Monetary Policy in a Channel System*

Aleksander Berentsen
Department of Economics, University of Basel

Cyril Monnet
DG-Research, European Central Bank

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Abstract

This paper studies monetary policy when the central bank operates a channel system of interest control. We conduct our analysis in a dynamic general equilibrium model with infinitely-lived agents where money is essential for trade. We characterize the equilibrium allocation and optimal policy when collateral is costless to hold and when it is costly to hold. Under the first assumption, it is optimal to set deposit and lending rates equal. Under the second assumption, it is optimal to choose a strictly positive interest-rate corridor. A central bank that must tighten its policy in response to a change in economic conditions has two options. It can shift the interest-rate band up while keeping the band constant. Or, it can increase the interest-rate band.

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1 Introduction

In this paper we analyze monetary policy when the central bank operates a channel system of interest-rate control similar to ones used by the Bank of Canada, the European Central Bank, the Reserve Bank of Australia, and the Reserve Bank of New Zealand. Under a channel system the central bank stands ready to supply an arbitrary amount of money against collateral at a given borrowing rate and commercial banks that clear transactions through the central bank can deposit excess money overnight with the central bank at a deposit rate (see Woodford (2000) for more details). For example, the European Central Bank offers a so called standing facility with a borrowing rate that is 100 basis points higher than its policy interest rate and a deposit rate, which is 100 basis points below its policy rate. For each loan it requires collateral which are typically low-risk and low-yield assets. All central banks that use the channel system react to changing economic conditions by shifting the interest-rate corridor - defined by the difference between the borrowing and the deposit rates. Figure 1 in the Appendix illustrates such shifts using data from the interest rate corridor operated by the Reserve Bank of New Zealand. The solid black line is the overnight interbank cash rate that the Reserve Bank targets via its channel system. The dotted red curve is the borrowing rate and the dotted blue line the deposit rate.

We consider three questions in this paper. First, what is the optimal interest-rate corridor? Second, why is it optimal for a central bank to shift its corridor while leaving its size constant when reacting to changing economic conditions? Third, what do collateral requirements imply for the optimal monetary policy?

To answer these questions we construct a dynamic general equilibrium model with infinitely-lived agents and a central bank. Agents face random production and consumption opportunities so that at any point in time some are liquidity constraint while other agents have idle balances. Once agents have learnt their liquidity needs, they can use the central bank's standing facility to either borrow or deposit money at the specified rates. Central bank credit is subject to a default constraint and so the central bank only provides collateralized loans.

We characterize the optimal policy for two cases. In the first case the rate of return of the collateral is such that it is costless to acquire collateral for commercial banks. In the second case collateral is costly to hold. The following results emerge from the model. Under the first assumption, it is optimal to set deposit and lending rates equal. Under the second assumption, it is optimal to choose a strictly positive interest-rate corridor. A central bank that must tighten its policy in response to a change in economic conditions has two options. It can shift the interest-rate band up while keeping the band constant as illustrated in Figure 1. Or, it can increase the interest-rate band. For instance, it can keep the deposit rate constant and increase the borrowing rate.[1]

An interesting aspect of our model is that money growth and hence inflation is endogenous unlike in most theoretical analysis of monetary policy that characterize optimal policy in terms of a path for the money supply. In practice, however, monetary policy involves rules for setting nominal interest rates and most central banks specify operating targets for overnight interest rates. This paper therefore is an attempt to break the apparent dichotomy (Goodhard, 1989) between theoretical analysis and central bank practices, by investigating interest rate policies as in Woodford (2005).

Literature: Woodford (2003) summarizes the literature that investigates optimal interest rate rules. The channel system is investigated in Woodford (2000, 2001).

The paper is structured as follows. Section 2 outlines the environment. The equilib-

[1]Since 2004 the US Federal Reserve System im moving towards a channel system since it now encourages the use of the discount window by commercial banks. Its sets the deposit rate to zero (which is equivalent of not allowing to deposit) and sets the discount window rate consistently with its target federal fund rate.

rium with costless collateral is characterized in Section 3. Section 4 defines and characterizes the equilibrium when collateral is costly. Finally, Section 6 concludes.

2 The environment

There is a [0,1] continuum of infinitively-lived agents. Time is discrete and in each period two perfectly competitive markets open sequentially. The first market is a settlement stage where all agents produce and consume a general good and settle their claims from the previous period with the central bank. General goods are produced solely from inputs of labor according to a constant return to scale production technology where one unit of the consumption good is produced with one unit of labor generating one unit of disutility. Thus, producing h units of the general good implies disutility $-h$, while consuming h units gives utility h.[2]

In the second market agents produce or consume a perishable good. At the beginning of the second market, agents receive liquidity shocks.[3] These arise from preference shocks which determine whether agents consume or produce in market 2. With probability $1-n$ an agent can consume and cannot produce. We refer to these agents as buyers. With probability n, an agent can produce and cannot consume. These are sellers. Agents get utility $u(q)$ from q consumption in the second market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. Producers incur a utility cost $c(q) = q$ from producing

[2]The environment is similar to the one introduced by Berentsen, Camera and Waller (2004). The linear preferences in market 1, first introduced by Lagos and Wright (2005) to get a degenerate distribution of money holdings at the beginning of a period, allows us to interpret transactions that are taking place in the first market as settlement transactions, as in Koeppl, Monnet and Temzelides (2005). Like them, we abstract from modelling commercial banks explicitly. Rather, we construct a model where agents face random needs for liquidity and random opportunities to acquire liquidity.

