Forecasting with Judgment and Models

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Abstract

This paper proposes a parsimonious and model-consistent method for combining forecasts generated by structural micro-founded models and judgmental forecasts. The goal is to produce forecasts that are model-based, and therefore disciplined by the rigor of the economic model, but that can also incorporate judgmental information. In our set-up, there are three actors: the economic agents and two types of forecasters, the purely-model based and the judgmental forecasters. They all know the true model of the economy, but their information sets differ. The economic agents observe shocks as they realize and make their decisions consequently, while the forecasters do not observe current shocks. The judgmental forecasters however have access to more timely information than the purely model-based forecasters, but their forecasts are affected by some noise (i.e. they are not perfectly rational). Thus, the idea is to extract such information from the judgmental forecasts. This method also allows interpreting the judgmental forecasts through the lens of the model. We illustrate the proposed methodology with a real-time forecasting exercise, using a stripped-to-the-bone version of an RBC model and the Survey of Professional Forecasters.

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1 Introduction

Much of the macroeconometric literature of the last decade has focused on making micro-founded dynamic stochastic general equilibrium (DSGE) models a viable option for policy analysis and forecasting. Since Smets and Wouters (2004) have shown that DSGE models estimated with Bayesian techniques seem to perform quite well in forecasting relative to standard benchmark models such as VARs, DSGE models are playing a more relevant role in practice and have indeed become an increasingly important tool for policy analysis and forecasting at central banks. The attractiveness of using these models to forecast derives from the fact that they are theoretically consistent and based on first principles. The micro-foundations imply that the parameters are more likely to be truly structural and allow interpreting the forecasts in an economically intuitive way. Moreover, the structural nature of the model allows computing forecasts conditional on a policy path and allows examining the structural sources of the forecast errors and their implications for monetary policy.

Despite their growing employment in practice, model-based forecasts still seem to be outperformed at short horizons -and particularly in the nowcast[1]- by forecasts produced by institutional and professional forecasters, such as the Federal Reserve's Greenbook (e.g. Sims, 2003) or the Survey of Professional Forecasters.[2] Where does this advantage come from?

Professional forecasters monitor and analyze literally hundreds of data series, using informal methods to distill information from the available data. Not only they access what is generally called *hard* data (data series that are released by the statistical agencies, as for example, GDP, industrial production, etc), they also gather so-called *soft* information, *i.e.* things like the quantity of goods transported by railway in each month (Bruno and Lupi, 2004) or the electricity consumed each month (Marchetti and Parigi, 1998). Moreover, professional forecasters are able to incorporate new data and new information as it becomes available throughout the month or the quarter and therefore are can take advantage of the *timeliness* of this information. Indeed, as Giannone, Reichlin, Small (2005) point out, timely information seems to play a very important role in improving the quality of the forecasts, and of the nowcasts in particular. Finally, in their forecasts, professional forecasters account also for all sheerly judgmental information. A typical example is the adjustments of the forecasts made in 1999 in order to account for the fear of the Y2K bug. Indeed this seemed at the time a very important event, but since it had never happened no model could be expected to encompass it, while the institutional forecasters could.

[1]Nowcasts are estimates of the current value of variables, such as GDP, that are unknown in the current period due to information lags

[2]This view has recently been challenged by Edge, Kiley and Laforte (2006), who suggest that a richly specified DSGE models have a forecasting performance that is comparable to that of the Greenbooks. We believe that their results are very much related to the sample they choose for their out of sample exercise, i.e. 1996-2001. We will discuss this in more detail in the empirical application.

Hence, judgement - *i.e.* information, knowledge and views outside the scope of a particular model [3] - strongly informs the institutional forecasts. The empirical evidence at hand suggests that the ability to account for more, more timely and "softer" information is what makes the professional or judgmental (I will use the two terms interchangeably from now on) forecasts better at nowcasting and forecasting short horizons.

The introduction of DSGE models in a policy and projection environment has given rise to a literature on how the model's outcomes should be combined with judgmental input and off-model information. The aim of this paper is to propose a method for combining judgment - proxied by judgmental forecasts - with model-based forecasts, in order to make predictions that are more accurate but nevertheless disciplined by rigorous economic theory. In particular, we propose to interpret the judgmental forecasts as an estimate - made with a different, possibly more informative, information set - of the real signal, estimate which can be filtered in order to extract the information it possibly contains. We then use the model to generate another forecast that can now also account for judgmental information and therefore make more accurate predictions. The new forecast that we generate is a combination of the model-based forecast and the judgmental forecast: the Kalman filter will automatically associate weights to the two forecasts depending on the information content of the judgmental forecasts. Moreover, with the methodology we propose, we will be able to look at the judgmental forecasts through the lens of the model. Storytelling is difficult when it comes to judgmental forecasts; in our set-up we will be able to interpret the forecasts in light of the model and therefore somehow structuralize the forecasts. The approach we propose is similar in spirit to the one used by Coenen, Levin and Wieland (2005) and Koenig (2005) to deal with revisions. One of the key factors that distinguishes our approach from theirs is that we consider the judgmental forecasts as optimal forecasts made with a different information set, not a noisy signal of the actual variables.

Recently, other authors have addressed the issue of how to use soft data and judgment in models. Svensson (2005), Svensson and Tetlow (2005) and Svensson and Williams (2005) develop different frameworks that allow accounting for central-bank judgment when constructing optimal policy projections of the target variables and the instrument rate. They show that such monetary policy may perform substantially better than monetary policy that disregards judgment and follows a given instrument rule. Our approach differs quite substantially from theirs: our goal is solely to produce model-based forecasts that can account for judgmental and off-model information. Our approach leaves the structure of the DSGE model unchanged and combines the model-based forecasts with the judgmental forecasts.

In a Bayesian framework, Robertson, Tallman and Whiteman (2005) suggest a minimum relative entropy procedure for imposing moment restrictions on simulated forecasts distributions from a variety of models. This technique involves changing the initial predictive distribution to a new one that satisfies specific moment conditions that come from outside of the models, *i.e.* that are judgmental. Therefore, minimum-entropy methods allow adjusting the full posterior distribution of the DSGE models to match a given experts' assessment.

Another approach that can be used to incorporate judgmental and off-model

[3] This definition appears in Svensson (2005)

information is that of Boivin and Giannoni (2005). They build on the factor model literature, started by Stock and Watson (1999, 2002) and Forni, Hallin, Lippi and Reichlin (2000), and propose an empirical framework for the estimation of DSGE models that exploits the relevant information from a data-rich environment. Their methodology allows using as much information as possible to estimate the structural model and to update the estimates of the state variables featuring in the model. In this way, soft information can be used systematically to update the current assessment of the state variables as well as of the short-term forecast.

The paper is structured as follows. In Section 2 we outline the framework and describe the proposed methodology; we illustrate how to extract the weights given to the model-based and the judgmental forecast; and describe how to structuralize the professional forecasts. In Section 3 we apply the proposed methodology on a stripped-to-the-bone version of an RBC model using the Survey of Professional Forecasters' forecasts to extract eventual judgmental information. Section 4 presents the results of the empirical application described in the previous section. In Section 5 we give some conclusions and outline future extensions of this paper.

2 The Econometric Methodology

2.1 The Framework

Let us consider a general linear(ized) rational expectations model of the form

$$AE_t \begin{bmatrix} z_{t+1} \\ Z_{t+1} \end{bmatrix} = B \begin{bmatrix} z_t \\ Z_t \end{bmatrix} + Cx_t \tag{1}$$

$$x_t = Mx_{t-1} + \varepsilon_t \tag{2}$$

$$\varepsilon_t \sim WN(0, Q)$$

where z_t is a vector of non predetermined endogenous variables, Z_t is a vector of predetermined endogenous variables or of lagged exogenous variables such that $E_t Z_{t+1} = Z_{t+1}$, x_t is a vector of exogenous variables following the process (2), Q is diagonal and A, B, C and M are conformable matrices of coefficients that form a structural parameter space that we shall call Θ. Linear(ized) general equilibrium models containing additional lags, lagged expectations or expectations farther in the future can be cast in this form simply by expanding the vectors z_t and Z_t appropriately. Several numerical techniques have been developed to solve models of the form (1)-(2), see, e.g., Blanchard and Kahn (1980), Klein (1997) and Sims (2000). The solution of the model has the following state-space representation

$$S_t = \begin{bmatrix} Z_t \\ x_t \end{bmatrix} = F(\theta)S_{t-1} + H(\theta)\varepsilon_t \tag{3}$$

$$z_t = N(\theta)S_t, \tag{4}$$

where $F(\theta), H(\theta)$ and $N(\theta)$ are functions of the underlying structural parameters.

To allow for greater generality we augment each equation in (4) with a (possibly serially correlated) residual, or error term, as in Ireland (2004). The model now consists of the transition equation (3) and the new observation equation

$$y_t = z_t + v_t = N(\theta)S_t + v_t, \tag{5}$$

where

$$v_{t+1} = Dv_t + \xi_{t+1} \tag{6}$$

for all $t = 1, 2, ...$ and ξ_t is a vector of zero mean, serially uncorrelated innovations that is normally distributed with covariance matrix $E\xi_t\xi_t' = V$ and is uncorrelated with the innovation ε_t. There are two appealing features in this set-up. First, depending on the way in which the matrices D and V are defined, the residuals can be interpreted as measurement errors or as additional elements capturing all of the movements and co-movements in the data that the DSGE models, because of their elegance and parsimony, cannot explain. Second, the model consisting of (3),(5) and (6) overcomes the well-known stochastic singularity problem, pointed out by Ingram et al.(1994), that often appears in DSGE models, deriving from the assumption that few fundamental shocks drive all the dynamics of the model.

Model (3),(5) and (6) can be rewritten more compactly as:

$$s_{t+1} = \begin{bmatrix} S_{t+1} \\ v_{t+1} \end{bmatrix} = G(\theta)s_t + \nu_{t+1} \tag{7}$$

$$y_t = \Lambda(\theta)s_t \tag{8}$$

where $G(\theta) = \begin{bmatrix} F(\theta) & 0 \\ 0 & D \end{bmatrix}$, $\Lambda(\theta) = \begin{bmatrix} N(\theta) & I \end{bmatrix}$ and $\nu_t = \begin{bmatrix} H(\theta)\varepsilon_t \\ \xi_t \end{bmatrix}$ is serially uncorrelated, normally distributed with zero mean and covariance matrix $E\nu_t\nu_t' = Q = \begin{bmatrix} H(\theta)\Sigma H(\theta)' & 0 \\ 0 & V \end{bmatrix}$.

From now on, for notational simplicity, we will drop the indication that the matrices G, Λ, etc... are function of the structural parameters θ.

