preliminary and incomplete

Using Parameter Instability to Test Economic Theories

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Abstract: This paper develops new methods for testing structural hypotheses based on tests for parameter instability. It is shown that when the structure is of a particular type and its parameters change, predictable patterns of stability and instability in certain empirical relationships will result. These empirical characteristics are contingent on the type of structure involved. The paper develops different procedures for testing exogenous, recursive or long-run neutral structural relationships. And since the results are necessary and sufficient conditions, measurement could precede theory; Patterns of stable and unstable empirical relationships could be used to determine if these three types of structure are consistent with the evidence.

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Introduction

Testing macroeconomic theories is a fairly complicated task. Of course, that hasn't stopped economists from trying, but there is not a consensus view about which methods are successful in this endeavor. One significant problem is that macroeconomic experiments are prohibitively costly to run. As is commonly done in other non-experimental fields of study, empirical macroeconomic research typically uses a theory to select an empirical specification, then runs a regression to estimate the model and finally sees if parameter estimates have the correct sign, and perhaps reasonable magnitudes, according to the theory in question.

A well-known problem with this approach is that correlation does not necessarily imply causation. While one theory may predict that the dependent variable is caused by the explanatory variable, it also possible that theory implies reverse causation --- the case in which an explanatory variable is caused by the dependent variable. Thus the coefficient may be large and statistically significant even when causality runs in the opposite direction. And it is difficult, if not impossible, to use the standard regression results to address reverse causation issues. Economists sometimes use economic theory to argue against a reverse causation explanation for a result. But economics is often ambiguous about the direction of causal relationship — one theory may make one claim about causal structure while another theory makes a different claim — which can allow people with very different views of the world to sometimes come to opposing conclusions about what the evidence says. An example of this would be the old debate about why money and output are positively correlated ....

Recent research has been developing new methods that may allow us to empirically determine directions of causal influence. One of these approaches is based on tests for structural breaks. Structural break tests are becoming ever more powerful and increasingly useful in empirical economics. Such breaks reflect changes in the parameters of a model and may serve as natural experiments. If we estimate a fixed parameter model over a period in which a break occurs, then, in general, inconsistent estimates for all the structural parameters are obtained. Consistent estimates of the structure are essential if we want to use an empirical model to address questions about economic structure or if we want to correctly assess the consequences of alternative policies. Accounting for breaks might also improve the forecasting performance of an empirical model.

The term "structural break tests" is, however, a bit misleading. These tests can be applied as easily to a structural model as to a reduced-form (i.e. non-structural) model. I believe that these tests have actually been applied more often to reduced-forms models. On the other hand, Hoover (2001 and in various papers) has developed one of the first truly "structural" applications of the structural break tests. As Stock and Watson (1996) point out, structural breaks are a rather common occurrence. However, Hoover finds that if the causal relationship between two variables only goes in one direction, and the structural process for one of these variables undergoes a structural break, then certain empirical distributions will exhibit instability and certain other distributions will remain stable in the face of this structural break. His idea can be used to test if causation runs one way, or if it runs both ways or if there is no causation at all between a pair of variables.

My work builds on Hoover's pioneering efforts. However, there are important differences between my work and previous research. My methods can analyze systems with an arbitrary number of variables or equations while previous studies are limited to testing structural relationships between only two variables. I develop results for systems of structural equations that can be written in terms of blocks of equations whereas prior results apply to individual equations. My results encompass earlier ones, are significantly more general and in some cases are fundamentally new.

So what exactly are the identification results? The basic ideas in two primary theorems are: (1) If the structure is block recursive and breaks occur in one or more equations from a particular block, then certain empirical distributions are stable; and (2) If certain empirical distributions are stable in the face of instability in some structural equations, then the structure is block recursive.

My analysis has many applications because the theoretical results investigate block recursive structures with an arbitrary number of variables in each block and where the number of blocks is unlimited. Far from being a narrow class of structures, block recursiveness is pervasive in economics. It is possible that a majority of economic theories fall into this class, but even if not the majority, block recursive structures are common to virtually all fields of economics. Another reason for wide applicability comes from the fact that my identification results apply to models with time series, panel or cross sectional data. A new method for testing exogeneity is also developed.

I present two examples of how the results can be applied to structural vector autoregressions. Structural VARs are a very popular tool in empirical macro. A limitation to that line of research is that the theories used to motivate identification restrictions are often assumed true without being subjected to rigorous tests. This important gap in the literature is what my research intends to fill. One important topic that I will investigate is the way that monetary policy affects the economy. This question has received a lot of attention in the literature. Almost all of the key papers in this research have made the assumption that the structure is block recursive, although rarely have two papers used precisely the same block recursive structure. Hence, the models imply different stability and instability relationships following a break in a structural equation, and this will allow us to test the models against one another, even when these models are not nested. A natural place to look for breaks is in the monetary policy process. Plenty of institutional evidence can be found which is consistent with changing policy rules. Of course, parameters in other structural equations may also undergo a change. Most of the theories in this area are just-identified, meaning a theory that does not impose testable restrictions on the data. An interesting implication of my research is that structural breaks can make a just-identified structural model testable. I show how to apply the techniques developed here to testing the identification assumptions in Christiano, Eichenbaum and Evans (1996, 2000).

A second area of interest is theories that address long-run structural assumptions. In contrast, the monetary policy research has typically used short-run identification assumptions. There are a number of long-run neutrality propositions that have emerged from economic theory. For example, permanent changes in the level of the money supply are neutral in the long run with respect to output and many other real variables in most economic theories. Permanent changes in the growth rate of the money supply are also long-run neutral for output and some other real variables, although this sort of neutrality is sensitive to certain theoretical assumptions. Researchers have frequently used neutrality assumptions to identify a structural model, but formal tests of the neutrality hypothesis have almost never been done. Long-run neutrality typically yields a long-run block- recursive structure, and I use this example to illustrate how to use structural breaks to test this hypothesis.

Why are tests for structural breaks useful?