[3]This is an approximation for the money market where banks receive liquidity shocks at the end of the day. Since there is no trading of reserves feasible after this market, banks who need liquidity have no choice but to use the standing facility offered by the central bank.

q units of output. All trades are anonymous and agents' trading histories are private information. Since sellers require immediate compensation for their production effort money is essential for trade.[4] The discount factor is β where for technical reasons we assume that $\beta > n$.

Standing facility In the second market a central bank operates a standing facility. It offers nominal loans ℓ at an interest rate i and promises to pay interest $i_d d$ on nominal deposits d with $i \geq i_d$.[5] Since we focus on standing facilities, we restrict financial contracts to overnight contracts. An agent who borrows ℓ units of money from the central bank in market 2, repays $(1 + i)\,\ell$ units of money in market 1 of the following period. Also, an agent who deposits d units of money at the central bank in market 2 of period t receives $(1 + i_d)\,d$ units of money in market 1 of the following period.

Accordingly, in the absence of open market operations, the money stock evolves endogenously as follows

$$M_{+1} = M - (1 - n)i\ell + ni_d d, \tag{1}$$

where M denotes the per capita stock of money at the beginning of period t. In the first market total loans $(1 - n)\ell$ are repaid. Since interest rate payments by the agents are $(1 - n)i\ell$, the stock of money shrinks by this amount. Interest payments by the central bank on total deposits are $ni_d d$. The central bank simply prints additional money to make these interest payments so the stock of money increases by this amount. The central bank operates the standing facility at zero cost. Consequently, the central bank cannot make profits or losses.

[4] By essential we mean that the use of money expands the set of allocations (Kocherlakota 1998 and Wallace 2001).

[5] This restriction eliminates the possibility for arbitrage where agents borrow and subsequently make a deposit at interest $i_d > i$, thus increasing their money holdings at no cost.

Default In any model of credit, default is a serious issue. Since production is costly, those agents who borrow in market 2 have an incentive to default in market 1 of the following period. To prevent default the central bank requires general goods as collateral for each loan. We assume that general goods that are produced in market 1 can be stored at the central bank with a constant return to scale technology that yields $R \geq 1$ units of general goods in market 1 of the following period. We also impose $\beta R \leq 1$ since when $\beta R > 1$ agents would store infinite amounts of goods which is inconsistent with equilibrium. General goods can *only* be stored at the central bank. Consequently, general goods cannot be used to issue collateralized IOU's among private agents.

First-best allocation The expected lifetime utility of the representative agent for a stationary allocation (q, b) at the beginning of a period is given by

$$(1-\beta)\,\mathcal{W} = (1-n)\,[u(q) - q] + (\beta R - 1)\,b \tag{2}$$

The first term on the right-hand side is the expected utility from consuming and producing the market 2 good. The second term is the utility of producing collateral and receiving the return in the following period.

It is obvious that the first-best allocation (q^*, b^*) satisfies $q = q^*$ where q^* is the value of q that solves $u'(q) = 1$. Moreover, $b^* = 0$ if $\beta R < 1$. Thus, with a negative real interest rate a social planner would never choose a positive amount of collateral.

3 Symmetric stationary equilibrium

In period t, let $\phi \equiv 1/P$ be the real price of money in market 1. We study equilibria where beginning-of-period real money balances are time invariant

$$\phi M = \phi_{+1} M_{+1}. \tag{3}$$

We refer to it as a stationary equilibrium. This implies that $\phi_{+1}/\phi = P_{+1}/P = M/M_{+1} = \gamma$. Moreover, we restrict our attention to stationary equilibria where γ is time invariant.

We let $V(m, b)$ denote the expected value from entering market 2 with m units of money and b collateral. $W(m, b, \ell, d)$ denotes the expected value of entering the first market with m units of money, b collateral, ℓ loans, and d deposits. For notational simplicity we suppress the dependence of the value function on the time index t.

In what follows we look at a representative period t.

3.1 The settlement market

In the first market, the problem of a representative agent is:

$$W(m, b, \ell, d) = \max_{h, m_2, b_2} -h + V(m_2, b_2)$$
$$s.t. \quad \phi m_2 + b_2 = h + \phi m + Rb + \phi(1 + i_d)d - \phi(1 + i)\ell.$$

where h is hours worked in market 1. Using the budget constraint to eliminate h in the objective function, one obtains the first-order conditions

$$V_m \leq \phi \ (= \text{ if } m > 0\ ) \tag{4}$$
$$V_b \leq 1 \ (= \text{ if } b > 0\ ) \tag{5}$$

$V_m \equiv \frac{\partial V(m_2,b_2)}{\partial m_2}$ is the marginal value of taking an additional unit of money into the second market in period t. Since the marginal disutility of working is one, $-\phi$ is the utility cost of acquiring one unit of money in the first market of period t. $V_b \equiv \frac{\partial V(m_2,b_2)}{\partial b_2}$ is the marginal value of taking additional collateral into the second market in period t. Since the marginal disutility of working is 1, -1 is the utility cost of acquiring one unit of collateral in the first market of period t. The implication of (4) and (5) is that all agents enter the following period with the same amount of money and the same quantity of collateral (which can be zero). This is the reason why, as in Koeppl, Monnet and Temzelides (2005), we interpret this market as a settlement stage. By itself, this market does not increase social welfare. Rather, it involves a mere transfer of an asset between participants in order to erase claims from the previous period. The settlement market gives the opportunity to all agents to start afresh since, after settling his obligation, an agent is no longer liable to the central bank.