Associated with the state space representation (7)-(8) is the innovations representation[4]

$$\hat{s}_{t|t} = G\hat{s}_{t-1|t-1} + Ku_t \tag{9}$$

$$y_t = \Lambda G\hat{s}_{t-1|t-1} + u_t \tag{10}$$

where $\hat{s}_{t|t} = E[s_t|y_t, y_{t-1}, ..., y_0, \hat{s}_0]$ is the estimate of the state vector s_t based on the observations of y_τ up to date t, $u_t = y_t - y_{t|t-1} = y_t - E[y_t|y_{t-1}, ..., y_0]$ the forecast error made when forecasting y_t given the observations of y_τ up to date *t-1*, $K = GP\Lambda'(\Lambda P\Lambda')^{-1}$ is the steady state Kalman gain, and P is unique positive semidefinite solution that satisfies the algebraic Riccati equation

$$P = Q + GPG' - GP\Lambda'(\Lambda P\Lambda')^{-1}\Lambda PG'. \tag{11}$$

P is the steady-state covariance matrix of the innovations $s_t - s_{t|t-1}$ given the information in period $t-1$. It is useful to rewrite the innovations representation

[4]The conditions for the existence of this representation are stated carefully, among other places, in Anderson, Hansen, McGrattan, and Sargent (1996). The conditions are that that (F,H,N) be such that iterations on the Riccati equation for $\Sigma_t = E(x_t - \hat{x}_t)(x_t - \hat{x}_t)'$ converge, which makes the associated Kalman gain K_t converge to K. Sufficient conditions are that (F, N') is stabilizable and that (F', H') is detectable. See Anderson, Hansen, McGrattan, and Sargent (1996, page 175) for definitions of stabilizable and detectable.

given by (9) and (10) as the sum of two components: one that is forecastable given the information set containing all observations of y_τ up to date *t-1*,

$$M_t = Sp\{y_{t-1}, y_{t-2}, ...y_0\}, \tag{12}$$

and a sequence of innovation terms. That is,

$$\hat{s}_{t+h|t+h} = G^{h+1}\hat{s}_{t-1|t-1} + \sum_{j=0}^{h-1} G^j K u_{t+h-j} \tag{13}$$

$$y_{t+h} = \Lambda G^{h+1}\hat{s}_{t-1|t-1} + \Lambda G \sum_{i=1}^{h} G^i K u_{t+h-i} + u_{t+h}. \tag{14}$$

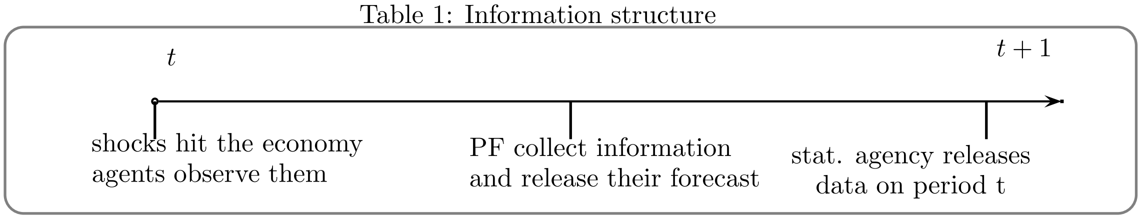
2.2 Model of the Professional Forecasts

The goal of this section is to show how to incorporate judgmental forecasts into models of the form (7)-(8), or equivalently in (9)-(10). In order to do so, we need to somehow formalize these forecasts. More specifically, we need to make assumptions on the model and the information set that the professional forecasters use to generate their forecasters. In this paper, we will make the assumption that the professional forecasters know the structure of the economy, *i.e.* know the model (7)-(8) and can use it to forecast, but they access an information set, I_t, which is more informative than M_t.

The assumptions on the information available in each period t are outlined in Table 1. We assume that the shocks hit the economy at the beginning of period t. The agents observe the shocks and base their decisions on this knowledge. We then assume that there are three types of forecasters. The first type generates his forecasts on the basis of the model and the data released by the statistical agency. The data reporting agency releases data about the current period at the end of the period, so in period t there is data available only up to t-1. Therefore the information set available to the first type of forecaster in time t comprises exclusively information up to time t-1: his information set is M_t, as defined in (12). From now on I will call the first type of forecaster 'purely' model-based forecaster.

The second type of forecaster also knows the model of the economy and uses it to make its forecasts, but accesses another information set I_t, which comprises M_t but is possibly more informative. This type represents the professional forecasters (PF from now on). As highlighted in the Introduction, their information set is plausibly richer than M_t: they collect soft, intra-period extra-model information, such as monthly electricity consumption or quantity of goods transported by railway in each month. In what follows, we assume that professional forecasters have extra-model information only on the current period's shock but not on future shocks. In appendix we illustrate the case in which we assume that they have some extra-model information, not exclusively on the current shocks, but possibly also on future shocks. Considering the way the professional forecasts are made and the type of extra-information they use - i.e., information about this month electricity consumption, but also information on future tax raises or on the possible effects of extraordinary events like the Y2K bug or the World Cup- , both cases are credible. However, the possible inconsistency arising from assuming that the Professional Forecasters know more

Table 1: Information structure



about the future that the agents, but that the agents don't incorporate them in their solution persuaded me to relegate this part to the appendix.

Finally, the third type of forecaster will use the method I propose: I will define the forecasts they produce augmented forecasts, since the use the PF to augment their information set.

Let us formalize rigorously the second type of forecaster, the professional forecasters. PF are able to obtain information on the *current* period's shocks. At any given time T their information set I_T comprises M_T but is such that, for $h = 1, 2, 3, 4$

$$E[u_T I_T'] \neq 0, \qquad (15)$$
$$u_{T+h} \perp I_T$$

The forecasters know the model of the economy and produce their forecasts as linear least squares forecasts given their information set plus an error term.[5] Notice that if we just modelled the PF's forecasts as a noisy version of the actual signal, *i.e.*

$$s_{t+1+h} = G s_{t+h} + \nu_{t+1+h}$$
$$y^{PF}_{t+h} = \Lambda s_{t+h} + e_{t+h}$$

this formulation would be totally inconsistent with our assumptions. First of all, the e_{t+h}'s would not be forecast errors - since they are not orthogonal to the past - and therefore the PF would not be forecasts, but rather noisy signals of actual future variables. This would mean that we are assuming that the PF have crystal balls through which they see the future. This is neither realistic, nor model-consistent. Instead we want to model the output of the professional forecasters as forecasts, as shown in detail below. Sargent (1989) first distinguished among these two possible modelizations, discussing two models of a statistical agency that is collecting and reporting observations on a dynamical linear stochastic economy.

For $t = 1, 2, ..., T - 1$ both purely model-based forecasters and professional forecasters are going to construct the innovations representation (9)- (10). For $t \geqslant T$, professional forecasters will report the following: for h=0,

$$\hat{s}_{T|T} = G\hat{s}_{T-1|T-1} + K u_T \qquad (16)$$
$$y^{PF}_t = E[y_T|I_T] + \eta_{T|T} \qquad (17)$$

where $E[y_T|I_T] = \Lambda G \hat{s}_{T-1|T-1} + E[u_T|I_T]$ is the least squares forecast made by the PF with their information set I_t and $\eta_{T|T}$ is the measurement error (the

[5] For a recent review of the debate on the rationality (unbiasedness and efficiency) of macroeconomic forecasts see Schuh(2001)

typo) made by the professional forecasters in T while reporting their forecast. This can be cast in state-space form as follows. For h=0,

$$\begin{aligned} \hat{s}_{T|T} &= G\hat{s}_{T-1|T-1} + Ku_T \\ y_t^{PF} &= \Lambda G\hat{s}_{T-1|T-1} + w_{T|T}, \end{aligned} \tag{18}$$

where

$$w_{T|T} = E[u_T|I_T] + \eta_{T|T},$$

and

$$\begin{bmatrix} Ku_T \\ w_{T|T} \end{bmatrix} \sim WN\left(\; 0 \quad , \quad \Sigma_0 \; \right)$$

with

$$\Sigma_0 = \begin{bmatrix} KE(u_T u_T')K' & KE(u_T w_{T|T}') \\ E(w_{T|T} u_T')K' & Ew_{T|T} w_{T|T}' \end{bmatrix}.$$

For $h = 1, 2, 3, 4$ we have

$$\begin{aligned} \hat{s}_{T+h|T+h} &= G\hat{s}_{T+h-1|T+h-1} + Ku_{T+h} \\ y_{T+h}^{PF} &= \Lambda G\hat{s}_{T-1|T-1} + \Lambda G^h KE[u_T|I_T] + \eta_{T+h|T}, \\ &= \Lambda G\hat{s}_{T+h-1|T+h-1} + w_{T+h|T} \end{aligned} \tag{19}$$

where

$$w_{T+h|T} = -\Lambda \sum_{j=1}^{h-1} G^j Ku_{T+h-j} - \Lambda G^h K(u_T - E[u_T|I_T]) + \eta_{T+h|T} \tag{20}$$

$$\begin{bmatrix} Ku_{T+h} \\ w_{T+h|T} \end{bmatrix} \sim WN\left(\; 0 \quad , \quad \Sigma_h \; \right)$$

and

$$\Sigma_h = \begin{bmatrix} KE(u_{T+h} u_{T+h}')K' & 0 \\ 0 & Ew_{T+h|T} w_{T+h|T}' \end{bmatrix}.$$

As above, $\eta_{T+h|T}$ is the measurement error (the typo) made by the professional forecasters in T while reporting their forecast for period $T+h$. We assume that $\eta_{s|T} \perp u_\tau$, for any s and τ, and that $\eta_{T+h|T} \perp E(u_T|I_T)$ for $h = 0, 1, 2, 3, 4$. Clearly, the form of the matrices Σ_h depends crucially on the assumptions we made on the information set of the professional forecasters. Here Σ_h will be block diagonal for all $h \neq 0$, since we have modeled the PF as having some extra-information only on the current shock. In appendix we present the case in which the PF can have extra-information up to 4 periods ahead. In that case, the Σ_h will not necessarily be block diagonal: the value of the off-diagonal terms will depend on the information on future shocks actually carried by the professional forecasts.

We want to extract the optimal linear projection of y_{T+h} given I_T, i.e $E[y_{T+h}|I_T]$, from y_{T+h}^{PF}. To understand how the augmented forecasts are built, consider the fact that models (18) and (19) can be seen as a new state space model in which the new observables are the professional forecasts. By filtering model (18)-(19) with a time-varying Kalman smoother one can obtain optimal estimates of the state variables $s^{+}_{T+h|I_T}$ that comprise the extra-information contained in the professional forecasts and employ it optimally within the model.[6] Having $\hat{s}^{+}_{T+h|I_T}$ for we can finally construct the augmented forecasts $y^{+}_{T+h|I_T}$

$$y^{+}_{T+h|I_T} = \Lambda \hat{s}^{+}_{T+h|I_T} \tag{21}$$

that incorporates optimally in the model-based framework the judgemental information coming from the conjunctural forecasters.

In order to implement the Kalman filter, we need to be able to recover the exact form of the covariance matrices Σ_0 and Σ_h for h=1,2,3,4. To recover all the elements of Σ_0, we proceed as follows. First of all let us point out that, since $\eta_{T|T} \perp u_T$ by assumption, then

$$E(u_T w'_{T|T}) = E[u_T E(u_T|I_T)'] + E[u_T \eta'_{T|T}] = E[u_T E(u_T|I_T)'].$$

The element on the right hand side of this equation can be calculated as follows. First notice that the following equality holds

$$\begin{aligned} E[u_T y_T^{PF\prime}] &= E[u_T(\Lambda G \hat{s}_{T-1|T-1})'] + E[u_T E(u_T|I_T)'] + E[u_T \eta'_{T|T}] \\ &= E[u_T E(u_T|I_t)\prime], \end{aligned}$$

The second equality derives from the fact that $u_T \perp \Lambda G \hat{s}_{T-1|T-1}$ by construction and that $\eta_T \perp u_T$ by assumption. Finally, as the series for the u_T's are readily available via the Kalman filter, we are able to recover empirically the value of $E[u_T y_T^{PF\prime}]$, and therefore of $E[u_T E(u_T|I_T)']$. Moreover, notice that, since $E(u_T|I_T)$ is a linear projection of u_T on $Sp(I_T)$, the space spanned by I_T, then

$$u_T = E(u_T|I_T) + \mu_T$$

where μ_T is orthogonal to the space spanned by I_T. Therefore,

$$E[u_T E(u_T|I_T)'] = E[E(u_T|I_T)E(u_T|I_T)'], \tag{22}$$

i.e. we have determined also the variance of the expected value of the current shock given the information set I_T, $E(u_T|I_T)$, and we showed it is equal to the covariance among the shock and its expected value.