Structural breaks provide a way to model permanent changes in the economy

Unit roots are a popular alternative way of modeling permanent changes

Here permanent shocks occur in every period

Structural breaks are modeled as less frequent permanent shocks

If permanent shocks do not occur every period, we should obtain better estimates of the effects of permanent changes by modeling them as infrequent events. (Balke and Fomby JME)

If a structural break occurs within the sample period over which a fixed parameter model is estimated, inconsistent estimates are obtained

To get consistent estimates of all the structural parameters requires:

breaking the sample into subsamples with stable parameters;
or
specifying the actual way structural parameters vary over time

Consistent estimates of the structure are usually required if one wants to use an empirical model to:

address questions about economic structure;
or
conduct a valid analysis of alternative policies

Appropriate accounting for breaks might also lead to improved forecasts

In a sequence of papers and a 2001 book Kevin Hoover develops a method for using breaks in empirical distributions to infer causal direction

Changes in structural parameters essentially serve as natural experiments
My "paper" builds on Hoover's pioneering efforts

How is my work different from Hoover's research?

I extend the analysis to multivariate system of equations;

I develop new and general results for blocks of structural equations;
Results are applicable to cross-sectional, panel or time series models.

I show how these results can be applied to dynamic structural models:

Structural VAR models based on:

Short-run identification assumptions;

Long-run identification assumptions.

I develop a new method for testing "structural exogeneity"

Testing for Structural Exogeneity

Granger Causality testing is a commonly used procedure in empirical work

Granger, Sims, Geweke, etc. have developed tests

Granger has been careful to explain what his test can accomplish:

The Granger Causality test is a test for marginal predictive content, not necessarily a test for exogeneity

But if a theory implies patterns of predictiveness and/or non-predictiveness, Granger Causality tests may be used to test a structure.

In spite of Granger's warnings and the caveats of many other researchers, it is not too difficult to find work that draws excessively strong structural conclusions from the test results.

Important limitations with Granger Causality as a test for exogeneity include:

the test is performed on a reduced form model and

it is likely that the model omits some relevant variables.

The problem with using a reduced-form model:

Even if there are not any omitted variables, failure to Granger Cause is necessary, but not sufficient for exogeneity

Necessary: If a variable is exogenous to another variable the second variable will not be Granger Caused by the first variable

Not Sufficient: A configuration of structural parameters could force the reduced-form coefficients of one variable in another variable's equation to be zero. (Hoover 2001, provides a good policy example)

And even if these coefficients are not made precisely zero, sampling error could make them statistically indistinguishable from zero.

The multivariate extension of Granger Causality is "block exogeneity"

Which tests that a block of reduced-form coefficients is equal to zero.

This test is subject to similar concerns as with Granger's test

Definition: "structural block exogeneity" means a block recursive system of structural equations,

Failure to reject "block exogeneity" is necessary, but not sufficient for there to be "structural block exogeneity"

In practice, Granger causality, or its multivariate extension, is in virtually all cases applied to a subset of the relevant macro variables:

In this case failure to Granger Cause is neither necessary or sufficient for exogeneity.

For example, one variable could Granger Cause a second variable, even if it has no structural effect, because the model has an omitted explanatory variable that is correlated with the Granger causing variable.

Alternatively, a variable may fail to Granger Cause another variable, even if it is an important causal factor, because the empirical relationship is confounded by a particular choice of included/omitted variables (e.g. Simpson's Paradox)

The Results

Suppose the economy is written as a linear structural VARMA:

$$A(L)X_t = C(L)\varepsilon_t$$

X=vector of variables,

ε=vector of structural shocks,

A(L) and C(L) are structural parameters:

$$A(L) = A_0 + A_1 L + A_2 L^2 + ... + A_p L^p$$

and $$C(L) = C_0 + C_1 L + C_2 L^2 + ... + C_q L^q$$

with arbitrary finite values for p and q and unconstrained A_i and C_i matrices for all i

Zellner and Palm (1974) show that this multivariate system of structural equations can be transformed into:

$$|A(L)| X_t = A^+(L)C(L)\varepsilon_t$$

where $|A(L)|$ is the determinant of A(L)

and

$A^+(L)$ is the transpose of the cofactor matrix of A(L)

This equation implies a univariate time series model for each variable in X.

In general, all parameters from every structural equation in the multivariate system is a factor in each univariate representation.

Zellner and Palm provide the intuition for why Stock and Watson (1996) were not surprised to find wide-spread evidence of parameter instability in their tests with univariate and bivariate time series models

But the prevalence of instability does not necessarily imply that structural breaks have occurred in many different sectors of in the economy: A break in one structural equations could, in principle, cause every univariate distribution to exhibit parameter instability.

Definition: A distribution is <u>stable</u> if its parameters are invariant to change when a break occurs in a particular structural equation.

Now consider a bivariate structural VARMA with money (m), output (y) and structural shocks to the money supply (ε_m) and the real economy (ε_y):

$$\begin{bmatrix} A_{mm}(L) & A_{my}(L) \\ A_{ym}(L) & A_{yy}(L) \end{bmatrix} \begin{bmatrix} m_t \\ y_t \end{bmatrix} = \begin{bmatrix} C_{mm}(L) & C_{my}(L) \\ C_{ym}(L) & C_{yy}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{mt} \\ \varepsilon_{yt} \end{bmatrix}$$

It is easy to solve for m in the first equation and eliminate m in the second equation to obtain the univariate expression for y:

$$\rho_y(L)y_t = \alpha_{ym}(L)\varepsilon_{mt} + \alpha_{yy}(L)\varepsilon_{yt}$$

where: $\rho_y(L) = A_{yy}(L) - A_{ym}(L)A_{mm}(L)^{-1}A_{my}(L)$

$$\alpha_{ym}(L) = C_{ym}(L) - A_{ym}(L)A_{mm}(L)^{-1}C_{mm}(L)$$

$$\alpha_{yy}(L) = C_{yy}(L) - A_{ym}(L)A_{mm}(L)^{-1}C_{my}(L)$$

Note: Solving in this way allows y and m to be vectors.

Later we will make use of this generalization

Suppose there is a change in the money supply process:

$A_{mi}(L)$ and $C_{mi}(L)$ for i= m,y and the distribution for ε_m are all subject to change

Changes in these parameters will induce changes in the univariate y process

Unless $A_{ym}(L) = C_{ym}(L) = 0$.