The envelope conditions are

$$W_m = \phi; W_b = R; W_\ell = -\phi\left(1+i\right); W_d = \phi\left(1+i_d\right) \tag{6}$$

where W_j is the partial derivative of $W(m, b, \ell, d)$ with respect to $j = m, b, \ell, d$.

3.2 The second market

Let q and q_s respectively denote the quantities consumed by a buyer and produced by a seller in market 2. Let ℓ_b (ℓ_s) and d_b (d_s) respectively denote the loan obtained and the amount of money deposited by a buyer (seller) in market 2. An agent who has m money and b collateral at the opening of market 2 has expected lifetime utility

$$\begin{aligned} V(m,b) &= (1-n)[u(q)+\beta W(m-pq-d_b+\ell_b,b,\ell_b,d_b)] \\ &\quad +n[-q_s+\beta W(m+pq_s-d_s+\ell_s,b,\ell_s,d_s)] \end{aligned}$$

where q, q_s, ℓ_s, ℓ_b, d_s and d_b are chosen optimally as follows.

It is obvious that buyers will never deposit funds in the bank and sellers will never take out loans and therefore $d_b = 0$ and $\ell_s = 0$. It is also straightforward to show that sellers will deposit as much money as they can if $i_d > 0$ and therefore $d_s = m_1 + pq_s$. If $i_d = 0$, they are indifferent and we simply assume $d_s = m + pq_s$. Accordingly, we get

$$\begin{aligned} V(m,b) &= (1-n)[u(q)+\beta W(m-pq+\ell,b,\ell,0)] \\ &\quad +n\left[-q_s+\beta W\left(0,b,0,m+pq_s\right)\right] \end{aligned}$$

where q_s and q solve the following optimization problems.

A seller's problem is $\max_{q_s}\left[-q_s+\beta W\left(0,b,0,m+pq_s\right)\right]$. Using (6), the first-order condition reduces to

$$\beta p\phi_{+1}\left(1+i_d\right)=1. \tag{7}$$

If an agent is a buyer, he solves the following maximization problem:

$$\begin{aligned} \max_{q,\ell} &\quad u(q)+\beta W(m-pq+\ell,b,\ell,0) \\ \text{s.t.} &\quad pq \le m+\ell \text{ and } \ell \le \bar{\ell} \end{aligned}$$

where $\bar{\ell}$ is the maximal amount that a buyer can borrow from the central bank. If the central bank requires collateral, then $\bar{\ell} = Rb/\left[\phi_{+1}\left(1+i\right)\right]$.

Using (6) and (7) the buyer's first-order conditions can be written as

$$u'(q) = p\beta\phi_{+1}(1 + \lambda_q) \tag{8}$$

$$\lambda_q = \lambda_\ell + i \tag{9}$$

where $\beta\phi_{+1}\lambda_q$ is the multiplier of the buyer's budget constraint and $\beta\phi_{+1}\lambda_\ell$ the one of the borrowing constraint. Combining (8) and (9) yields

$$u'(q) = \frac{1 + i + \lambda_\ell}{1 + i_d} \tag{10}$$

If the borrowing constraint is not binding and the central bank sets $i = i_d$, trades are efficient. If the borrowing constraint is binding, then $u'(q) > 1$ which means trades are inefficient even when $i = i_d$.

Using the envelope theorem and (8), the marginal value of money in market 2 is

$$V_m = (1 - n)u'(q)/p + n\beta\phi_{+1}(1 + i_d) \tag{11}$$

The marginal value of money has a straightforward interpretation. An agent with an additional unit of money becomes a buyer with probability $1-n$ in which case he acquires $1/p$ units of goods yielding additional utility $u'(q)/p$. With probability n he becomes a seller in which case he deposits overnight his money yielding the nominal return $1 + i_d$. Note that the standing facility increases the marginal value of money because agents can earn interest on idle cash.

Since in equilibrium there is no default the real return of collateral is $(1 - n)\beta R + n\beta R = \beta R$. The real return differs from the marginal value if $\lambda_\ell > 0$. To see this, use the envelope theorem to derive the marginal value of collateral in the second market

$$V_b = (1-n)\left(\beta\phi_{+1}\lambda_\ell \frac{\partial \bar{\ell}}{\partial b} + \beta R\right) + n\beta R \tag{12}$$

Here, an agent becomes a buyer with probability $1-n$ in which case the additional collateral relaxes the borrowing constraint yielding marginal utility $\beta\phi_{+1}\lambda_\ell \frac{\partial \bar{\ell}}{\partial b}$ in addition to the real return βR which is obtained for both buyers and sellers.