In order to recover

$$E[w_{T|T} w'_{T|T}] = E[E(u_T|I_T)E(u_T|I_T)'] + E[\eta_{T|T}\eta'_{T|T}], \tag{23}$$

(the equality holds because $\eta_{T|T} \perp E(u_T|I_T)$ by assumption), we will first define the forecasters' forecast error as

$$\begin{aligned} e_T &= y_T - y_T^{PF} \\ &= u_T - w_{T|T}. \\ &= u_T - E(u_T|I_T) - \eta_{T|T}. \end{aligned}$$

Its variance, whose value can be recovered from sample data, is:

$$\begin{aligned} E(e_T e'_T) &= E(u_T u'_T) - E[u_T E(u_T|I_T)'] + \\ &- E[u_T E(u_T|I_T)']' + E[w_{T|T} w'_{T|T}] \end{aligned} \tag{24}$$

[6]The notation $\hat{s}^{+}_{T+h|I_T}$ mean the estimate of the state s_{T+h} made in T give the augmented information set I_T.

$E(u_T u_T')$ can be obtained by the Kalman filter on the system of equations (7) and (8) as follows

$$E(u_T u_T') = \Lambda P \Lambda'$$

Where P is the solution of the Riccati equation defined in (11). Using the above equations, we can finally recover $E[w_{T|T} w_{T|T}']$ and we therefore have pinned down all the values of the matrix Σ_0.

Moreover, it is possible to recover the value of the variance of the typo $E[\eta_{T|T}\eta_{T|T}']$; from (23) and (24) we infer the following equation, which can be reshuffled to obtain $E[\eta_{T|T}\eta_{T|T}']$.

$$E(e_T e_T') = E(u_T u_T') - E[E(u_T|I_T)E(u_T|I_T)'] + E\eta_{T|T}\eta_{T|T}'. \tag{25}$$

The procedure to recover Σ_h, for h=1,2,3,4 is very similar.

2.3 Model-consistent weights for forecast pooling

Now we discuss how the time-varying Kalman smoother we use to generate the "judgment-augmented" forecasts actually combines the judgmental forecasts with the purely model-based forecasts. Let us turn back to the system of equations we smooth to obtain the augmented forecasts, *i.e.*

$$\begin{aligned} \hat{s}_{T+h|T+h} &= G\hat{s}_{T+h-1|T+h-1} + Ku_{T+h} \\ y^{PF}_{T+h} &= \Lambda G \hat{s}_{T+h-1|T+h-1} + w_{T+h|T}, \end{aligned} \tag{26}$$

where for h=0

$$w_{T|T} = E[u_T|I_T] + \eta_{T|T} \tag{27}$$

and for h=1,2,3,4,

$$w_{T+h|T} = -\Lambda \sum_{j=1}^{h-1} G^j K u_{T+h-j} - \Lambda G^h K(u_T - E[u_T|I_T]) + \eta_{T+h|T}$$

and

$$\left[\begin{array}{c} Ku_{T+h} \\ w_{T+h|T} \end{array}\right] \sim WN\left(0 \quad , \quad \left[\begin{array}{cc} \Sigma^h_{11} & \Sigma^h_{12} \\ \Sigma^h_{21} & \Sigma^h_{22} \end{array}\right] \right).$$

In period T we filter (26) and we generate a new innovations representation, initialized with $\hat{s}^+_{T-1|I_T} = \hat{s}_{T-1|T-1}$

$$\begin{aligned} \hat{s}^+_{T+h|I_T} &= G\hat{s}^+_{T+h-1|I_T} + K_{2h} a_{T+h} \\ y^{PF}_{T+h} &= \Lambda G \hat{s}_{T+h-1|I_T} + a_{T+h}, \end{aligned} \tag{28}$$

where

$$\hat{s}^+_{T+h|I_T} = E(\hat{s}^+_{T+h|I_T} | y^{PF}_{T+h}, y^{PF}_{T+h-1}, ..., y^{PF}_T, y_{T-1}, ...y_1, \hat{s}_{T-1|T-1}),$$

$$a_{T+h} = y^{PF}_{T+h} - E\left[y^{PF}_{T+h} | y^{PF}_{T+h}, y^{PF}_{T+h-1}, ..., y^{PF}_T, y_{T-1}, ...y_1, \hat{s}_{T-1|T-1}\right].$$

The Kalman gain K_{2h} is time-varying and takes the form: for h=0,

$$K_{20} = \Sigma^0_{12} {\Sigma^0_{22}}^{-1}$$

and

$$K_{2h} \quad = \quad (GP_{h|h-1}G'\Lambda' + \Sigma_{12}^{h})(\Lambda GP_{h|h-1}G'\Lambda' + \Sigma_{22}^{h})^{-1} \tag{29}$$

otherwise. In order to understand how the Kalman filter is combining the purely model-based forecast and the judgemental forecast, let us consider the case of the nowcast:

$$\hat{s}^{+}_{T|I_T} \quad = \quad (I - K_{20})G\hat{s}_{T-1|T-1} + K_{20}y_T^{PF}, \tag{30}$$

where $\hat{s}^{+}_{T-1|I_T} = \hat{s}_{T-1|T-1}$ and $G\hat{s}_{T-1|T-1}$ is the purely model-based forecast of the state at time T. Therefore the augmented nowcast for y_T is

$$\begin{aligned} y^{+}_{T|I_T} &= \Lambda\hat{s}^{+}_{T|I_T} \\ &= \Lambda(I - K_{20})G\hat{s}_{T-1|T-1} + \Lambda K_{20}y_{T+h}^{PF} \end{aligned} \tag{31}$$

Since K_{20} has the form described in equation (29), the weight given to the judgmental forecast y_T^{PF} is directly proportional to Σ_{12}^0, *i.e.* to the correlation among u_T and $w_{T|T}$, and inversely proportional to the variance of $w_{T|T}$ Σ_{22}^0. That is, the more the professional forecasters are able to gather information on the current period's shock, the more the Kalman filter will use the professional forecasts when combining the two forecasts, but it will down-weigh them if the variance of their forecast errors is too large. Similarly for higher horizons.

The assumption that the professional forecasters have information only on the current period shocks is crucial in determining the negligible weights assigned to the professional forecasts for $h \neq 0$. In appendix we present some results for the case in which the PF are assumed to have some off-model information on current and future shocks up to four periods ahead. In that case, the weight associated to the judgmental forecast will be sizeable -depending on the informational content, of course- at all horizons considered.

2.4 Using the model to interpret judgemental forecasts

Another interesting aspect of this procedure is that it also allows to see the judgmental forecasts through the lens of the model. Storytelling is difficult when it comes to judgmental forecasts; in our set-up we will be able to interpret the forecasts in light of the model and therefore somehow structuralize the forecasts.

Let us have another look at model (28). The element $K_{20}a_T$, with

$$a_T = y_T^{PF} - E\left[y_T^{PF}|\hat{s}^{+}_{T-1|I_T}\right],$$

is the estimate of the current period's structural shocks made by the professional forecasters with their information set I_T. That is, these are the structural shocks the professional forecasters perceive given their information set; they do not necessarily coincide with the "real" structural shocks. Indeed

$$K_{20}a_T = E[Ku_T|I_T].$$

Moreover, given this information, we can also derive $[u_T|I_T]$ as follows:

$$\begin{aligned} E[u_T|I_T] &= (K'K)^{-1}K'E[Ku_T|I_T] \\ &= (K'K)^{-1}K'(K_{20}a_T). \end{aligned}$$

Now, recalling that the professional's forecasts are represented by equations (18)-(19), we can evaluate exactly how much of the forecast is due to extra information on the current shocks and how much of it is instead due to measurement errors. Moreover, we can construct different scenarios - e.g. assume that the professional forecasters have extra information only on certain types of shocks but not on others - and compare them among each other. As we will show in Section 4, this can be very informative.

3 An application

In order to illustrate the methodology proposed in the previous section, we will apply it to a stripped-to-the-bone version of an RBC model with unit root technology. Let us briefly outline the model. A representative consumer has preferences defined over consumption C_t during each period t=1,2,..., as described by the expected utility function

$$E\sum_{i=1}^{\infty}\beta^t \ln(C_t) \tag{32}$$

where the discount factor satisfies $0 < \beta < 1$. In this economy there is only one final good Y_t and it is produced using capital K_t and labor N_t according to the constant-returns-to-scale technology

$$Y_t = (A_t N_t)^{\alpha} K_t^{(1-\alpha)}, \tag{33}$$

where $0 < \alpha < 1$ and A_t is a labor-augmenting technological change process. We will assume that labor is supplied inelastically and that there is no population growth: in such case N_t is constant for any t and we can normalize it to 1, *i.e.* $N_t = 1$ for any t. We there for rewrite equation (33) as

$$Y_t = A_t^{\alpha} K_t^{(1-\alpha)}. \tag{34}$$

The logs of the technology shock A_t follow a first order autoregressive process of the form:

$$\ln(A_t) = \ln(A_{t-1}) + \gamma + \varepsilon_t, \tag{35}$$

where the innovation ε_t is serially uncorrelated and normally distributed with mean zero and standard deviation σ.

In each period the representative agent decides how much of output Y_t to consume and how much to invest, subject to the resource constraint

$$Y_t = C_t + I_t. \tag{36}$$

By investing I_t units of output in period t, the agent increased the capital stock K_{t+1} available in period $t+1$ according to

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{37}$$

where δ is the depreciation rate and satisfies $0 < \delta < 1$.

The standard method of analyzing models with steady state growth is to transform the economy into a stationary one where the dynamics are more tractable. The transformation, which is shown in great detail in King, Plosser and Rebelo (1988a, 1988b), involves dividing all variables in the system by the growth component, which in our setting corresponds to A_t.The stationarized model is very similar to the untransformed model, with some exceptions that we will highlight in what follows.

Let us start defining $E[\frac{A_{t+1}}{A_t}] = \gamma_A$, $y_t = \frac{Y_t}{A_t}$, $c_t = \frac{C_t}{A_t}$, $k_t = \frac{K_t}{A_t}$ and so on. The stationary version of the model defined by equations (32), (34), (35), (36) and (37) is

$$\max E \sum_{i=1}^{\infty} \beta^t \ln(C_t) \tag{38}$$

subject to

$$y_t = k_t^{(1-\alpha)}, \tag{39}$$

$$y_t = c_t + i_t, \tag{40}$$

and

$$(\gamma_A e^{\varepsilon_{t+1}})k_{t+1} = k_t + i_t. \tag{41}$$

The first order conditions for this problem are

$$R_t = [(1-\alpha)k_t^{-\alpha} + (1-\delta)] \tag{42}$$

where R_t is the gross rate of return of capital in t and

$$c_t^{-1} = \beta E_t[\frac{c_{t+1}^{-1} R_{t+1}}{e^{\varepsilon_{t+1}}}]. \tag{43}$$

Equation (43) equates the marginal rate of substitution to the marginal product of capital for all t=1,2,..... Equations (39)-(43) form a system of five non-linear stochastic difference equations in the model's five variables y_t, c_t, k_t, i_t, r_t. A linear approximation of this system can be derived log-linearizing it around its steady state, as shown in Appendix A. The system of equations (39)-(43) has an approximate solution of the form:

$$x_{t+1} = \begin{bmatrix} k_{t+1} \\ k_t \end{bmatrix} = G(\theta)x_t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \varepsilon_{t+1} \tag{44}$$

$$z_t = \begin{bmatrix} \Delta y_t \\ c_t - y_t \\ i_t - y_t \end{bmatrix} = \Lambda(\theta)x_t, \tag{45}$$

where the expressions for the matrices $G(\theta)$ and $\Lambda(\theta)$ in terms of the structural parameters θ can be found in Appendix A. A particular feature of this model is that one shock - the aggregate technology shock ε_t - drives all business cycle fluctuations.

Any equation of the form (45) can be rewritten in terms of differences, simply by premultiplying everything with a suitable matrix:

$$y_t = \begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1-L & 0 \\ 1 & 0 & 1-L \end{bmatrix} z_t = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1-L & 0 \\ 1 & 0 & 1-L \end{bmatrix} \Lambda x_t. \tag{46}$$

In general, equation (44) will change conformably to take into account that (46) is now defined in terms of current and lagged states s_t. In this specific case the state equation will not need to be rewritten, since it already contains the lagged state.