In this case, y becomes: $A_{yy}(L)y_t = C_{yy}(L)\varepsilon_{yt}$

This condition which guarantees the univariate output process is immune to changes in the money supply equation's parameters is that output is exogenous to the money supply process

On the other hand, if output is not exogenous to money, then a change in any money supply parameter will cause instability in the univariate y process

Conclusion: Univariate tests of parameter instability can be used to test for structural exogeneity

Example: Money is always neutral in traditional RBC models, We can test these models with this structural break method.

The Multivariate Extension of the Result:

Reinterpret m and y as vectors of nominal and real variables, respectively, in the preceding structural model.

All previous equations continue to hold, except now each shock is a vector of shocks and each scalar lag polynomial is a matrix of lag polynomials.

Thus the same analytical result obtains: if $A_{ym}(L) = C_{ym}(L) = 0$, the vector process for y becomes

$$A_{yy}(L)y_t = C_{yy}(L)\varepsilon_{yt}$$

and so the VARMA process for y would not exhibit parameter instability.

Parameter stability tests could, in principle, be applied to the VARMA for y, but a less cumbersome approach exists:

Since each univariate representation of a variable in y is derived from the previous system of equations, each variable in y has a stable univariate representation

A direct consequence of the Zellner and Palm (1974) result

Conclusion: Univariate tests for parameter instability are able to test for structural block exogeneity.

Structural change in the univariate process for any $M \in m$ coming from changes in money supply will not be found in the univariate process for any $Y \in y$ if and only if y is structurally block exogenous to m

Omitted variables do not create a problem for this test.

Analogously we could test for M being exogenous to Y

Of course, we could reinterpret m and y according to any theory that implies exogenous structural relationships.

The method is summarized in the following table:

Test for a Parameter Instability in M and Y Equations, with $M \in m$ and $Y \in y$

	Break in the M Process	**No Break in M**
Break in the Y Process	***Inconclusive***	***M is Exogenous to Y***
No Break in Y	***Y is Exogenous to M***	***Uninformative***

If there are no breaks the approach can not be used and some other means of testing for exogeneity would be required.

A brake in one univariate process, but not the other, means that the variable with the stable process is exogenous to the variable with the unstable process.

When breaks occur in both variables the test alone is inconclusive because it is unable to determine whether or not there is exogeneity.

> However, if we can draw from other sources of information we might be able to interpret causal direction from these test results. Hoover and others call this additional information "narrative evidence".
>
> If the breaks appear at the same time for the M and Y equations, and if the timing of these breaks coincides with known changes in a particular structural equation that would indicate a causal direction.
>
> > For example, suppose coincident breaks are associated with change in the money supply process. This finding of coincident breaks implies output is responding to changes in money supply:
> >
> > > Conclusion: Y is Not Exogenous to M.
>
> Breaks in M and Y equations may not be coincident. That evidence may be consistent with the two variables being mutually exogenous.
>
> A finding that some breaks are coincident and some are non-coincident could be difficult to interpret.
>
> Cautionary note on the timing of breaks: Breaks may not line up exactly in time due to structural delays. E.g. if output reacts to money with a lag, a break in the money supply equation could cause instability in the univariate M process to occur at an earlier point in time than it occurs in univariate representation for Y.

Can these findings be used to interpret the existing body of evidence from structural break tests?

By the way: The result still holds under certain conditions when the structural VARMA has expected future variables in it. For example, if the structural equation for y also had expected future y and expectations are based on all currently available information, then the conclusions would not be changed.

Hoover's Approach:

> Source: *Causality in Macroeconomics,* 2001, Cambridge University Press

Suppose G causes T, but T does not cause G:

$$T \Leftarrow \alpha G + \varepsilon \text{ where } \varepsilon \sim N(0, \sigma_\varepsilon^2)$$

$$G \Leftarrow \beta + \eta \text{ where } \eta \sim N(0, \sigma_\eta^2)$$

β and α are parameters and $E\varepsilon\eta = 0$.

Notice that Hoover uses "$\Leftarrow$" instead of "$=$",

This new operator is a one-directional equals sign

causation has a direction, it's a one-way relationship,

The standard equals sign works in both directions

The structural mechanism for T is: $T = \alpha G + \varepsilon$ and while

G is not caused by T, $G = \dfrac{T - \varepsilon}{\alpha}$, is algebraically true

Given the 4 structural equations, basic statistical theory says that:

$$D(G) \sim N(\beta, \sigma_\eta^2)$$

$$D(T \mid G) \sim N(\alpha G, \sigma_\varepsilon^2)$$

$$D(T) \sim N(\alpha\beta, \alpha^2\sigma_\eta^2 + \sigma_\varepsilon^2)$$

$$D(G \mid T) \sim N\left(\frac{\alpha\sigma_\eta^2 T + \beta\sigma_\varepsilon^2}{\alpha^2\sigma_\eta^2 + \sigma_\varepsilon^2}, \frac{\sigma_\eta^2\sigma_\varepsilon^2}{\alpha^2\sigma_\eta^2 + \sigma_\varepsilon^2}\right)$$

Think of these distributions as linear regressions.

If we are certain G causes T and T does not cause G, we would simply estimate the first two distributions, since the last two would be irrelevant

Estimating a model and testing to see if the parameters are consistent with an assumed theory is a standard approach in economics

But this approach has trouble discriminating between different theories, particularly when, as is often the case theory doesn't give very precise values for parameters

This method is very different from the experimental methods commonly used in many other fields of science

Hoover's idea amounts to using a change in structural parameters essentially as a natural experiment

The implications of this one-way causal relationship from G to T:

A. If the structural process for T experiences a break in parameters (changes in σ_ε^2 and/or α occur), then D(G) will be stable while the other three D(.|.) will exhibit instability.

B. If the structural process for G experiences a break (changes in σ_η^2 and/or β occur), then D(T|G) will be stable while the other 3 D(.|.) will exhibit instability.

Two New Results

Result 1: If the structure is block recursive and breaks occur in one or more equations from a particular block, then certain empirical distributions are stable.

Result 2: If certain empirical distributions are stable in the face of instability in some structural equations, then the structure is block recursive.