It is critical for the working of the model that the real return of collateral is different from its marginal value. It implies that agents are willing to hold collateral because of its liquidity value as expressed by the shadow price λ_ℓ even though the real interest rate $\beta R - 1$ is negative. One can also see that in any stationary equilibrium where agents hold collateral the shadow value λ_ℓ must be strictly positive otherwise $V_b = \beta R < 1$ where the right-hand side of this inequality is the disutility of producing one unit of collateral.

4 Costless collateral

In this section, we characterize the equilibrium when the rate of return for the collateral satisfies $R = 1/\beta$. In this case a unit of collateral is costless to acquire since the marginal cost is equal to the discounted marginal benefit. As a consequence, if $\lambda_\ell > 0$ agents are strictly better off by increasing their collateral holdings up to the amount where $\lambda_\ell = 0$.[6] Consequently, in equilibrium the borrowing constraint does not bind ($\lambda_\ell = 0$) and from (10) we have

$$u'(q) = \frac{1+i}{1+i_d} \equiv \Delta, \tag{13}$$

[6] Agents are indifferent between any amount of collateral that yields λ_ℓ. In this case without loss of generality we assume that they acquire the smallest amount consistent with $\lambda_\ell = 0$.

where $\Delta/\beta R$ is the price of collateral in terms of goods in market 2.[7] Monetary policy affects the allocation and welfare by its choice of Δ. An increase in Δ increases the cost of acquiring money and hence market 2 goods with collateral.

The endogenous rate of money growth satisfies

$$\frac{\gamma - \beta}{\beta} = (1-n)i + ni_d \tag{14}$$

Moreover, from (1) z_ℓ and z_m satisfy

$$\gamma = (1+i_d) - (1-n)(i-i_d)\frac{z_\ell}{z_m}, \tag{15}$$

where

$$q = z_m + z_\ell \tag{16}$$

We can use these four equations to define a symmetric stationary equilibrium with costless collateral. They determine recursively the endogenous variables (γ, q, z_m, z_ℓ). The first two equations yield q and γ. Once we have solved for q equation (15) yields z_m and (16) z_ℓ. Note that all other endogenous variables can be derived from these equilibrium values.

Definition 1 *A symmetric stationary equilibrium with costless collateral and borrowing is a list (γ, q, z_m, z_ℓ) satisfying (13)-(16) with $z_\ell > 0$ and $z_m \geq 0$.*

Proposition 1 *For any (i, i_d) with $i \geq i_d \geq 0$ there exists a critical value*

$$\tilde{\Delta} = \frac{1-\beta n}{\beta - \beta n} > 1$$

[7]To see this note that an agent with one unit collateral can acquire $\frac{\beta R(1+i_d)}{1+i}$ units of goods in market 2 of the current period.

such that if $1 \leq \Delta < \tilde{\Delta}$*, a unique symmetric stationary equilibrium with collateral exists. If* $\Delta = 1$*, a symmetric stationary equilibrium exists with* $z_m = 0$*.*

Proof of Proposition 1. The critical elements to verify are $\gamma \geq \beta$ and $z_\ell > 0$. The first inequality holds for any (i, i_d) with $i \geq i_d \geq 0$. Consider now the inequality $z_\ell > 0$. Assume $i > 0$, then from (20) we need $1 + i_d > \gamma$. Replace γ to get

$$1 + i_d > \beta(1-n)(1+i) + n\beta\,(1+i_d)$$

Rearranging yields (24). Evidently, for a given i_d if i is too high, this constraint cannot be satisfied. ■

The critical elements to verify in the proof is whether $z_\ell > 0$ and $z_m \geq 0$. The first inequality holds if $\Delta < \tilde{\Delta}$. It requires that the borrowing rate is not too high, otherwise borrowing is too costly and agents do not use the standing facility to borrow. The second inequality holds if $i \geq 0$.

We now derive the optimal policy. The central bank's objective is to maximize welfare (2). It does so by choosing the interest corridor i and i_d. Suppose first that the central bank is constraint to set i_d equal to some positive value. What should be the optimal band? Given i_d the optimal standing facility involves choosing $i = i_d$. This result follows immediately from equation (13) where q is decreasing in i.

We define the allocation that is attained under the optimal policy as the limiting allocation that is attained when $i \to i_d$. With a zero band we find the following results.

Proposition 2 *With costless collateral, the optimal policy* $i \to i_d$ *implements the first-best allocation* q^**. The price level approaches infinity.*

The proof of the first part is an immediate consequence of equation (13). To understand why the price level approaches infinity under the optimal policy note that if

$i = i_d > 0$, then money is strictly dominated in return by collateral. The reason is that the collateral can costlessly be transformed into money and so any consumption level that can be achieved with money can be achieved with collateral at no additional cost. However, the collateral has the intrinsic return $\beta R = 1$ while the return on money is $\frac{\beta}{\gamma} < 1$. Consequently, the demand for money approaches zero. To encourage agents to hold the stock of money its price must approach zero. This immediately implies that $p \to +\infty$ and therefore $z_m = M_{+1}/p \to 0$. This however does not mean that money is not used since it still plays the role of a medium of exchange in market 2. It only means that agents do not want to hold it across periods. Only at the Friedman rule $i = i_d = 0$ the returns are equal and so agents are indifferent between holding money, collateral or both.