We also augment each equation in (46) with a serially correlated residual, or error term, so that the model now consists of (44),

$$y_t = \begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \end{bmatrix} = \Lambda^* x_t + v_t, \tag{47}$$

where $\Lambda^* = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1-L & 0 \\ 1 & 0 & 1-L \end{bmatrix} \Lambda$ and

$$v_{t+1} = D v_t + \xi_{t+1} \tag{48}$$

for all $t = 1, 2, ...$ and ξ_t is a vector of zero mean, serially uncorrelated innovations that is normally distributed with covariance matrix $E\xi_t\xi_t' = V$ and is uncorrelated with the innovation ε_t.

We have considered three possible specifications of v_t, the first one assumes v_t is white noise, the second one allows for autocorrelation (but not serial correlation) in v_t, while in the third specification we allow v_t to be a VAR(1). The results were very similar for each of the three specifications, therefore we will present results only for the model with autocorrelated but not cross-correlated residuals, which is our best performing one. All results are robust to the specification of the measurement error v_t.

We calibrated all the model's parameters, with the exclusion of the variance of the technological shock and and the parameters describing the measurement errors, using values from King, Plosser and Rebelo (1988a, 1988b) and Ireland (2004). All parameters are reported in Table 7 in Appendix A. We estimate the variance of technological shock and the parameters of the measurement error only once, using maximum likelihood, and then we take them as calibrated. The covariance matrices Σ_i in (20) are instead estimated using a rolling window.

For the estimation and the forecasting exercise we use real-time quarterly data for real GDP and real consumption for the US from the Philadelphia Fed real-time dataset. The dataset covers the period 1947 through 2005 and the first available vintage is 1965:Q4. Due to the unavailability of real-time data on population, we have made the somewhat heroic assumption that the population has been constant throughout the period considered. In what follows, we will perform an out-of-sample real-time forecasting exercise, using as evaluation sample the period 1987-2004.

We use the Survey of Professional Forecasters (SPF) as example of judgmental forecast. The Survey of Professional Forecasters, conducted by the Federal Reserve Bank of Philadelphia, is based on many individual commercial forecasts, which are then grouped in mean or median forecasts. The Survey is conducted near the end of the second month of each quarter and publishes forecasts for the current quarter and the next 4 quarters in the future. We consider a sample period going from the second quarter of 1987 to the fourth quarter of 2004.

An important data-related issue regards the appropriate "actual" series to use when comparing the various forecasts. Because macroeconomics data is continuously revised, we need to make a choice about which revision to use. Following Romer and Romer (2000), we choose to use the second revision, *i.e.*

Table 2: Relative MSFE of forecasts of GDP growth with respect to naive benchmark. Asterisks denote forecasts that are statistically more accurate than he naive benchmark at 1% ($***$), 5%($**$) and 10%($*$)

relative to constant growth		
EVALUATION SAMPLE: 1987:2 - 2004:4		
Forecast horizon	RBC	SPF
Q0	0.86	0.68 $**$
Q1	0.88 $**$	0.77
Q2	0.92	0.88
Q3	0.94 $**$	0.94
Q4	0.97	0.95
relative to constant growth		
EVALUATION SAMPLE: 1996:2 - 2004:4		
Forecast horizon	RBC	SPF
Q0	0.81$**$	0.80$**$
Q1	0.85 $***$	0.94
Q2	0.84$***$	1.01
Q3	0.84 $***$	1.01
Q4	0.87 $**$	1.01

the one done at the end of the subsequent quarter. The second revision seems to be the appropriate series to use because it is based on relatively complete data, but it is still roughly contemporaneous with the forecasts we are analyzing. This series does not include rebenchmarking and definitional changes that occur in the annual and quinquennial revisions and should, therefore, be conceptually similar to the series being forecast.

Let us now present some forecasting results that will highlight the motivation of this paper. We will compare all the forecasts of GDP and Consumption growth to a naive benchmark: the constant growth model, *i.e.* random walk in levels. Tables 2 and 3 report the performance in out-of-sample forecasting of the forecasts generated with the RBC model, specified as having autocorrelated, but not serially correlated residuals (i.e. D is diagonal) and of the SPF relative to the naive benchmark, for GDP growth and consumption growth respectively. Each table is divided in two subtables which consider the full sample period 1987:Q2-2004:Q4 and the subsample 1996:Q2-2004:Q4. In the first column of each table one can see the ratio of the mean square error of the purely model-based forecast (RBC) against the mean square error of the naive benchmark, while the second column reports the ratio of the mean square error of the SPF against the mean square error of the naive benchmark. Asterisks indicate a rejection of the test of equal predictive accuracy between each forecast and the naive benchmark. [7]

[7] Following Romer and Romer (2000), our inference is based on the regression: $(z_{ht}-\hat{z}_{ht}^{m})^2-(z_{ht}-\hat{z}_{ht}^{naive})^2=c+u_{ht}$ where z is the variable to be forecasted at horizon h using model -m . The estimate of c is simply the difference between forecast-m and a Naive model MSFEs, and the standard error is corrected for heteroskedasticity and serial correlation over h-1 months. This testing procedure falls in the Diebold-Mariano-West framework, and Giacomini and White (2006, Section 3.2, see in particular Comment 4) show that by using rolling window

Table 3: Relative MSFE of forecasts of Consumption growth with respect to naive benchmark. Asterisks denote forecasts that are statistically more accurate than he naive benchmark at 1% (∗ ∗ ∗), 5%(∗∗) and 10%(∗)

relative to constant growth		
EVALUATION SAMPLE: 1987:2 - 2004:4		
Forecast horizon	RBC	SPF
Q0	0.97	0.70
Q1	0.92 ∗∗	0.89
Q2	0.94 ∗∗	0.94
Q3	0.94 ∗	0.99
Q4	0.94 ∗	1.00
relative to constant growth		
EVALUATION SAMPLE: 1996:2 - 2004:4		
Forecast horizon	RBC	SPF
Q0	0.89 ∗∗	0.96
Q1	0.89 ∗∗	1.06
Q2	0.88 ∗∗	1.03
Q3	0.87 ∗∗	1.03
Q4	0.86 ∗∗	1.08

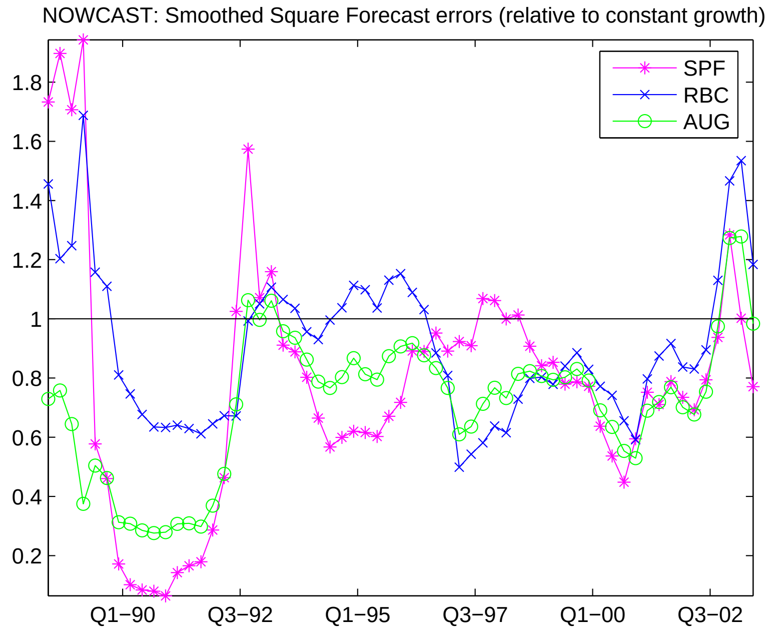
Over the full sample period 1987:Q2-2004:Q4, SPF forecasts of GDP growth outperform the model-based forecasts at all horizons, but their advantage reduces as the forecasting horizon grows. Considering the subsample 1996:Q2-2004:Q4, we notice the model seems to perform better at all horizons excluding the nowcast. The model-based forecasts of consumption growth are poorer in the very short run than in the medium term (4 quarters ahead). Over the full sample period 1987:Q2-2004:Q4, SPF forecasts of consumption growth outperform the model-based forecasts in the nowcast and 1 period ahead, and never in the subsample 1996:Q2-2004:Q4. The two results on consumption growth point to the added-value of enforcing accounting identities, especially in the medium-term.

Figure 1 reports the smoothed forecast errors for the nowcast of GDP (centered moving average 4 quarters on each side) over the full sample period. Model and judgement seem to contain information that is useful in different points in time: in some periods the model does better than the benchmark and in others it does worse, and the same holds for SPF. Particularly during recessions, judgement seems to fare much better than the model (a similar result can be found in Giannone, Reichlin and Sala, 2005).

In the last part of the sample, and particularly between 1996 and 2000, judgmental forecasts have performed quite bad, while the model seems to fare acceptably. A similar result can be found in Edge, Kiley and Laforte (2006), where the authors compare the forecasting performance of a richly specified DSGE models to the Greenbooks in the sample 1996-2000 and find that the model's forecasting performance is comparable to the one of the Greenbooks.

estimators, as we do here, the limiting behavior of this type of tests is standard, and therefore standard asymptotic theory can be used for inference on the difference in predictive accuracy.

Figure 1: **NOWCASTS: Smoothed Square forecast errors**



In fact, not only the richly specified DSGE model that Edge, Kiley and Laforte (2006) propose, but also the toy model I use seem to fare better than judgment in that period.[8]

Furthermore, Figure 1 highlights how performances of the purely model-based and the judgmental forecasts do not seem positively correlated, but rather they seem to somehow counterbalance each other. Therefore, it is plausible that combining the two forecasts can be advantageous. The method I propose indeed allows for a model-based and hence interpretable averaging of the two forecasts.

In the following section we present the results obtained when applying the methodology proposed in Section 2 to the toy model presented above and using the SPF forecasts.

4 Results of the Forecasting Exercise

The main goal of this section is to present model-based forecasts for real GDP and real consumption that can account for the judgmental information contained in the SPF forecasts. We will compare their performance on the basis of their mean square forecast error, *i.e.* deeming better a forecast with a smaller MSE. We will perform out-of-sample forecasting exercises on the full evaluation sample 1987:Q2-2004:Q4 and on the subsample 1996:Q2-2004:Q4.

[8]It is worth stressing that the naive benchmark model Edge, Kiley and Laforte use for GDP growth, *i.e.* a random walk, is not model compatible.

Table 4: Relative MSFE of forecasts of GDP growth with respect to naive benchmark. Asterisks denote forecasts that are statistically more accurate than he naive benchmark at 1% ($***$), 5%($**$) and 10%($*$)

relative to constant growth			
EVALUATION SAMPLE: 1987:2 - 2004:4			
Forecast horizon	RBC	SPF	AUG
Q0	0.86	0.68 $**$	0.68 $**$
Q1	0.88 $**$	0.77	0.84 $**$
Q2	0.92	0.88	0.91
Q3	0.94 $**$	0.94	0.94 $*$
Q4	0.97	0.95	0.97

relative to constant growth			
EVALUATION SAMPLE: 1996:2 - 2004:4			
Forecast horizon	RBC	SPF	AUG
Q0	0.81 $**$	0.80 $**$	0.75 $***$
Q1	0.85 $***$	0.94	0.85 $**$
Q2	0.84 $***$	1.01	0.83 $***$
Q3	0.84 $***$	1.01	0.85 $***$
Q4	0.87 $**$	1.01	0.88 $**$

The purely model-based forecasts are generated with model (44), (47) and (48), with v_t is autocorrelated, but not serially correlated, *i.e.* D is diagonal. The results however are robust to the specification of the measurement error. We construct the augmented forecasts following the procedure described in previous Section - assuming that the professional forecasters have extra-information on the current period. In appendix we present some results for the case in which we assume that the professional forecasters have some extra-information also on future shocks.

Table 8 reports the mean square forecast error (MSE) of the purely model-based forecasts, the SPF and the augmented forecasts relative to the naive model (random walk in levels), when forecasting GDP growth.