An example of how the results might be used:

Many papers have assumed the structure is recursive or block recursive

E.g. Christiano, Eichenbaum and Evans (e.g. 1996 RESTAT)

Their empirical model consists of:

O=real output (GDP), P=the GDP deflator,
C=commodity price index, F=Fed Funds interest rate,
R=total reserves of depository institutions, N=nonborrowed reserves,
D=some additional variable.

The CEE dynamic structural system:

$$O_t = a_{op}P_t + a_{oc}C_t \qquad\qquad + g_o(L)Z_{t-1} + \varepsilon_{1t}$$
$$P_t = a_{po}O_t + a_{pc}C_t \qquad\qquad + g_p(L)Z_{t-1} + \varepsilon_{2t}$$
$$C_t = a_{co}O_t + a_{cp}P_t \qquad\qquad + g_c(L)Z_{t-1} + \varepsilon_{3t}$$
$$F_t = a_{fo}O_t + a_{fp}P_t + a_{fc}C_t \qquad\qquad + g_f(L)Z_{t-1} + \varepsilon_{4t}$$
$$R_t = a_{ro}O_t + a_{rp}P_t + a_{rc}C_t + a_{rf}F_t + a_{rn}N_t + a_{rd}D_t + g_r(L)Z_{t-1} + \varepsilon_{5t}$$
$$N_t = a_{no}O_t + a_{np}P_t + a_{nc}C_t + a_{nf}F_t + a_{nr}R_t + a_{nd}D_t + g_n(L)Z_{t-1} + \varepsilon_{6t}$$
$$D_t = a_{do}O_t + a_{dp}P_t + a_{dc}C_t + a_{df}F_t + a_{dr}R_t + a_{dn}N_t + g_d(L)Z_{t-1} + \varepsilon_{7t}$$

where:

$Z_{t-1} = (O_{t-1}, P_{t-1}, C_{t-1}, F_{t-1}, R_{t-1}, N_{t-1}, D_{t-1})'$ and

$g_i(L)$ is a 1×7 vector of lag polynomials for i =o,p,c,f,r,n,d.

The key identifying assumptions for CEE are:

Monetary policy does not react immediately to R, N and D, but does react immediately to O, P and C;

O, P and C do not react immediately to F, R, N and D;

Structural shocks are uncorrelated with each other.

They show that a recursively ordered VAR, with O, P and C ordered ahead of F and with R, N and D ordered after F will identify the effects of monetary policy shocks.

I will show that if there structure is correct and the F equation (the policy reaction function) experiences structural breaks, the following linear projections are stable:

OLS regressions of O, P and C on all lagged variables

OLS regressions of R, N and D on all lags and current O, P, C and F

while all other regressions would in general have parameter instability

Block Recursive Structure Implies Stable Empirical Distributions

Suppose there exists a block recursive structure:

$$\begin{bmatrix} \alpha_{xx} & 0 & 0 \\ \alpha_{vx} & \alpha_{vv} & 0 \\ \alpha_{yx} & \alpha_{yv} & \alpha_{yy} \end{bmatrix} \begin{bmatrix} X \\ V \\ Y \end{bmatrix} = \begin{bmatrix} \gamma_x \\ \gamma_v \\ \gamma_y \end{bmatrix} Z + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_y \end{bmatrix} \qquad (*)$$

X, V and Y are the endogenous variables

Z = pre-determined (e.g. lagged X, V and Y) and exogenous variables

ε_x, ε_v and ε_y are structural shocks to the X, V and Y equations

The assumptions for shocks are fairly typical:

Shocks in a block are uncorrelated with shocks in all other blocks:

$$E\varepsilon_x\varepsilon_v' = E\varepsilon_x\varepsilon_y' = E\varepsilon_y\varepsilon_v' = 0.$$

Shocks are uncorrelated with Z:

$$E\varepsilon_j Z = 0 \text{ for } j=x,v,y.$$

However, structural shocks within a particular block can be correlated

THEOREM 1: If the structure is block recursive of the form given in equation (*), the shocks satisfy the previous conditions and the structural equations for $V_j \in V$ experience a structural break, then

I. $D(X_i \mid Z)$ is stable for all $X_i \in X$;

II. $D(Y_k \mid X, V, Z)$ is stable and for every $Y_k \in Y$;

III. All other $D(\cdot \mid \cdot)$ are subject to instability, unless we impose further constraints on the parameters in α

Linear projections are used in this analysis:
Think of projections as OLS regressions

Proof of Theorem 1:

(I) The projection of X onto Z is stable:

Solve for X as a function of Z and structural shocks. $X = \alpha_{xx}^{-1}\gamma_x Z + \alpha_{xx}^{-1}\varepsilon_x$.

Given all structural shocks are orthogonal to Z,

Coefficients from projecting X onto Z are given as: $\alpha_{xx}^{-1}\gamma_x$,

Errors from projecting X onto Z are: $\alpha_{xx}^{-1}\varepsilon_x$.

Since these coefficients and errors are not functions of $\alpha_{vx}, \alpha_{vv}, \gamma_v$ or ε_v

$D(X \mid Z)$ is stable when any V equation is subject to structural breaks

(II) The projection of Y onto Z, X and V is stable:

Solve the subsystem of equations for Y to obtain:

$$Y = \alpha_{yy}^{-1}\left(\gamma_y Z - \alpha_{yx} X - \alpha_{yv} V\right) + \alpha_{yy}^{-1}\varepsilon_y \ .$$

The structural errors in the Y equations are orthogonal to Z, X, and V. Hence, the projection coefficients can be taken directly from this equation:

$$P\left(Y \mid Z, X, V\right) = \alpha_{yy}^{-1}\gamma_y Z - \alpha_{yy}^{-1}\alpha_{yx} X - \alpha_{yy}^{-1}\alpha_{yv} V$$

and the projection errors are: $\alpha_{yy}^{-1}\varepsilon_y$.

Clearly V is a factor in this distribution, but the structural parameters from the V process are not involved. Thus this conditional distribution is stable following a structural change to V.