5 Costly collateral

We now analyze the case when collateral is costly ($\beta R < 1$). Under this condition in any equilibrium where agents hold collateral the borrowing constraint is binding implying from (12) that the marginal value of collateral is[8]

$$V_b = (1-n)u'(q)\beta R/\Delta + n\beta R. \tag{17}$$

According to (17) an agent with an additional unit of collateral becomes a seller with probability n in which case the real return is βR. He becomes a buyer with probability $1-n$. In this case he uses the collateral to borrow $\frac{R}{\phi_{+1}(1+i)}$ units of money which allows him

[8]If the borrowing constraint is non-binding ($\lambda_\ell = 0$), equation (12) reduce to $V_b = \beta R$ implying from (5) that $b = 0$ since we have $\beta R < 1$. Consequently, in any equilibrium where agents hold collateral it must be the case that the constraint is binding ($\lambda_\ell > 0$) and so $\bar{\ell} = Rb/\left[\phi_{+1}(1+i)\right]$ implying $\frac{\partial \bar{\ell}}{\partial b} = R/\left[\phi_{+1}(1+i)\right]$.

to acquire $\frac{R}{p\phi_{+1}(1+i)}$ units of goods in market 2. From (7) this amount is $\frac{\beta R(1+i_d)}{1+i} = \beta R/\Delta$.

Use the first-order condition (5) to get

$$\frac{1-R\beta}{R\beta} = (1-n)\left[u'(q)/\Delta - 1\right]. \tag{18}$$

Using (4), (7), (11), and taking into account that in a stationary equilibrium, $M_{+1}/M = \phi/\phi_{+1} = \gamma$, we obtain

$$\frac{\gamma - \beta\left(1+i_d\right)}{\beta\left(1+i_d\right)} = (1-n)\left[u'(q) - 1\right]. \tag{19}$$

Also from (1) we get

$$\gamma = 1 + i_d - (1-n)(i-i_d)\frac{z_\ell}{z_m}, \tag{20}$$

where $z_m = m/p$ and $z_\ell = \ell/p$. To derive this equation we use $d = m + pq_s$, $nq_s = (1-n)q$ and we set $m = M$ since in symmetric equilibrium all agents hold identical amounts of money when they enter the second market. Finally, from the budget constraint of the buyer we have

$$q = z_m + z_\ell \tag{21}$$

We can use these four equations to define an equilibrium. They determine the endogenous variables (γ, q, z_ℓ, z_m). The first two equations yield q and γ. Once we have solved for q and γ equations (20) and (21) yield z_ℓ and z_m. Note that all other endogenous variables can be derived from these equilibrium values.

Definition 2 *A symmetric stationary equilibrium with binding collateral constraint is a list (γ, q, z_ℓ, z_m) satisfying (18)-(21) with $z_\ell > 0$, $z_m \geq 0$ and $\lambda_\ell > 0$.*

The system of equations (18)-(21) can be reduced to the following two equations in b

and q

$$\frac{1}{R\beta} = (1-n)u'(q)/\Delta + n \tag{22}$$

$$q = \beta RbF(\Delta) \tag{23}$$

where

$$F(\Delta) = \frac{1}{\Delta}\left[1 + \frac{(1-n)(\Delta-1)}{1+\beta n(\Delta-1)-\Delta/R}\right].$$

Proposition 3 *For any (i, i_d) with $i \geq i_d \geq 0$ there exists a critical value*

$$\overline{\Delta} = \frac{1-n\beta}{1/R - n\beta} \tag{24}$$

such that if $1 \leq \Delta < \overline{\Delta}$, a unique equilibrium exists. If $\Delta = 1$, an equilibrium exists with $z_m = 0$.

Proof of Proposition 3. The critical elements to verify are $\gamma \geq \beta$ and $z_\ell > 0$. The first inequality holds for any (i, i_d) with $i \geq i_d \geq 0$. Consider now the inequality $z_\ell > 0$. Assume $i > 0$, then from (20) we need $1 + i_d > \gamma$. Replace γ to get

$$1 + i_d > (1/R - \beta n)(1+i) + n\beta\,(1+i_d)$$

Rearranging yields (24).

The system of equations (18)-(21)

$$z_m = M/p; z_\ell = Rb/[p\phi_{+1}(1+i)] = \beta Rb(1+i_d)/(1+i).$$

$$
\begin{aligned}
\frac{1}{R\beta} &= (1-n)u'(q)(1+i_d)/(1+i)+n && (25)\\
\frac{\gamma-\beta(1+i_d)}{\beta(1+i_d)} &= (1-n)[u'(q)-1] && (26)\\
\gamma &= 1+i_d-\frac{(1-n)z_\ell}{z_m}(i-i_d) && (27)\\
q &= z_m+z_\ell && (28)
\end{aligned}
$$

can be reduced as follows.