In all samples considered, the augmented forecasts outperform quite consistently the model-based forecasts. The greatest gain is achieved in the nowcast, since that is where the judgmental forecasts help. For higher horizons, the weight associated to the judgmental forecast is very small, and therefore the augmented forecast is very similar to the purely model-based one. This result crucially depends on the assumptions made on the information set of the professional forecasters.

Table 9 reports the weights that the filter gives to the judgmental forecasts of real GDP growth and real consumption growth when constructing the augmented forecasts. In particular, the first column gives the weights associated to the SPF forecasts of real GDP growth when constructing the augmented forecast of real GDP growth; the second column instead gives the weights associated to the SPF forecasts of real consumption growth when constructing the augmented forecast of real consumption growth. Clearly, since the Professional Forecasters are assumed to have information only on current shock, the judgmental forecast receives a significant weight in the construction of the augmented forecasts only

Table 5: Weight given to the SPF forecast of ΔGDP and $\Delta CONS$ in the augmented forecast . 2004:4

	ΔGDP	$\Delta CONS$
nowcast	0.3941	0.2574
1 step ahead	-0.1038	-0.0020
2 step ahead	-0.0341	0.0018
3 step ahead	-0.0185	0.0019
4 step ahead	-0.0078	0.0014

in the nowcast. The reason is that the weights given by the Kalman filter to the SPF tend to zero as their information content goes to zero. Finally, notice that the weight associated to the SPF forecast of consumption growth is smaller than those associated to the SPF forecast of GDP growth. This indicates that the extra-model information accessed by the professional forecasters is more informative on GDP than on consumption.

Let us briefly compare these results with the ones reported in appendix for the case in which the SPF are assumed to have extra some extra-information up to 4 periods ahead. In the latter case, the augmented forecasts beat both the SPF forecasts and the model-based forecasts at all horizons over the full evaluation sample 1987:Q2-2004:Q4. Moreover, in the full sample, the augmented forecasts built under the assumption that the PF have some extra-information on the current and future shocks perform much better that the augmented forecast obtained by assuming that the PF have extra-model information only on the current shock. Interestingly, the opposite is true in the subsample 1996:Q2-2004:Q4. This indicates that the SPF are better modeled by assuming that they have extra-model information on current and future shocks in the full evaluation sample, while in the subsample 1996:Q2-2004:Q4 they are better represented by assuming they have extra-information only on the current shock. The above results point out to the decline in predictability highlight in D'Agostino, Giannone and Surico (2006).

Table 6 reports the mean square forecast error (MSE) of the purely model-based forecasts, the SPF and the augmented forecasts relative to the naive model (random walk in levels), when forecasting consumption growth.

Figures (2)-(8) report the nowcasts and the forecasts, plotted against the data. In each figure, the green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the purely model-based forecasts; the dashed line corresponds to the augmented forecasts. The latter forecasts becomes more and more similar to the model-based forecast as we increase the forecasting horizon. This is not surprising, since we have assumed that the SPF have extra information only regarding the current period; thus the informational value of their forecast is much lower at horizons higher than the nowcast and will therefore be given less weight. This would clearly not be the case under different assumptions for the information

Table 6: Relative MSFE of forecasts of Consumption growth with respect to naive benchmark. Asterisks denote forecasts that are statistically more accurate than he naive benchmark at 1% (∗ ∗ ∗), 5%(∗∗) and 10%(∗)

relative to constant growth			
EVALUATION SAMPLE: 1987:2 - 2004:4			
Forecast horizon	RBC	SPF	AUG
Q0	0.97	0.70	0.84
Q1	0.92 ∗∗	0.89	0.91∗∗
Q2	0.94 ∗∗	0.94	0.93∗∗
Q3	0.94 ∗	0.99	0.93 ∗
Q4	0.94 ∗	1.00	0.93 ∗

relative to constant growth			
EVALUATION SAMPLE: 1996:2 - 2004:4			
Forecast horizon	RBC	SPF	AUG
Q0	0.89 ∗∗	0.96	0.88∗
Q1	0.89 ∗∗	1.06	0.89 ∗∗
Q2	0.88 ∗∗	1.03	0.87 ∗∗
Q3	0.87	∗∗ 1.03	0.87 ∗∗
Q4	0.86 ∗∗	1.08	0.86∗∗

set of the SPF. If, as in appendix, we assumed that they have some extra-model information not only on current shocks, but also on future ones, then the SPF could be given sizeable weights (see Tables in Appendix) also at horizons higher than than the current quarter.

Finally, we report few illustrative results on the "structural" analysis of the SPF forecasts. First of all, we can make some considerations regarding information content of the SPF forecasts, according to the extent that they will reduce uncertainty surrounding the estimation problem faced by the agents (as in Coenen, Levin and Wieland, 2005). Figure 9 plots the nowcasts and their confidence bands[9]. The black solid line is the purely model-based nowcast, while the black dashed-dotted lines are its confidence bands. Similarly, the red lines represent the augmented augmented nowcast (and its confidence bands). It is clear from the picture that the confidence bands for the augmented nowcast are smaller than the ones of the purely model-based nowcast. This means that there is less uncertainty when nowcasting using also the SPF, and therefore that the SPF are indeed somewhat informative on the current state of the economy. Similarly for higher horizons.

Second, we can infer the type of shocks the Professional Forecasters saw when performing their forecasts. Figure 10 reports actual real GDP growth and the different shocks they perceived while doing their nowcast. The dotted line represented actual real GDP growth. The blue line is the technological shock as perceived by the SPF, the green line is the measurement error they perceive on GDP growth, while the red line is the measurement error they perceive on real consumption growth. In this very simple model the only shocks different

[9]While constructing these confidence bands, we consider only the uncertainty in the estimation of the state and assume that the parameters, estimated previously, are known

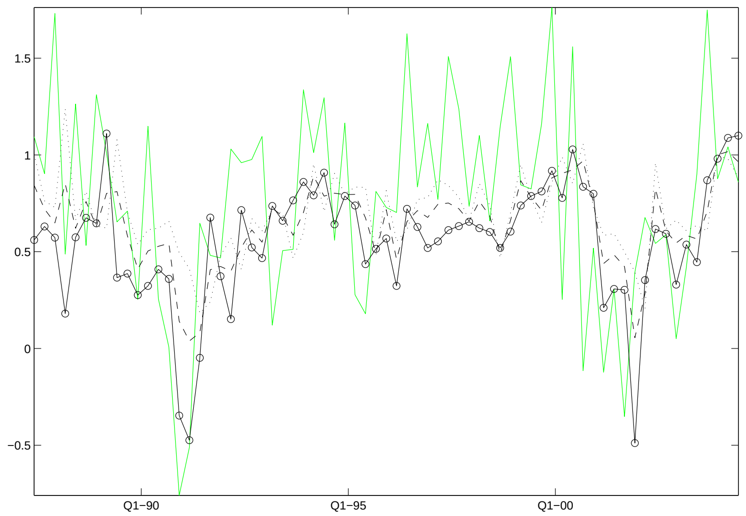


Figure 2: **Nowcast for GDP - EVALUATION SAMPLE 1987:2-2004:4**. The green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the fully model-based forecasts; the dashed line corresponds to the the augmented forecasts

from the technological shocks are the shocks to the residual term that is meant to describe all the dynamics that is not captured by the RBC model. However the proposed procedure would, when applied to a more elaborate DSGE model, allow for understanding the perception of the different shock - monetary, fiscal, etc.. - that the SPF have.

Finally, interesting counterfactual exercises can be done in this setup. Figure 11 reports, for the period 1987:2-1994:4, the the actual SPF nowcast and the nowcasts they would have done if they had only parts of the information they actually have. In particular, the black solid line represents actual real GDP growth; the starred line is the actual SPF nowcast. The line marked with plus signs is the nowcast the SPF would have made if they would have had no information on any of the shocks, the line with squares plots the nowcast they would have made if they had extra-information only on the technological shock, while the line with diamonds is the nowcast they would have made if they had extra-information only on the measurement error shocks.

From this figure we can extract some very relevant information. For example, consider the fall in actual GDP growth in the last quarter of 1989. The actual SPF nowcast for that quarter is very close to the actual figure and, interestingly, coincides with the nowcast the SPF would have made if they only saw the technology shock (green line). On the other hand, the nowcast the SPF would have made if they only saw the "measurement error" shock (red line) coincides with the one they would have made if the had no extra-information at all (blue line). This means that this specific movement in GDP growth was most probably due to a technological shock. Similarly, in the third quarter of

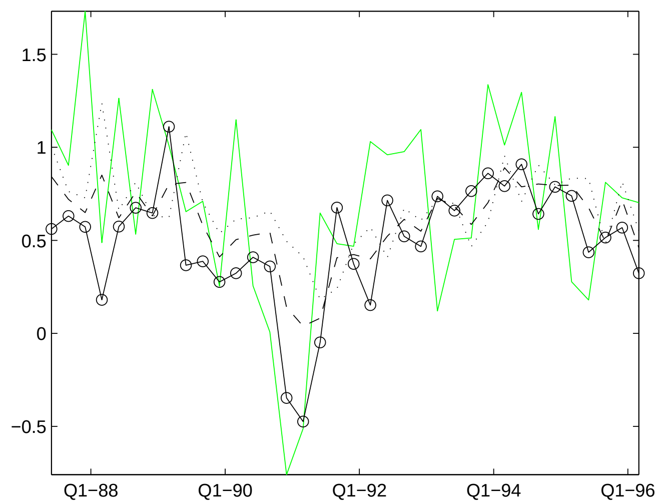


Figure 3: **Nowcast for GDP - PERIOD I 1987:2-1996:1**. The green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the fully model-based forecasts; the dashed line corresponds to the the augmented forecasts

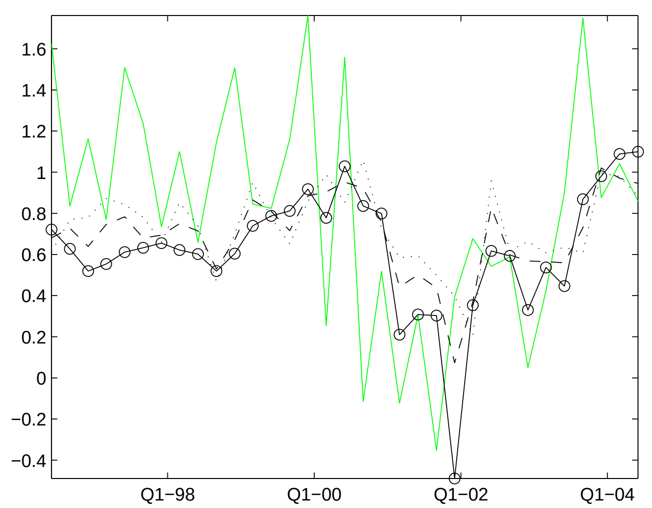


Figure 4: **Nowcast for GDP - PERIOD II 1996:2-2004:4**. The green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the fully model-based forecasts; the dashed line corresponds to the the augmented forecasts

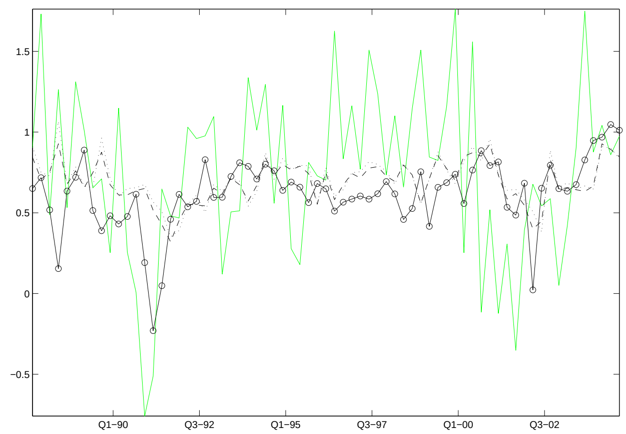


Figure 5: **Forecasts 1 step ahead for GDP - EVALUATION SAMPLE 1987:2-2004:4**. The green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the fully model-based forecasts; the dashed line corresponds to the the augmented forecasts