(III) Outline of the proof that all other distributions are subject to instability, unless we impose further constraints on the parameters in α:

Every projection of V is unstable because the structural equations for V are the source of instability

All projections of X onto endogenous variables V and Y are unstable (unless further restrictions are placed on α)

e.g. $P(X \mid V, Z)$ and $X - P(X \mid V, Z)$ are, in general, functions of structural parameters for V

$P(X \mid V, Z)$ can always be decomposed into two projections as follow:

$$P(X \mid Z, V) = P(X \mid Z) + P\left\{\left(X - P(X \mid Z)\right) \mid \left(V - P(V \mid Z)\right)\right\}$$

The first one, X projected onto Z, was already calculated.

The second one involves projecting $X - P\left(X \mid V\right)$ onto $Y - P\left(Y \mid V\right)$.

Thus the projection of X onto Z and V can be re-written as:

$$P\left(X \mid Z, V\right) = \alpha_{xx}^{-1}\gamma_x Z + \pi_{xv}\left(V - P(V \mid Z)\right)$$

and π_{xv} are coefficients from the projection involving the errors

V as a function of Z and shocks is obtained from the structural equations for X and V:

$$V = \alpha_{vv}^{-1}\left(\gamma_v - \alpha_{vx}\alpha_{xx}^{-1}\gamma_x\right)Z + \left[\alpha_{vv}^{-1}\varepsilon_v - \alpha_{vv}^{-1}\alpha_{vx}\alpha_{xx}^{-1}\varepsilon_x\right]$$

Given structural shocks are orthogonal to Z, the projection of V onto Z is:

$$\alpha_{vv}^{-1}\left(\gamma_v - \alpha_{vx}\alpha_{xx}^{-1}\gamma_x\right)Z,$$

and

$$V - P\left(V \mid Z\right) = \alpha_{vv}^{-1}\varepsilon_v - \alpha_{vv}^{-1}\alpha_{vx}\alpha_{xx}^{-1}\varepsilon_x .$$

Given this error and the error from projecting X onto Z, $\alpha_{xx}^{-1}\varepsilon_x$, it is easy to calculate the projection coefficients:

$$\pi_{xv} = \alpha_{xx}^{-1}\Sigma_x\alpha_{xx}'^{-1}\alpha_{vx}'\alpha_{vv}'^{-1}\left(\alpha_{vv}^{-1}\Sigma_v\alpha_{vv}'^{-1} + \alpha_{vv}^{-1}\alpha_{vx}\alpha_{xx}^{-1}\Sigma_x\alpha_{xx}'^{-1}\alpha_{vx}'\alpha_{vv}'^{-1}\right)^{-1}$$

This matrix of coefficients is a function of parameters from V's structure.

Another way of writing this projection of X onto V and Z is:

$$P\left(X \mid Z, V\right) = P\left(X \mid Z\right) - \pi_{xv}P\left(V \mid Z\right) + \pi_{xv}V ,$$

and plugging in the projections of X and V onto Z yields:

$$P\left(X \mid Z, V\right) = \left(\alpha_{xx}^{-1}\gamma_x - \pi_{xv}\alpha_{vv}^{-1}(\gamma_v - \alpha_{vx}\alpha_{xx}^{-1}\gamma_x)\right)Z + \pi_{xv}V .$$

The error from projecting X onto Z and V can be calculated by noting

$$X - P(X \mid Z,V) = \left[X - P(X \mid Z)\right] + \left[P(X \mid Z) - P(X \mid Z,V)\right].$$

and using our previous results we obtain:

$$X - P(X \mid Z,V) = \alpha_{xx}^{-1}\varepsilon_x - \pi_{xv}\left(\alpha_{vv}^{-1}\varepsilon_v - \alpha_{vv}^{-1}\alpha_{vx}\alpha_{xx}^{-1}\varepsilon_x\right)$$

The projection of X onto Z and V and resulting error are functions of the parameters in the V process, unless π_{xv} is equal to zero.

Therefore, as long as π_{xv} is not equal to zero, the distribution of X given Z and Y will be unstable following structural breaks in V.

The only way that $\pi_{xv} = 0$ is if $\alpha_{vx} = 0$.

Implication: When a distribution of X conditioned on other endogenous variables is stable in the face of a break in V's structure, that implies additional restrictions on the block recursive α matrix.

We can also show that the following projections of X are unstable:

Projection of X onto Z and Y

Projection of X onto Z, V and Y

unless further restrictions are imposed on the block recursive α matrix.

Also, we can show that the following projections of Y are unstable:

Projection of Y onto Z

Projection of Y onto Z and X

Projection of Y onto Z and V

unless further restrictions are imposed on the block recursive α matrix.

How can we use Theorem 1?

If a theory implies a particular block recursive structure, and if we have reason to believe that equations from one block have been subject to structural change, then we can test block recursiveness by:

1. Regressing each X_i onto Z, for all $X_i \in X$;

2. Regressing each Y_k onto Z, X and V for all $Y_k \in Y$;
3. And testing the hypothesis that each of these regressions is stable.
4. Every other regression involving X, V or Y would have unstable parameters unless the structure was subject to further restrictions

A rejection of the hypothesis of no structural change in any one of the regressions in 1 and 2 would provide evidence against the block recursive structure.

What provides "reason to believe" structural changes have occurred in V?

1. Instability in the distributions involving V:
   E.g. regress V_j onto Z and reject stability for all $V_j \in V$
2. narrative evidence of a structural break in at least one of the structural equations associated with variables V

Application: Contemporaneous Identification Restrictions in VAR Models

Suppose Z is lagged values of X, V, and Y.

$$\begin{bmatrix} \alpha_{xx} & 0 & 0 \\ \alpha_{vx} & \alpha_{vv} & 0 \\ \alpha_{yx} & \alpha_{yv} & \alpha_{yy} \end{bmatrix} \begin{bmatrix} X_t \\ V_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \gamma_{xx}(L) & \gamma_{xv}(L) & \gamma_{xy}(L) \\ \gamma_{vx}(L) & \gamma_{vv}(L) & \gamma_{vy}(L) \\ \gamma_{yx}(L) & \gamma_{yv}(L) & \gamma_{yy}(L) \end{bmatrix} \begin{bmatrix} X_{t-1} \\ V_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{vt} \\ \varepsilon_{yt} \end{bmatrix}$$

Current and lagged values of exogenous variables could also be included WLOG.