From the last equation, we get $z_m = q - \beta Rb(1+i_d)/(1+i)$. Multiplying both sides of (27) by z_m and replacing the expression for z_m, we get

$$(q-\beta Rb/\Delta)\left[\gamma-(1+i_d)\right] = -(1-n)z_\ell(i-i_d)$$

We can then use (26) to get rid-off γ. And we obtain

$$
\begin{aligned}
&(q-\beta Rb/\Delta)\,(1+i_d)(1-\frac{\gamma}{1+i_d}) = (1-n)z_\ell(i-i_d)\\
&(q-\beta Rb/\Delta)\,(1+i_d)(1-(1-n)\beta[u'(q)-1]-\beta) = (1-n)z_\ell(i-i_d)\\
&(q-\beta Rb/\Delta)\,(1-(1-n)\beta[u'(q)-1]-\beta) = (1-n)\frac{\beta Rb}{(1+i)}(i-i_d)
\end{aligned}
$$

Hence, an equilibrium with collateral is defined by the following two equations:

$$
\begin{aligned}
&\frac{1}{R\beta} = (1-n)u'(q)(1+i_d)/(1+i)+n\\
&(q-\beta Rb/\Delta)\,(1-(1-n)\beta[u'(q)-1]-\beta) = (1-n)\frac{\beta Rb}{(1+i)}(i-i_d)
\end{aligned}
$$

We can use the first equation to replace for $u'(q)$ in the second. We then get (22) and (23).

If we substitute (23) into (22), we get

$$\frac{1}{R\beta} = (1-n)u'\left[\beta R b F(\Delta)\right]/\Delta + n \equiv RHS \tag{29}$$

The left-hand side of (29) is constant while the right-hand side is decreasing in b for a given $1 \leq \Delta < \overline{\Delta}$. Moreover, we have $\lim_{b \to 0} RHS = +\infty$ and $\lim_{b \to \infty} RHS = n < \frac{1}{R\beta}$. Hence, for any policy Δ with $1 \leq \Delta < \overline{\Delta}$ a unique $b > 0$ exists. Then, from (23) a unique value for q exists. Accordingly a unique symmetric stationary equilibrium exist.

Finally, we have $\lim_{\Delta \to \overline{\Delta}} F(\Delta) = +\infty$ and so $b \to 0$. ■

Note that $\overline{\Delta} < \tilde{\Delta}$. Thus if the rate of return of collateral is decreasing, the central bank has to decrease the largest borrowing rate which is still consistent with the existence of an equilibrium with collateral.

Corollary 4 *Given an allocation* $(q(\Delta), b(\Delta))$ *any pair* (i, i_d) *satisfying* $\Delta = \frac{1+i}{1+i_d}$ *is consistent with this allocation.*

According to Corollary 4 there are many ways to implement a given policy Δ since any interest rate pair (i, i_d) satisfying $\Delta = \frac{1+i}{1+i_d}$ will do it. Note that the allocation is affected by a change in the band $\delta = i - i_d$ since increasing δ reduces both q and b. A central bank that wants to tighten its policy can simply increases δ. Another way to tighten monetary policy is to keep the band δ constant but to increase both interest rates (see the policy of the Reserve Bank of New Zealand in Figure 1) since this increases Δ as well.

5.1 Optimal policy

We now derive the optimal policy. The central bank's objective is to maximize the welfare of the representative agent. It does so by choosing the quantities consumed and produced in each market subject to market clearing. The policy is implemented by choosing the interest rates i and i_d.

Since the central bank is constrained to choose allocations that satisfy (22) and (23) we can use (22) to substitute Δ in (23). Moreover, since an equilibrium with a positive amount of collateral requires that $1 \leq \Delta < \overline{\Delta}$ we can use (18) to define the following constraint on q. Denote $\hat{q}$ be the level of consumption when $\Delta = 1$. From (18), it satisfies $u'(\hat{q}) = \frac{1/(\beta R)-n}{1-n}$. Let $\overline{q}$ be the level of consumption when $\Delta = \overline{\Delta}$. From (18), it satisfies $u'(\overline{q}) = \frac{1/\beta - n}{1-n}$.Accordingly, the central bank's problem is

$$\max_{q,b} \;\; (1-n)\left[u(q)-q\right] + (\beta R - 1)\, b \text{ s.t.}$$

$$q = \beta b R F\left(\frac{R\beta(1-n)u'(q)}{1-nR\beta}\right)$$

$$\hat{q} \geq q \geq \overline{q}$$

Substituting the constraint into the objective function the bank's problem becomes

$$\max_{q} \;\; (1-n)\left[u(q)-q\right] + (\beta R - 1)\frac{q}{\beta R F\left(\frac{R\beta(1-n)u'(q)}{1-nR\beta}\right)} \text{ s.t.}$$

$$\hat{q} \geq q \geq \overline{q}$$

After rearranging the first-order condition is

$$(1-n)\left[u'(q)-1\right] + \frac{1-\beta R}{\beta R F(\Delta)}\left[\frac{F'(\Delta)\,\Delta}{F(\Delta)}\frac{u''(q)\,q}{u'(q)} - 1\right] = \hat{\lambda} - \overline{\lambda}$$

where $\hat{\lambda}$ is the multiplier of the first inequality and $\overline{\lambda}$ the one of the second inequality.