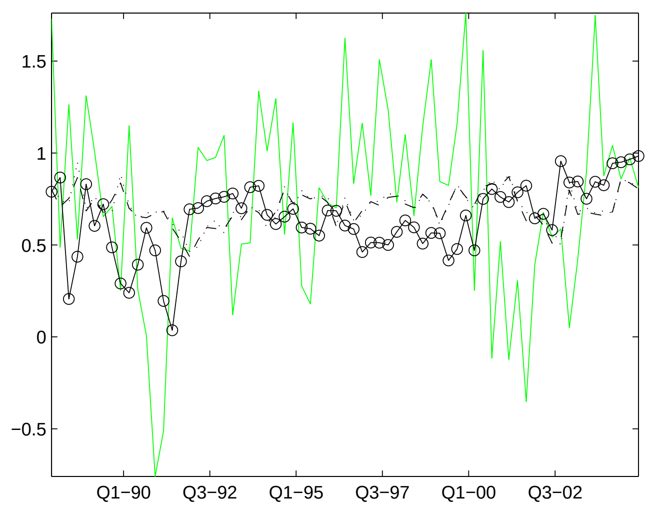


Figure 6: **Forecasts 2 step ahead for GDP - EVALUATION SAMPLE 1987:2-2004:4**. The green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the fully model-based forecasts; the dashed line corresponds to the the augmented forecasts

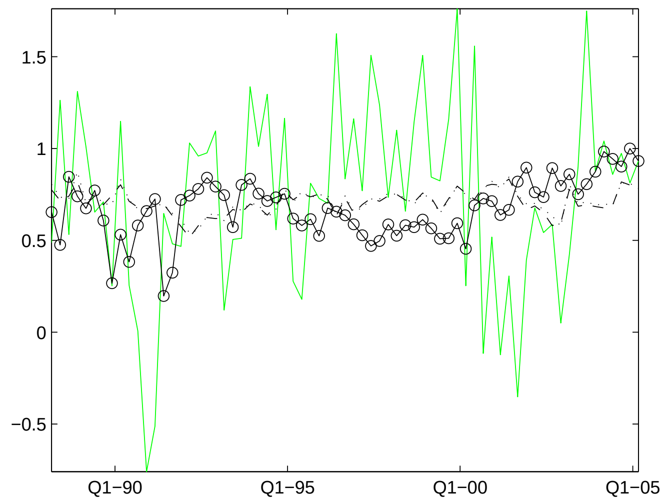


Figure 7: **Forecasts 3 step ahead for GDP - EVALUATION SAMPLE 1987:2-2004:4**. TThe green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the fully model-based forecasts; the dashed line corresponds to the the augmented forecasts

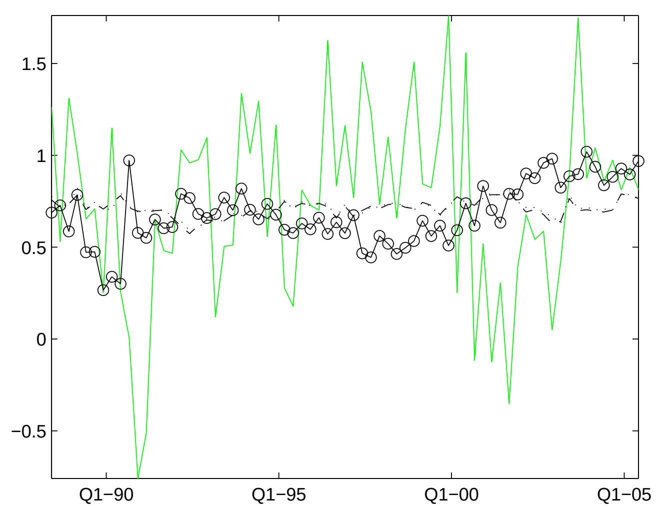


Figure 8: **Forecasts 4 step ahead for GDP - EVALUATION SAMPLE 1987:2-2004:4**. The green/grey solid line represents the actual data; the solid line with circles is the series of the SPF forecasts, the dotted line portrays the fully model-based forecasts; the dashed line corresponds to the the augmented forecasts

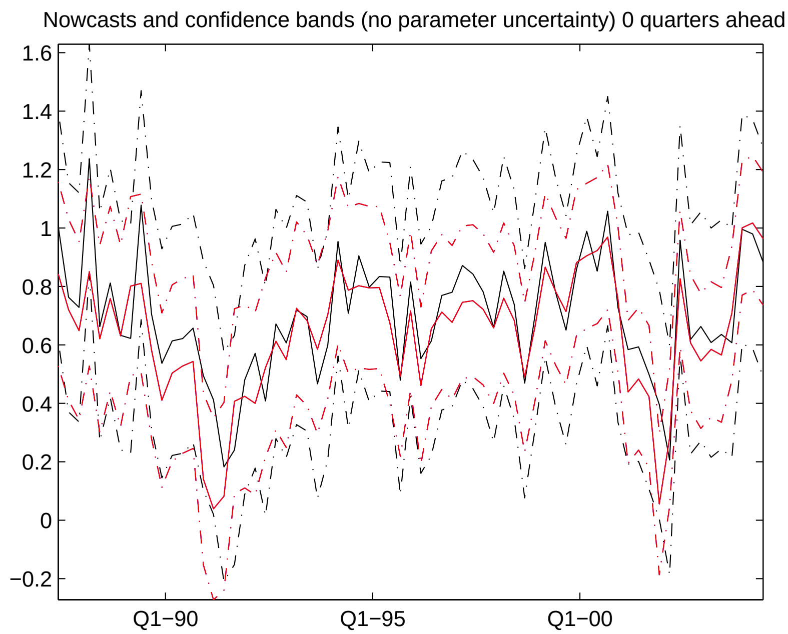


Figure 9: **Nowcasts with confidence bands (no parameter uncertainty) 1987:2-2004:4**. The black solid line is the purely model-based nowcast, while the black dashed-dotted lines are its confidences bands. Similarly, the blue lines represent the augmented nowcast (and its confidence bands). When constructing the confidence bands, we only consider the uncertainty in the estimation of the state, but no parameter uncertainty. the current period's shock.

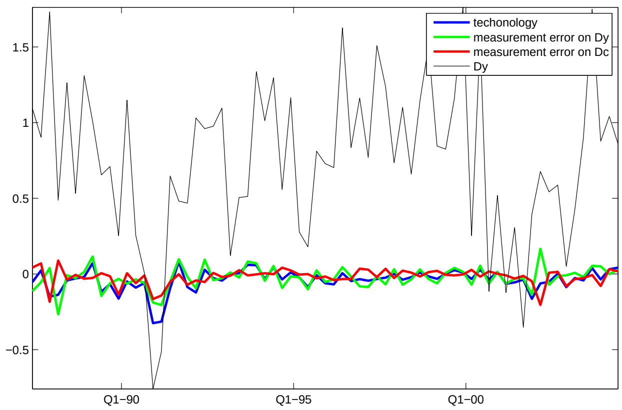


Figure 10: **Current structural shocks perceived by the SPF when nowcasting 1987:2-2004:4**. The dotted line represents actual real GDP growth. The blue line is the technological shock the SPF see, the green line is the measurement error they perceive on GDP growth, while the red line is the measurement error they perceive on real consumption growth.

1992, the SPF nowcast coincides with the nowcast the SPF would have made if they only saw the "measurement error" shock (red line), while have information on the technological shock is like having no information. In this case, then, the variation of the GDP was certainly not due to technology, but due to the "measurement errors" and some other factors. Of course, this exercise would be much more interesting if we developed it in a setup in which we were able to distinguish, for example, monetary or fiscal shocks, but it is any how quite informative.

5 Conclusions and Extensions

In this paper we have proposed a method to incorporate judgmental information, proxied by professional forecasts, into model-based forecasts. We suggested modeling the professional forecasts as optimal estimates of the variables of interest, made with a different, possibly more informative, information set; we then have shown how they can be accounted for in the framework of a linearized and solved DSGE model. The methodology we propose allows generating forecasts that are more accurate than the purely model-based ones, but that are still disciplined by the economic rigor of the model.

We have also highlighted how to infer the information content of the SPF forecasts from the weights that the Kalman filter assigns to them. In particular, the weights given by the Kalman filter to the SPF go to zero as their information content goes to zero. More precisely, the more the professional forecasters are able to gather information on the shocks, the more the Kalman filter will use the professional forecasts when combining them with the predictions from the

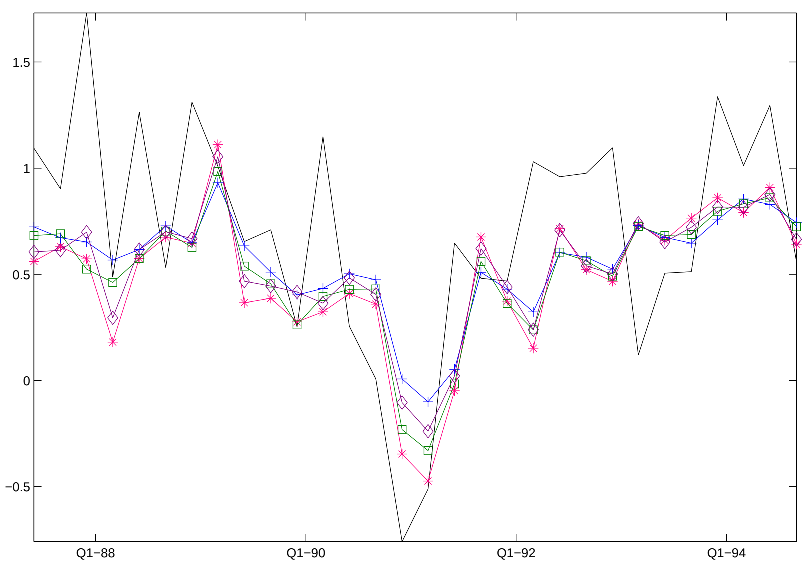


Figure 11: **Counterfactual exercise -nowcast- 1987:2-1994:4**. The black solid line represents actual real GDP growth; the dashed-dotted line is the actual SPF nowcast. The blue line is the nowcast the SPF would have made if the would have had no information on any of the shocks, the green line plots the nowcast they would have made if they had extra-information only on the technological shock, while the red line is the nowcast they would have made if they had extra-information only on the measurement error shocks.

model, but it will down-weigh them if the variance of their forecast errors is too large.

Finally we have described how to interpret the forecasts through the lens of the model. We were able to extract the structural shocks as they were perceived by the professional forecasters and to make several counterfactual exercises on the forecasts the professional forecasters would have done if they saw only some of the shocks.

We working on several extensions to this paper. First, in a joint paper with Domenico Giannone and Lucrezia Reichlin we allow for the timely information to enter the model directly, not processed by the professional forecasters. We differ from Boivin and Giannoni (2005) in that we consider a large dataset containing intra-period data, in order to really capture the effects of timely information.

Then, we are also working on reformulating the problem so to be able to account for the possibility that the forecasts feedback into the model. If for example we wanted to include a policymaker that targeted current inflation and current output gap, the estimate of inflation and output made using judgmental information should feedback into the model via the policy rule. This extension can be implemented using the extension of the Kalman filter proposed by Svensson and Woodford (2003) that allows to do signal-extraction with forward-looking obsverables[10].

Once we have reformulated the problem, we will be able to apply the methodology we propose to richer DSGE models, with more shocks. This can be very interesting from a storytelling perspective, because, as we have sketched in the simple application we consider in this paper, it will allows us to understand which types of shocks the professional forecasters perceived. More importantly, we will be able to understand, in the cases in which the professional forecasters forecasts where very close to the actual data, the extent to which the perception of certain shocks helped them forecasting so well and infer if the movements in the data were due to, say, a technology or a monetary shock.

[10]Since the forecasts feedback into the model, we cannot solve the model first, as we did in our simple RBC case, and then forecast. For this reason we will need to deal with forwarding-looking observables.