Assume this system of equations is the structure underlying the reduced-form VAR model of (X,V,Y)

α are the structural parameters on current variables

$\gamma(L)$ are the structural parameters on lagged variables.
The SVAR literature rarely imposes restrictions on $\gamma(L)$

If each $\gamma_{ij}(L)$ lag polynomial is of lag order ℓ, pre-multiplying the equation above by α^{-1} yields a reduced-form VAR in which each variable in X, V and Y is a function of $\ell+1$ lags of every variable in X,V and Y.

Initial VAR research, following Sims (1980), performed Cholesky decompositions of the residuals to identify VARs.

Every possible Cholesky decomposition is equivalent to selecting a specific ordering of variables such that α is a lower triangular matrix, and then estimating each equation by OLS.

Subsequent structural VAR research has tried to tie identification restrictions more closely to a particular theory, or to a class of structural models.

Many have used a specific equation-by-equation recursive specification:

Christiano, Eichenbaum and Evans (1996,RESTAT) estimate the following recursive model:

$$
\begin{aligned}
O_t &= && g_o(L)Z_{t-1} + \varepsilon_{1t} \\
P_t &= a_{po}O_t && + g_p(L)Z_{t-1} + \varepsilon_{2t} \\
C_t &= a_{co}O_t + a_{cp}P_t && + g_c(L)Z_{t-1} + \varepsilon_{3t} \\
F_t &= a_{fo}O_t + a_{fp}P_t + a_{fc}C_t && + g_f(L)Z_{t-1} + \varepsilon_{4t} \\
R_t &= a_{ro}O_t + a_{rp}P_t + a_{rc}C_t + a_{rf}F_t && + g_r(L)Z_{t-1} + \varepsilon_{5t} \\
N_t &= a_{no}O_t + a_{np}P_t + a_{nc}C_t + a_{nf}F_t + a_{nr}R_t && + g_n(L)Z_{t-1} + \varepsilon_{6t} \\
D_t &= a_{do}O_t + a_{dp}P_t + a_{dc}C_t + a_{df}F_t + a_{dr}R_t + a_{dn}N_t && + g_d(L)Z_{t-1} + \varepsilon_{7t}
\end{aligned}
$$

Letting X=(O,P,C), V=F, and Y=(R,N,D) and using Theorem 1, we see that when the equation for F has a shift in parameters, the following are stable:

OLS regressions of O, P and C on all lagged variables

OLS regressions of R, N and D on all lags and current O, P, C and F

Theorem 1 is proven using block recursive structures, and therefore if:

Each of Y, P and C were a function of these other two variables,
or
Each of R, N and D were a function of these other two variables,

these same testable implications would obtain.

An implication is that structural breaks can make a just-identified structural model testable via patterns of stability and instability in the face of these breaks

All equation by equation recursive models are block recursive, but block recursive systems encompass a broader set of structural relationships.

A large number of papers in the structural VAR literature have used block recursive systems of equations.

For example:

Bernanke (1986): Defense Spending is ordered ahead of all other variables

Testable implications based on Theorem 1:

If there are structural breaks in defense spending (DE), a regression of each other variable in this model on lags of all variables and current DE will be stable.

If there are structural breaks in any or all of the other equations, then the regression of DE on all lagged variables is stable.

Sims (1986): Investment ordered ahead of all other variables in the model

Testable implications based on Theorem 1:

Same as Bernanke (1986) above, except replace DE by investment.

Hoover's Bivariate Approach is a Special Case of this General Result

Recall Hoover's example assumes that G causes T, but T does not cause G.

This is a bivariate recursive system of structural equations.

Suppose there is a structural shift in the T process:

Let V=T and X=G and drop Y from my analysis, to obtain his result:

D(G|Z) is stable while D(T|Z), D(T|Z,G) and D(G|Z,T) exhibit instability.

Alternatively, suppose there is a structural shift in the G process, then

Let V=G and Y= T, and drop X from my analysis, to obtain his result:

D(T|Z,G) is stable while D(T|Z), D(G|Z,T) and D(G|Z) are unstable

In summary, Theorem 1 covers a much broader array of structures:

Multivariate systems;

A bivariate system may omit important explanatory variables or relevant structural mechanisms

Block recursive structures;

Equation-by-equation recursive structures are a special case, block recursive includes many more types of structure.

The analysis implies that instability could arise in any of the blocks:

The result is proven for instability in a middle block;

Eliminating X implies instability in the first block;

Eliminating Y implies instability in the last block.

How do results extend to systems with > 3 recursive blocks ?

Application: Long-Run Identification Restrictions in VARs

Suppose the following system of structural equations:

$$A(L)\Delta w_t = \varepsilon_t$$

Assume there is no cointegration in the system;

Cointegration would not change anything significantly in what follows: It could most easily be handled using the triangular representation of Phillips, modifying the vector of variables in the model to include the stationary linear combinations of variables

Using the multivariate Beveridge and Nelson decomposition, we can always write

$$A(L) = A(1) + (1-L)A^*(L) \text{ where } A(1) \equiv A(L)|_{L=1}.$$

If A(L) is a lag polynomial of length ℓ, then $A^*(L)$ is of length ℓ-1, by construction.

Use this representation of A(L) to reformulate the original system of equations:

$$A(1)\Delta w_t = -A^*(L)\Delta^2 w_t + \varepsilon_t$$

This representation is convenient because it separates the structural effects into

Parameters that describe long run structural relationships: A(1),

and

Parameters that characterize transitory dynamics: $A^*(L)$,

$$\text{with } A^*(L) = \sum_{j=0}^{\ell-1} A_j^* L^j \text{ and } A_j^* = \sum_{i=0}^{j} A_i$$

Constrained versions of A(1) have been estimated.

Blanchard and Quah (1989): A(1) is lower triangular

Shapiro and Watson (1988): an IV procedure (more on this in a moment)

One way to test hypotheses about A(1) is to estimate the matrix

This requires enough restrictions on A(1) to identify parameters.