Proposition 5 *There exists a critical value $\overline{R}$ such that if $R < \overline{R}$, then the optimal policy is $\Delta = \overline{\Delta}$. Otherwise the optimal policy is $\Delta \in \left(1, \overline{\Delta}\right)$.*

Proof of Proposition 5. Consider the first-order condition[9]

$$\Theta(q,R) = (1-n)\left[u'(q)-1\right] + \frac{1-\beta R}{\beta R F(\Delta)}\left[\frac{F'(\Delta)\Delta}{F(\Delta)}\frac{u''(q)\,q}{u'(q)} - 1\right] = \hat{\lambda} - \overline{\lambda}$$

where

$$\Delta(q) = \frac{R\beta(1-n)\,u'(q)}{1-nR\beta}$$

Suppose that the optimal q is such that $\Delta = 1$, i.e., $q = \hat{q}$ where $\hat{q}$ satisfies $u'(\hat{q}) = \frac{1/(\beta R)-n}{1-n}$. In this case $\overline{\lambda} = 0$ and $\hat{\lambda} > 0$ implying that $\Theta(\hat{q},R) > 0$. Then we have $F(1) = 1$, $F'(1) = \frac{1-nR}{R-1}$ and so

$$\Theta(\hat{q},R) = \frac{1-\beta R}{\beta R}\frac{1-nR}{R-1}\frac{u''(\hat{q})\,\hat{q}}{u'(\hat{q})} < 0$$

which is a contradiction. Thus, in any equilibrium $q < \hat{q}$ implying $\Delta > 1$.

Now suppose that the optimal q is such that $\Delta = \bar{\Delta} = \frac{1-\beta n}{1/R-\beta n}$, i.e., $q = \bar{q} = \frac{1/\beta - n}{1-n}$. In this case $\overline{\lambda} > 0$ and $\hat{\lambda} = 0$ implying that $\Theta(\bar{q},R) < 0$. One can show that $\lim_{\Delta\to\bar{\Delta}} F(\Delta) = \infty$, $\lim_{\Delta\to\bar{\Delta}} F'(\Delta) = \infty$, $\lim_{\Delta\to\bar{\Delta}} \frac{F'(\Delta)\Delta}{F(\Delta)} = \infty$ and $\lim_{\Delta\to\bar{\Delta}} \frac{F'(\Delta)\Delta}{F(\Delta)F(\Delta)} = \frac{(1-1/R)}{(1/\Delta)^2(1-n)(\Delta-1)^2}$. Moreover, $(1-n)\left[u'(q)-1\right] = 1/\beta - 1$. Accordingly, we get

$$\Theta(\bar{q},R) = 1/\beta - 1 + \frac{1-\beta R}{\beta R}\frac{R(1-\beta n)^2}{(R-1)(1-n)}\frac{u''(\bar{q})\,\bar{q}}{u'(\bar{q})}$$

Consider first $R \to 1$. Then we have $\lim_{R\to 1}\Theta(\bar{q},R) = -\infty$. Now consider $R \to 1/\beta$. Then we have $\lim_{R\to 1/\beta}\Theta(\bar{q},R) = 1/\beta - 1 > 0$. Since $\frac{1-\beta R}{\beta}\frac{(1-\beta n)^2}{(R-1)(1-n)}$ is monotonically decreasing in R we have a unique critical value $\overline{R}$ such that $\Theta\left(\bar{q},\overline{R}\right) = 0$. Thus if $R < \overline{R}$, $q = \bar{q} = \frac{1/\beta - n}{1-n}$ and if $R > \overline{R}$, q solves $\Theta(q,R) = 0$. ■

[9] The following proofs omit many intermediate steps. A file containing the full proof is available upon request.

6 APPENDIX

In this Appendix we show that if the central bank's objective is to maximize the expected discounted utility of the representative agent, the central banks objective is to maximize (2). To derive (2) we must first calculate hours worked in market 1. The money holdings at the opening of the first market are $m = 0$ having bought and $m = m_{-1} + pq_s$ having sold. Hence, hours worked are

$$\begin{aligned} h_b &= \phi[m + (1+i)\ell] - (R-1)b \\ h_s &= \phi[m - (1+i_d)(m_{-1} + pq_s)] - (R-1)b \end{aligned}$$

Since $h = nh_s + (1-n)h_b$, we get

$$\begin{aligned} h &= -(R-1)b + \phi m + (1-n)\phi(1+i)\ell - n\phi(1+i_d)(m_{-1} + pq_s) \\ &= -(R-1)b + \phi m + \phi m_{-1} - \phi m_{-1} + (1-n)\phi(1+i)\ell - n\phi(1+i_d)(m_{-1} + pq_s) \\ &= -(R-1)b + \varphi + (1-n)\phi\ell - n\phi(m_{-1} + pq_s) + \phi m_{-1} \\ &= -(R-1)b + (1-n)\phi\ell - n\phi(m_{-1} + pq_s) + \phi m_{-1} \end{aligned}$$

where the last equality follows from (1) and the fact that $d = m_{-1} + pq_s$, so that

$$\varphi = \phi m - \phi m_{-1} + (1-n)\phi i\ell - n\phi i_d(m_{-1} + pq_s) = 0.$$

Hence we get

$$h = -(R-1)b + (1-n)\phi\ell + (1-n)\phi m_{-1} - n\phi pq_s = -(R-1)b.$$

where the last equality follows from the fact that $pq = m_{-1} + \ell$ and market clearing requires $q_s = \frac{1-n}{n}q$. Then, welfare is given by

$$\begin{aligned} \mathcal{W} &= -b + (1-n)\left[u\left(q\right) - q\right] + \sum_{j=1}^{\infty} \beta^j \left\{(1-n)\left[u\left(q\right) - q\right] + (R-1)\, b\right\} \\ &= \frac{(1-n)[u(q) - q] + (\beta R - 1)b}{1-\beta} \end{aligned}$$

To calculate welfare, it is also useful to consider the economy that starts at date $t = 0$, at the beginning of the centralized market when agents having no financial obligations toward the central bank. From then on, the economy is in steady state. At $t = 0$, agents do not hold any collateral and have to produce the steady state level b. Hence, at $t = 0$, $h(0) = b$, while for all $t \geq 1$, $h(t) = -(R-1)b$.