References

[1] Alvarez-Lois, P., R. Harrison, L. Piscitelli and A. Scott (2005), 'Taking DSGE Models to the Policy Environment'

[2] Anderson B.D.O. and J.B. Moore (1979), *Optimal Filtering*

[3] Anderson, E., L. P. Hansen, E. R. MCGrattan and T. J. Sargent (1996): "Mechanics of Forming and Estimating Dynamic Linear Economies," in *Handbook of Computational Economics*, Volume 1, ed. by D. A. K. Hans M. Amman, and J. Rust, pp. 171–252. North-Holland

[4] Aruoba, B. (2005), "Data Revisions are not Well Behaved" EABCN/CEPR Working Paper Series 21/2005.

[5] Blanchard, O.J. and C.M. Kahn (1980), "The Solution of Linear Difference Models under Rational Expectations,", Econometrica 48(5), 1305-1311

[6] Boivin, J. and M. Giannoni (2005), "DGSE in a Data-Rich Environment"

[7] Bruno, G. and C. Lupi (2004), "Forecasting industrial production and the early detection of turning points", Empirical Economics, 29, 647-671

[8] Campbell, John Y. (1994), " Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model," Journal of Monetary Economics 33, 463-506.

[9] Coenen, G., Levin and V. Wieland, (2005). "A Data Uncertainty and the Role of Money as an Information Variable for Monetary Policy," European Economic Review, 49(4), 975-1006.

[10] Cogley, T., S. Morozov and T. Sargent (2005), "Bayesian fan charts for U.K. inflation: Forecasting and sources of uncertainty in an evolving montary system," Journal of Economic Dynamics and Control 29, 1893-1925.

[11] D'Agostino, A., D. Giannone and P. Surico (2006), "(Un)predictability and Macroeconomic Stability", ECB Working Paper No 605

[12] Diebold, F.X. and R.S. Mariano (1995),"Comparing Predictive Accuracy,", Journal of Business Economics and Statistics, 13, 253-265

[13] Edge, R.M., M.T. Kiley, J. Laforte (2006) ,"A Comparison of Forecast Performance between Federal Reserve Staff Forecasts, Simple Reduced-Form Models and a DSGE Model," Federal Reserve Board (mimeo)

[14] Forni, M., M. Hallin, M. Lippi, and L. Reichlin (2000), "The Generalized Dynamic Factor Model: Identification and Estimation," Review of Economics and Statistics 82:4, 540—554.

[15] Friedman, M (1961), "The lag in effect of monetary policy", Journal of Political Economy, 69, 447-66.

[16] Giacomini, R. and H. White (2006), "Tests of Conditional Predictive Ability", Econometrica, vol. 74(6=, 1545-1578

[17] Giannone, D., Reichlin, L. and Sala, L., (2005). "Monetary Policy in Real Time," CEPR Discussion Papers 4981, C.E.P.R. Discussion Papers.

[18] Giannone, D., Reichlin, L. and Small, D., (2005). "Nowcasting GDP and Inflation: The Real Time Informational Content of Macroeconomic Data Releases," CEPR Discussion Papers 5178, C.E.P.R. Discussion Papers.

[19] Hamilton,J.D. (1994) Time Series Analysis, Princeton University Press,Princeton,NJ.

[20] Ingram,B.F.,Kocherlakota,N.R.,Savin,N.E. (1994), "Explaining business cycles:a multiple-shock approach. Journal of Monetary Economics" 34,415 –428.

[21] Ireland, P.N., (2004), "A method for taking models to the data," Journal of Economic Dynamics and Control, Elsevier, vol. 28(6), pages 1205-1226

[22] Hansen,G.D.,(1985), "Indivisible labor and the business cycle," Journal of Monetary Economics 16,309 –327.Journal of Monetary Economics 16,309 –327.

[23] King, R. G., C. I. Plosser et S. T. Rebelo (1988a), "Production, Growth and Business Cycles: 1. The Basic Neoclassical Model," Journal of Monetary Economics, 21, 195-232.

[24] King, R. G., C. I. Plosser et S. T. Rebelo (1988b), "Production, Growth and Business Cycles: 2. New Directions," Journal of Monetary Economics, 21, 309-341.

[25] Klein, P. (2000), "Using the Generalized Schur Form to Solve a System of Linear Expectational Difference Equations", Journal of Economic Dynamics and Control 24(10), 1405-1423

[26] Mankiw, N.G. and M.D. Shapiro, (1986), "News or Noise: An Analysis of GNP Revisions," Survey of Current Business, 66 , 20-25.

[27] Marchetti, D.J. and G. Parigi (1998), "Energy Cconsumption, Survey Data and the Prediction of Industrial Production in Italy", Tem di Discussione del Servizio Studi della Banca d' Italia, No 342

[28] McNees, S.K. (1990), "The role of judgment in macroeconomic forecasting accuracy," International Journal of Forecasting,. 6, 287-299.

[29] Österholm, P., (2006), "Judgement and Fan Charts - Incorporation and Evaluation"

[30] Reifschneider, D.,D.J. Stockton and D.W. Wilcox (1997),"Econometric Models and the Monetary Policy Process" , Carnegie-Rochester Conference Series on Public Policy, vol. 47, pp.1-37

[31] , Robertson, J., E. Tallman and C. Whiteman, (2005)"Forecasting using Relative Entropy," Journal of Money Credit and Banking 37, 383-402. Banking 37, 383-402.

[32] Romer, C. and D. Romer(2000), "Federal Reserve Information and the Behavior of Interest Rates", The American Economic Review, 90, 3, 429-457

[33] Sargent, T.J. (1989), "Two Models of Measurement and the Investment Accelerator", Journal of Political Economy, 97,2,251-287

[34] Schuh, S. (2001), "An Evaluation of Recent Macroeconomic Forecast Errors", New England Economic Review

[35] Smets, F. and R. Wouters, (2004) "Forecasting with a Bayesian DSGE model - an application to the euro area," Working Paper Series 389, European Central Bank

[36] Sims, C.A (2002), "Solving Linear Rational Expectations Models," Computational Economics, Springer, 20(1-2), 1-20.

[37] Sims, C.A (2003), "The role of models and probabilities in the monetary policy process," Brooking Papers on Economic Activity, 2002:2 1-63

[38] Stock, J.H., and M. W. Watson (1999), "Forecasting Inflation," Journal of Monetary Economics 44, 293-335.

[39] Stock, J.H., and M. W. Watson (2002), "Macroeconomic Forecasting Using Many Predictors".

[40] Svensson, L.E.O (2005), "Monetary Policy with Judgement: Forecast Targeting", NBER Working Paper 11167

[41] Svensson, L.E.O and R.J. Tetlow (2005), "Optimal Policy Projections", NBER Working Paper 11392

[42] Svensson, L.E.O. and N. Williams (2005), "Monetary Policy with Model Uncertainty: Distribution Forecast Targeting", NBER Working Paper 11733

[43] Svensson, L.E.O. and M. Woodford (2003), "Indicator Variables for Optimal Policy", Journal of Monetary Economics 50, 691-720

[44] Tinsley, P. A., Spindt, P. A. and Friar, M. E., 1980. "Indicator and filter attributes of monetary aggregates : A nit-picking case for disaggregation," Journal of Econometrics, Elsevier, vol. 14(1), 61-91

6 Appendix A: Solving the RBC model

The equilibrium conditions for the optimization problem are

$$y_t = k_t^{(1-\alpha)}, \tag{49}$$

$$y_t = c_t + i_t, \tag{50}$$

$$(\gamma_A e^{\varepsilon_t})k_{t+1} = k_t + i_t. \tag{51}$$

$$R_t = [(1-\alpha)k_t^{-\alpha} + (1-\delta)] \tag{52}$$

$$c_t^{-\eta} = \frac{\beta}{\gamma_A^{\eta}} E_t[\frac{c_{t+1}^{-\eta} R_{t+1}}{e^{\varepsilon_{t+1}}}]. \tag{53}$$

In absence of shocks, the economy converges to the following deterministic balanced growth path/steady state that can be obtained from equations (49)-(53) dropping the time subscripts and through some simple manipulation.

$$R = \frac{\gamma_A^{\eta}}{\beta}$$

$$\frac{Y}{K} = \frac{\frac{\gamma_A^{\eta}}{\beta} + 1 - \delta}{1-\alpha}$$

$$\frac{I}{K} = \gamma_A - (1-\delta)$$

$$\frac{C}{Y} = 1 - \frac{(1-\alpha)(\gamma_A - (1-\delta))}{R - (1-\delta)}$$

$$\frac{I}{Y} = \frac{(1-\alpha)(\gamma_A - (1-\delta))}{R - (1-\delta)}$$

A linear approximation of system (49)-(53) can be derived log-linearizing it around its steady state. Let us define, for and variable x, $\hat{x}_t = \ln(\frac{x_t}{X})$, *i.e.* the log-deviation of x_t from its steady state X. With some manipulation we obtained the following linearization for the system (49)-(53).

$$\begin{aligned}
\hat{y}_t &= \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{i}_t \\
\hat{k}_{t+1} &= \frac{I}{\gamma_A K}\hat{i}_t + \frac{1-\delta}{\gamma_A}\hat{k}_t - \varepsilon_t \\
\hat{y}_t &= (1-\alpha)\hat{y}_t \\
\hat{r}_t &= \frac{1-\alpha}{R}\frac{Y}{K}(\hat{y}_t - \hat{k}_t) \\
0 &= E_t[-\eta(\hat{c}_{t+1} - \hat{c}_t) + \hat{r}_{t+1}]
\end{aligned}$$

Manipulating it a bit, this system can also be rewritten all as a function of $\hat{k}_{t+1}$, $\hat{c}_t$ and ε_{t+1}.

$$\hat{k}_{t+1} = \lambda_1 \hat{k}_t + \lambda_2 \hat{c}_t - \varepsilon_t \tag{54}$$

$$E_t[\Delta \hat{c}_t] = \frac{\lambda_3}{\eta} E_t \hat{k}_{t+1} \tag{55}$$

$$\hat{y}_t = (1-\alpha)\hat{k}_t \tag{56}$$

where

$$
\begin{aligned}
\lambda_1 &= \frac{R}{\gamma_A} \\
\lambda_2 &= 1 - \frac{R + \alpha(1-\delta)}{\gamma_A(1-\alpha)} \\
\lambda_3 &= -\frac{\alpha(R - (1-\delta))}{R}
\end{aligned}
$$

Various methods are available for solving linear difference models like (54)-(56) under rational expectations, but given the simplicity of our model we do not need to resort to elaborate methods, we can simply apply the method of undetermined coefficients as described, e.g., in Campbell (1994). That is, we "guess" the functional form of $\hat{k}_{t+1}$ and $\hat{c}_t$ and then we verify it by finding parameters that satisfy the restrictions of the approximate loglinear model.