But if these restrictions are invalid the estimates are possibly useless

We can rewrite the Beveridge-Nelson decomposition in an equivalent two-block representation:

$$\begin{bmatrix} A_{11}(1) & A_{12}(1) \\ A_{21}(1) & A_{22}(1) \end{bmatrix} \begin{bmatrix} \Delta w_{1t} \\ \Delta w_{2t} \end{bmatrix} = - \begin{bmatrix} A^*_{11}(1) & A^*_{12}(1) \\ A^*_{21}(1) & A^*_{22}(1) \end{bmatrix} \begin{bmatrix} \Delta^2 w_{1t} \\ \Delta^2 w_{2t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \qquad (**)$$

I have developed a new way to test the hypothesis that $A_{12}(1)=0$. This procedure:

Does not require identifying restrictions on A(1),

But does require structural breaks to occur.

Theorem 3: If the structural equations for Δw_{2t} undergo a parameter change, then

i. $P\left(\Delta w_{2t} \mid \{\Delta^2 w_{t-j}\}_{j=0}^{k-1}\right)$ is unstable;

ii. $P\left(\Delta w_{1t} \mid \{\Delta^2 w_{t-j}\}_{j=0}^{k-1}\right)$ is stable if and only if $A_{12}(1)=0$.

Where P(.|.) is the linear projection of variables onto other variables.

Inverting the A(1) matrix in the 2-block system in equation (**) yields two expressions:

$$\Delta w_{it} = b_{i1}(L)\Delta^2 w_{1t} + b_{i2}(L)\Delta^2 w_{2t} + e_{it} \qquad \text{for } i=1,2.$$

These equations can NOT be estimated by OLS because some of the regressors are current endogenous variables in second differences.

Shapiro and Watson (1988) show how to estimate $P\left(\Delta w_{vt} \mid \{\Delta^2 w_{t-j}\}_{j=0}^{k-1}\right)$

For v=1,2

They develop an IV procedure with $\left\{\Delta w_{t-j}\right\}_{j=1}^{k}$ as instruments

Note: The number of instruments equals the number of regressors

When $A_{12}(1)=0$, the equations for Δw_{1t} are NOT functions of the structural parameters from the Δw_{2t} equations and therefore the equations for Δw_{1t} would be stable,

Otherwise $A_{12}(1)$ is not equal to zero and these structural parameters from the Δw_{2t} equations will affect the projection for Δw_{1t}

An analogous expression for Δw_{2t}, $P\left(\Delta w_{2t} \,|\, \{\Delta^2 w_{t-j}\}_{j=0}^{k-1}\right)$, is also estimated using precisely the same instruments: $\left\{\Delta w_{t-j}\right\}_{j=1}^{k}$

This expression is a function of the Δw_{2t} structural parameters

This estimate is used to confirm structural changes in the Δw_{2t} process.

To conduct this test we could estimate all equations using the full sample of data --- we don't need to break the data up into stable subsamples.

An Example: Testing Long-run Neutrality

Let $w_1=y$, a real variable (e.g. output) or a vector of real variables;

Let $w_2=p$, a nominal variable (e.g. price) or a vector of nominal variables.

Hence, in this framework $A_{12}(1)=0$ characterizes long-run neutrality: The absence of p effects in the long-run structural equation for y.

If there is a break in the structural process for nominal variables, then

$P\left(\Delta p_t \,|\, \{\Delta^2 w_{t-j}\}_{j=0}^{k-1}\right)$ will exhibit instability.

The test for stability in:

$$P\left(\Delta y_t \,|\, \{\Delta^2 w_{t-j}\}_{j=0}^{k-1}\right)$$

tests long-run neutrality.

Alternatively, suppose there have been breaks in the Δw_{1t} process and we want to test the null hypothesis that $A_{12}(1)=0$. One way to carry out such a test is:

1. Estimate $P\left(\Delta w_{1t} \mid \{\Delta^2 w_{t-j}\}_{j=0}^{k-1}\right)$ separately for each stable subsample,

Stable subsamples are needed to get consistent estimates of linear combinations of the structural shocks in the w_{1t} equations under the null.

2. Splice together the residuals from each subsample estimate to form a continuous series of residuals,

3. Use these residuals, and $\left\{\Delta w_{t-j}\right\}_{j=1}^{k}$ as instruments to estimate

$$P\left(\Delta w_{2t} \mid \Delta w_{1t}, \left\{\Delta^2 w_t\right\}_{j=0}^{k-1}\right).$$

This distribution is stable in the face of a structural break in the Δw_{1t} process if and only if:

$$A_{12}(1)=0$$

A potentially serious problem with this procedure is that a subsample in which Δw_{1t} is stable might be too short to yield reliable estimates of long run phenomena.

Stable Empirical Distributions Imply Block Recursiveness

Start with a potentially unconstrained structure:

$$\begin{bmatrix} \alpha_{xx} & \alpha_{xv} & \alpha_{xy} \\ \alpha_{vx} & \alpha_{vv} & \alpha_{vy} \\ \alpha_{yx} & \alpha_{yv} & \alpha_{yy} \end{bmatrix} \begin{bmatrix} X \\ V \\ Y \end{bmatrix} = \begin{bmatrix} \gamma_x \\ \gamma_v \\ \gamma_y \end{bmatrix} Z + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_y \end{bmatrix}$$

THEOREM 2: Suppose structural breaks occur in equations for some $V_j \in V$.

