpretation of ζ_t as a reduced-form sunspot shock. Determining the value for M_1 requires choosing one of two alternative identification schemes. If one assumes that the effects of fundamental and nonfundamental shocks on the forecast error are *orthogonal* to each other, then $M_1 = 0$. Otherwise, M_1 is chosen such that the impulse responses of the model $(\partial s_t/\partial\varepsilon_t)$ are *continuous* at the boundary between the determinacy and indeterminacy regions. Under indeterminacy, the impulse response is given by

$$B_1 + B_2M_1 = \left(\Upsilon^* - \Pi^* V_{\cdot 1}D_{11}^{-1}U_{\cdot 1}'\Upsilon^*\right) + \Pi^* V_{\cdot 2}M_1.$$

For a corresponding determinacy solution, the impulse response is given by

$$\tilde{B}_1 = \tilde{\Upsilon}^* - \tilde{\Pi}^*\tilde{V}_{\cdot 1}\tilde{D}_{11}^{-1}\tilde{U}_{\cdot 1}'\tilde{\Upsilon}^*$$

where a tilde denotes the fact that a different point in the parameter space is needed to alter the model dynamics. To get the indeterminate impulse responses as close as possible to the determinate ones, M_1 is computed by applying the least squares criterion

$$M_1 = \left[B_2'B_2\right]^{-1}B_2'\left[\tilde{B}_1 - B_1\right].$$

This result is substituted in (19) with $M_2 = 1$.

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