When agent start with m money holding in period 0 at the beginning of the centralized market, the expected discounted payoff of an agent from date 0 onward is

$$W(m, 0, 0, 0) = -h(0) + V(m_2, b_2)$$

In a steady state equilibrium the expected discounted payoff of an agent at the beginning of market 1 is

$$V(m, b) = (1-n)[u(q) + \beta W(m - pq + \ell, b, \ell, 0)] + n[-q_s + \beta W(m + pq_s - d, b, 0, d)]$$

where

$$\begin{aligned} W(m - pq + \ell, b, \ell, 0) &= -h_b + V(m_{+1}, b_{+1}) \\ W(m + pq_s - d, b_1, 0, d) &= -h_s + V(m_{+1}, b_{+1}) \end{aligned}$$

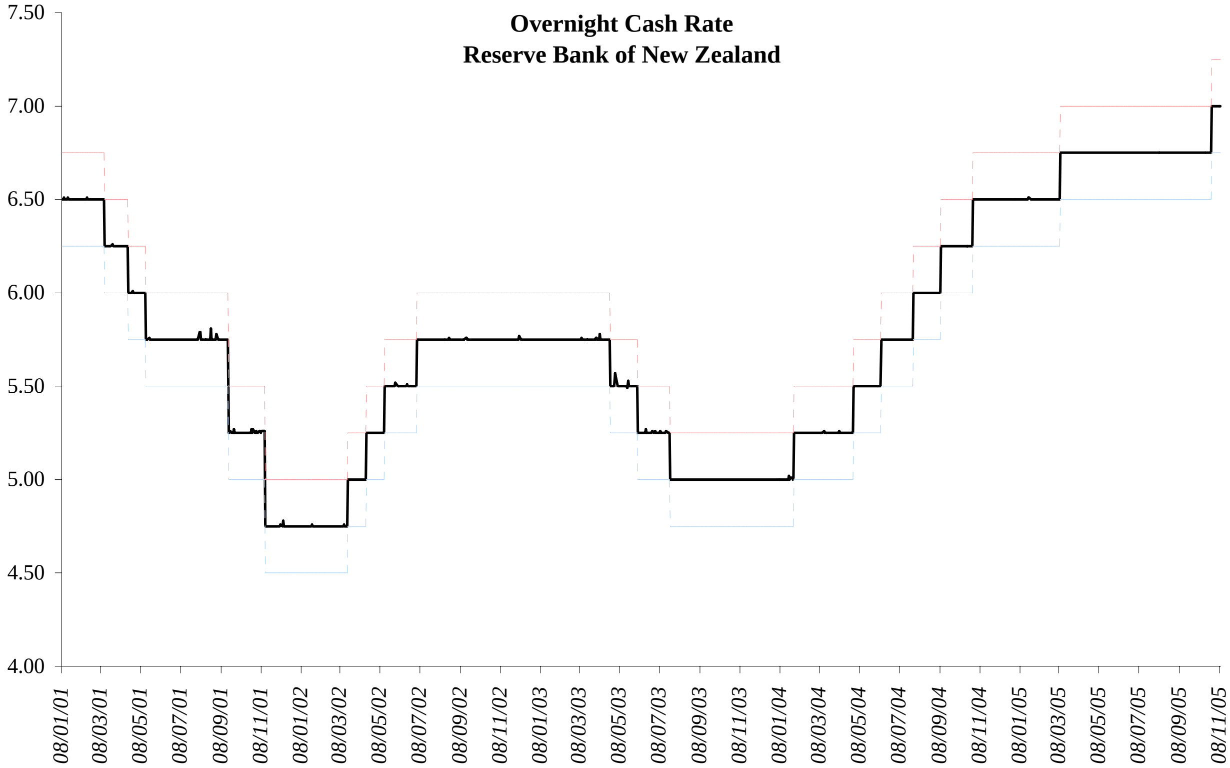
Using the definitions for $W(m - pq + \ell, b, \ell, 0)$ and $W(m + pq_s - d, b, 0, d)]$ we get

$$
\begin{aligned}
(1-\beta)V(m,b) &= (1-n)[u(q) - h_b] - n[q_s + h_s] \\
&= (1-n)[u(q) - q] + (R-1)b
\end{aligned}
$$

Hence, using the fact that $h(0) = b$, the expected discounted payoff of a representative agent is

$$
W(m_1, 0, 0, 0)(1-\beta) = (1-n)[u(q) - q] + (\beta R - 1)b
$$

which is equal to (2).


Overnight Cash Rate
Reserve Bank of New Zealand
7.50
7.00
6.50
6.00
5.50
5.00
4.50
4.00
08/01/01
08/03/01
08/05/01
08/07/01
08/09/01
08/11/01
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