I. If $D(X_i \mid Z)$ is stable then X_i is part of a block of structural equations that is ordered ahead of V;

II. $D(Y_k \mid X, V, Z)$ is stable then Y_k is part of a block of structural equations that is ordered after V;

Proof of Theorem 2:

(I) Projections of X onto Z, Y onto Z, V onto Z are obtained by tedious matrix algebra calculations

For example, an equation involving X, Z and the shocks:

$$A_1 X = A_2 Z + \tau_x$$

can be derived for which:

$$A_1 = \alpha_{xx} - \alpha_{xv}\alpha_{vv}^{-1}\alpha_{vx} - \left[\alpha_{xy} - \alpha_{xv}\alpha_{vv}^{-1}\alpha_{vy}\right]\left[\alpha_{yy} - \alpha_{yv}\alpha_{vv}^{-1}\alpha_{vy}\right]^{-1}\left[\alpha_{yx} - \alpha_{yv}\alpha_{vv}^{-1}\alpha_{vx}\right],$$

$$A_2 = \gamma_x - \alpha_{xv}\alpha_{vv}^{-1}\gamma_v - \left[\alpha_{xy} - \alpha_{xv}\alpha_{vv}^{-1}\alpha_{vy}\right]\left[\alpha_{yy} - \alpha_{yv}\alpha_{vv}^{-1}\alpha_{vy}\right]^{-1}\left[\gamma_y - \alpha_{yv}\alpha_{vv}^{-1}\gamma_v\right],$$

and $\tau_x = \varepsilon_x - \alpha_{xv}\alpha_{vv}^{-1}\varepsilon_v - \left[\alpha_{xy} - \alpha_{xv}\alpha_{vv}^{-1}\alpha_{vy}\right]\left[\alpha_{yy} - \alpha_{yv}\alpha_{vv}^{-1}\alpha_{vy}\right]^{-1}\left[\varepsilon_y - \alpha_{yv}\alpha_{vv}^{-1}\varepsilon_v\right].$

Multiplying the equation by the inverse of A_1 yields an expression for the coefficients obtained from projecting X onto Z as well as the projection error,
(remember structural shocks are orthogonal to Z)

If there is a break in the V process, the distribution of X given Z would also experience a break unless one of the following conditions holds:

(i) $\alpha_{xv} = 0$ and $\alpha_{xy} = 0$

Or

(ii) $\alpha_{xv} = 0$ and $\alpha_{yv} = 0$

In each case, the key implication of a stable D(X|Z) is that X is ordered ahead of V in a block recursive structure:

To see this, impose each set of restrictions on the system of equations:

For case (i):

$$\begin{bmatrix} \alpha_{xx} & 0 & 0 \\ \alpha_{vx} & \alpha_{vv} & \alpha_{vy} \\ \alpha_{yx} & \alpha_{yv} & \alpha_{yy} \end{bmatrix}\begin{bmatrix} X \\ V \\ Y \end{bmatrix} = \begin{bmatrix} \gamma_x \\ \gamma_v \\ \gamma_y \end{bmatrix} Z + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_y \end{bmatrix}.$$

Note: The block for X is ordered ahead of V and Y.

For case (ii), interchange the ordering of V and Y:

$$\begin{bmatrix} \alpha_{xx} & \alpha_{xy} & 0 \\ \alpha_{yx} & \alpha_{yy} & 0 \\ \alpha_{vx} & \alpha_{vy} & \alpha_{vv} \end{bmatrix} \begin{bmatrix} X \\ Y \\ V \end{bmatrix} = \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_v \end{bmatrix} Z + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_y \end{bmatrix}$$

Note: X and Y form a block that is ordered ahead of V.

The projection of X onto Z is also stable following a structural break in V if $\alpha_{xv} = 0$ and $\alpha_{xy} = 0$ and $\alpha_{yv} = 0$. These assumptions set α_{xy} equal to zero in case (ii) equations above, but X and Y still form a block ordered ahead of V.

(II) Proof of this: You'll have to trust me!

I won't bother you with the details today. Calculating the projection of Y onto Z, X, and V in the case of the unconstrained α matrix is extremely cumbersome. I'm looking for a cleaner, clearer and less tedious proof of the results that follow.

It is discovered that the projection of Y onto Z, X and V is stable under two conditions:

(iii) $\alpha_{vy} = 0$ and $\alpha_{xy} = 0$

(iv) $\alpha_{vy} = 0$ and $\alpha_{vx} = 0$

In each of these cases, the key implication is that V is part of a block of structural equations that is ordered ahead of Y.

Again, this is best seen by imposing these restrictions on the system.
For case (iii):

$$\begin{bmatrix} \alpha_{xx} & \alpha_{xv} & 0 \\ \alpha_{vx} & \alpha_{vv} & 0 \\ \alpha_{yx} & \alpha_{yv} & \alpha_{yy} \end{bmatrix} \begin{bmatrix} X \\ V \\ Y \end{bmatrix} = \begin{bmatrix} \gamma_x \\ \gamma_v \\ \gamma_y \end{bmatrix} Z + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_y \end{bmatrix}.$$

Note: X and V form a block ordered ahead of Y.

For case (iv), after interchanging V and X in the system:

$$\begin{bmatrix} \alpha_{vv} & 0 & 0 \\ \alpha_{xv} & \alpha_{xx} & \alpha_{xy} \\ \alpha_{yv} & \alpha_{yx} & \alpha_{yy} \end{bmatrix} \begin{bmatrix} V \\ X \\ Y \end{bmatrix} = \begin{bmatrix} \gamma_v \\ \gamma_x \\ \gamma_y \end{bmatrix} Z + \begin{bmatrix} \varepsilon_v \\ \varepsilon_x \\ \varepsilon_y \end{bmatrix}$$

Note: V is ordered ahead of a block formed by X and Y.

And having $\alpha_{vy} = 0$ and $\alpha_{xy} = 0$ and $\alpha_{vx} = 0$ would impose a restriction on case (iv) above, but would not alter the implication that V is ordered ahead of Y.

How could we use Theorem 2?

We could pre-test a vector of variables for block recursive structure.

1. Determine that structural breaks have occurred by projecting all variables onto Z and finding statistical evidence of some breaks;
2. Assign a variable to X if it has a stable projection onto Z. All variables not assigned to X must be members of either Y or V
3. For each variable not assigned to X, project that variable onto every other variable in the system. For each projection that is stable, assign the variable to Y.
4. All variables not assigned to X or Y are assigned to V.
5. Find narrative evidence supporting structural breaks in the sample for some of the variables assigned to V.

Naturally, if no breaks are found then this procedure would not be useful.

It is possible the procedure will not produce any X variables

It is possible the procedure may not produce any Y variables.

If all variables are assigned to V, X and Y are empty sets, that implies the structure does not have any block recursive features

(INSERT the proof that projection of a variable onto all variables in the system is stable if the variable belongs in the set of Y from our block recursive structure)

Conclusions

... forthcoming ...

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