As pointed out in King, Plosser and Rebelo (1988b), if technology is a logarithmic random walk with drift, then the solution to the transformed economy is particularly simple. The only impact of technological progress is to reset the transformed economy's capital stock relative to its long-sum stationary level. A positive 1% technological innovation in the untransformed economy leads to a 1% decline in the transformed economy's capital stock. Therefore we assume that

$$
\begin{aligned}
\hat{k}_{t+1} &= \mu \hat{k}_t - \varepsilon_{t+1} && (57) \\
\hat{c}_t &= \pi_{ck} \hat{k}_t && (58)
\end{aligned}
$$

and similarly, $\hat{y}_t = \pi_{yk}\hat{k}_t$, etcetera. Substituting (57) and (58) into (54)-(56) and manipulating, we obtain a second order equation for $\pi_c k$ which has only one positive solution (as required by the problem). Therefore we obtain

$$
\begin{aligned}
\hat{k}_{t+1} &= \mu \hat{k}_t - \varepsilon_{t+1} \qquad (59) \\
\begin{bmatrix} \hat{y}_t \\ \hat{c}_t \end{bmatrix} &= \begin{bmatrix} \pi_{ck} \\ (1-\alpha) \end{bmatrix} \hat{k}_t,
\end{aligned}
$$

where

$$
\begin{aligned}
\pi_{ck} &= \frac{-(\lambda_1 - 1 + \frac{1}{\eta})\lambda_2\lambda_3 + \sqrt{(}\lambda_1 - 1 + \frac{1}{\eta})\lambda_2\lambda_3)^2 + 4\lambda_2\frac{\lambda_1\lambda_3}{\eta}}{2\lambda_2} \\
\mu &= \lambda_1 + \lambda_2\pi_{ck}.
\end{aligned}
$$

In order to be able to bring the model to the data we still need to work on it a bit more. If we consider the variables in levels, we can write

$$
\begin{aligned}
\ln y_t &= a_t + \ln(Y) + \hat{y}_t \\
\ln c_t &= a_t + \ln(c) + \hat{c}_t \\
\ln i_t &= a_t + \ln(I) + \hat{i}_t
\end{aligned}
$$

Since we are in fact looking at a balanced growth path rather than a steady state, we cannot recover the steady state values Y, C, I, we can only pin down some ratios as $\frac{C}{Y}$ and $\frac{Y}{K}$. Therefore, in order to relate the model (59) to the data, we rather look at

$$\begin{aligned} \Delta(\ln y_t) &= \ln \gamma_A + (\alpha + \mu - 1)\hat{k}_{t-1} - \alpha \hat{k}_t \\ \ln c_t - \ln y_t &= log(\frac{C}{Y}) + [\pi_{ck} - (1-\alpha)]\hat{k}_t \\ \ln i_t - \ln y_t &= log(\frac{I}{Y}) + [\pi_{ik} - (1-\alpha)]\hat{k}_t. \end{aligned} \tag{60}$$

Finally we obtain

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{k}_t \end{bmatrix} = \begin{bmatrix} \mu & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{k}_{t-1} \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \varepsilon_{t+1} \tag{61}$$

$$\begin{bmatrix} \Delta \ln y_t \\ \ln c_t - \ln y_t \\ \ln i_t - \ln y_t \end{bmatrix} = \begin{bmatrix} \ln \gamma_A \\ log(\frac{C}{Y}) \\ log(\frac{I}{Y}) \end{bmatrix} + \begin{bmatrix} -\alpha & \alpha + \mu - 1 \\ \pi_{ck} - (1-\alpha) & 0 \\ \pi_{ik} - (1-\alpha) & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{k}_{t-1} \end{bmatrix},$$

in the form of a state-space econometric model, allowing the procedure for evaluating the likelihood function to continue using the Kalman filtering algorithms outlined, for example, by Hamilton (1994,Chapter 13).

Table 7 reports the parameters values for the model with the autocorrelated but not cross-correlated residual. All other specifications have the same calibrated parameters, but differ slightly in the estimated parameters.

Table 7: Parameter values for the model with the autocorrelated but not cross-correlated residual.

Parameter	value	
β	0.988	calibrated
α	0.667	calibrated
δ	0.01	calibrated
η	1 (log utility)	calibrated
γ	1.0072	calibrated
σ_ε^2	0.01	estimated
D_{yy}	0.6692	estimated
D_{cc}	0.2215	estimated
V_{yy}	0.01	estimated
V_{cc}	0.0075	estimated
V_{yc}	-0.0483	estimated

7 Appendix B: alternative modeling of the professional forecasters

Here I show how to construct an augmented forecast that extracts and accounts for the information contained in professional forecasts under the assumption that the professional forecasters have extra-model information on *current and future* shocks.[11] Their information set $I_T \supseteq M_T$ and is such that, for $h = 0, 1, 2, 3, 4$

$$E[u_{T+h} I'_{T+h}] \neq 0. \tag{62}$$

In this case, the forecasters will report the following state-space form. For h=0,

$$\begin{aligned} \hat{s}_{T|T} &= G\hat{s}_{T-1|T-1} + Ku_T \\ E[y_T|I_T] &= \Lambda G\hat{s}_{T-1|T-1} + w_{T|T}, \end{aligned} \tag{63}$$

where

$$w_{T|T} = E[u_T|I_T] + \eta_{T|T}$$

and

$$\begin{bmatrix} Ku_T \\ w_{T|T} \end{bmatrix} \sim WN\left(\; 0 \quad , \quad \Sigma_0 \; \right)$$

where

$$\Sigma_0 = \begin{bmatrix} KE(u_T u'_T)K' & KE(u_T w'_{T|T}) \\ E(w_{T|T} u'_T)K' & Ew_{T|T} w'_{T|T} \end{bmatrix},$$

$\eta_{T|T}$ is, as before, the measurement error (the typo) made by the forecasters in T while reporting their forecast for period T. The professional nowcast coincides with the one generated under the assumption that the forecasters have information only on the current period; the forecasts will instead be different. For $h = 1, 2, 3, 4$ we have

$$\begin{aligned} \hat{s}_{T+h|T+h} &= G\hat{s}_{T+h-1|T+h-1} + Ku_{T+h} \\ E[y_{T+h}|I_T] &= \Lambda G\hat{s}_{T-1|T-1} + \sum_{i=1}^{h} \Lambda G^i K E[u_{T+h-i}|I_T] + \eta_{T+h|T}, \\ &= \Lambda G\hat{s}_{T+h-1|T+h-1} + w_{T+h|T} \end{aligned} \tag{64}$$

where

$$w_{T+h|T} = \Lambda \sum_{i=1}^{h} G^i K \left[E(u_{T+h-i}|I_T) - u_{T+h-i}\right] + E[u_{T+h}|I_T] + \eta_{T+h|T} \tag{65}$$

$$\begin{bmatrix} Ku_{T+h} \\ w_{T+h|T} \end{bmatrix} \sim WN\left(\; 0 \quad , \quad \Sigma_h \; \right)$$

and

$$\Sigma_h = \begin{bmatrix} KE(u_{T+h} u'_{T+h})K' & KE(u_{T+h} w'_{T+h|T}) \\ (w_{T+h|T} u'_{T+h})K' & Ew_{T+h|T} w'_{T+h|T} \end{bmatrix}.$$

As above, $\eta_{T+h|T}$ is, as before, the measurement error (the typo) made by the forecasters in T while reporting their forecast for period $T + h$. We assume

[11] We acknowledge, but for now ignore the potential inconsistency arising from the fact of assuming that professional forecasters have information on future shocks, while agents do not and fail to look at the professional forecasters to have more information.

that $\eta_{s|T} \perp u_\tau$, for any s and τ, and that $\eta_{T+h|T} \perp E(u_{T+i}|I_T)$ for any i, h. Moreover we will make the following assumptions:

$$\begin{aligned} u_t &\perp E(u_\tau|I_T) \qquad \forall\tau \neq t \\ E(u_t|I_T) &\perp E(u_\tau|I_T) \qquad \forall\tau \neq t \\ u_t &\perp E(u_\tau|I_T) \qquad \forall\tau \neq t \end{aligned} \tag{66}$$

which will allow us to recover Σ_0 and Σ_h for h=1,2,3,4.

Once we have recovered the exact form of the covariance matrices Σ_0 and Σ_h we can then smooth (63) and (64) with a time-varying Kalman smoother, in order to obtain optimal estimates $s^+_{T+h|I_T}$,for h=0,1,2,3,4, that comprise the extra-information contained in the forecasts and employs it optimally within the model. Therefore, using $\hat{s}^+_{T+h|I_t}$ we can create a forecast $\hat{y}^+_{T+h|I_T}$,

$$y^+_{T+h|I_T} = \Lambda \hat{s}^+_{T+h|I_T}, \tag{67}$$

that incorporates optimally in the model-based framework the judgemental information coming from the conjunctural forecasters.

Since the nowcasts coincides with the ones of the previous subsection, Σ_0 can be recovered exactly in the same way. To recover all the elements of Σ_h, we proceed as follows. Given the assumptions we made in (66), it is easy to show that

$$E(u_{T+h}w'_{T+h|T}) = E[u_{T+h}E(u_{T+h}|I_T)'].$$

On the basis of assumptions (66), the following equality holds

$$E[u_{T+h}E(y_{T+h}|I_T)'] = E[u_{T+h}E(u_{T+h}|I_T)'],$$

As the series for the u_{T+h} are readily available via the Kalman filter, we are able to recover empirically the value of $E[u_{T+h}E(y_{T+h}|I_T)']$, and therefore of $E[u_{T+h}E(u_{T+h}|I_T)']$. Moreover, notice that, since $E(u_{T+h}|I_T)$ is a linear projection of u_{T+h} on the space spanned by I_T, $Sp(I_T)$, then

$$u_{T+h} = E(u_{T+h}|I_T) + \mu_T$$

where μ_T is orthogonal to the space spanned by I_T. Therefore,

$$E[u_{T+h}E(u_{T+h}|I_T)'] = E[E(u_{T+h}|I_T)E(u_{T+h}|I_T)'], \tag{68}$$

i.e. we have determined also the variance of the expected value of the current shock given the information set I_T, $E(u_{T+h}|I_T)$, and we showed it is equal to the covariance among the shock and its expected value.

In order to recover $Ew_{T+h|T+h}w'_{T+h|T+h}$ we will first define the forecasters' forecast error as

$$\begin{aligned} e_{T+h} &= y_{T+h} - E(y_{T+h}|I_T) \\ &= u_{T+h} - w_{T+h|T}. \end{aligned}$$

The variance of e_{T+h}, whose value can be recovered from sample data, is:

$$\begin{aligned} E(e_{T+h}e'_{T+h}) &= E(u_{T+h}u'_{T+h}) - E[u_{T+h}w'_{T+h|T}] + \\ &- E[u_{T+h}w_{T+h|T}]' + E[w_{T+h|T}w'_{T+h|T}] \end{aligned}$$

$E(u_{T+h}u'_{T+h})$ can be obtained by the Kalman filter as follows

$$E(u_{T+h}u'_{T+h}) = \Lambda P \Lambda' \tag{69}$$

Where P is the solution of the Riccati equation defined in (11). Using the above equations, we can finally recover $E[w_{T+h|T}w'_{T+h|T}]$ and we therefore have pinned down all the values of the matrix Σ_h.

We have shown how to construct model-based forecasts that incorporate extra-model information coming from the professional forecasts under two possible modelizations of the professional forecasters' information set. Similar results would hold however under all intermediate assumptions, as, for example, the assumption that the professional forecasters have extra-model information on the current and 1-step ahead shock. The only difference would be in the definition of $wT + h|T$, which depends on the assumptions made.

Table 8: Relative MSFE of forecasts of GDP growth with respect to naive benchmark. Asterisks denote forecasts that are statistically more accurate than he naive benchmark at 1% ($***$), 5%($**$) and 10%($*$)

relative to constant growth			
EVALUATION SAMPLE: 1987:2 - 2004:4			
Forecast horizon	RBC	SPF	AUG
Q0	0.86	0.68 $*$	0.68 $**$
Q1	0.88 $**$	0.77	0.76 $**$
Q2	0.92	0.88	0.82 $***$
Q3	0.94 $**$	0.94	0.90 $***$
Q4	0.97	0.95	0.92$***$

relative to constant growth			
EVALUATION SAMPLE: 1996:2 - 2004:4			
Forecast horizon	RBC	SPF	AUG
Q0	0.81 $**$	0.80 $**$	0.75 $***$
Q1	0.85 $***$	0.94	0.86 $**$
Q2	0.84 $***$	1.01	0.88 $***$
Q3	0.84 $***$	1.01	0.92 $***$
Q4	0.87 $**$	1.01	0.92 $**$

Table 9: Weight given to the SPF forecast of ΔGDP and $\Delta CONS$ in the augmented forecast . 2004:4

	ΔGDP	$\Delta CONS$
nowcast	0.3941	0.2574
1 step ahead	0.3606	0.1261
2 step ahead	0.3236	0.0931
3 step ahead	0.3387	0.0832
4 step ahead	0.3321	0.0818