Instability in U.S. Inflation: 1967 – 2005

James M. Nason
Research Department
Federal Reserve Bank of Atlanta
1000 Peachtree St, N.E.
Atlanta, GA 30309
jim.nason@atl.frb.org

First Draft: February 2, 2006
Current Draft: May 12, 2006

Abstract

This article studies U.S. monthly inflation, inflation growth, and price level dynamics from January 1967 to September 2005. Two rolling samples are constructed to recover evidence about instability in inflation, inflation growth, and price level persistence and volatility. Evidence is presented that changes in inflation, inflation growth, and price level persistence and volatility coincide with economic events such as the oil price shocks of the 1970s or the end of the 1990 – 1991 recession. A striking aspect of the persistence and volatility estimates is that the personal consumption expenditure deflator – excluding food and energy – approximates serially uncorrelated white noise when observations prior to 1992 are discounted.

JEL Classification Codes: *C220, E310*

Key Words: Persistence, Volatility, Expected Inflation, and Parameter Instability.

I thank Buz Brock, Tom Cunningham, Juan F. Rubio-Ramírez, Ellis Tallman, Mark Watson, and Tao Zha for their comments and suggestions. Andy Bauer, Michael Hammill, and Annie Tilden provided excellent research assistance. A special acknowledgement goes to Elaine Clokey for patiently assisting in preparing the draft for publication.

1. Introduction

Concerns about price stability and high, persistent, and volatile inflation are always and everywhere a worry of central bankers. This concern is institutionalized in the U.S. by the Federal Reserve Act in its Section 2A – Monetary Policy Objectives as

> *The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the economy's long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.*

Although the price stability goal is wedged in between mandates to strive for employment and to restrain long-term interest rates, maintaining a stable price level has come to dominate discussions among academic economists and many central bankers. Statements by Federal Reserve policymakers are remarkably consistent about what constitutes price stability objective over the last 20 years. For example, Alan Greenspan (1994) states

> *We will be at price stability when households and businesses need not factor expectations of changes in the average level of prices into their decisions.*

This suggests price stability occurs when the only source of inflation dynamics are unpredictable shocks whose size does not vary by 'too much' over time.

This article studies U.S. inflation, inflation growth, and price level dynamics. The analysis is disciplined with autoregressive (AR), moving average (MA), and unobserved components (UC) models. The models produce mean inflation, inflation and inflation growth persistence, and inflation, inflation growth, and price level volatility estimates on a sample that begins in January 1967 ($1967M01$) and ends with September 2005

(2005M09). Although this article is silent on the success of policies aimed at price stability, these estimates reveal if the persistence and volatility of inflation, inflation growth, and the price level have changed during the last 40 years.

The disinflation of the 1980s suggests inflation became less persistent and volatile in the 1990s and early 2000s compared to the inflation of the 1970s. For example, Stock and Watson (2005) report that quarterly U.S. inflation became less persistent and volatile since 1984. This suggests it is possible to better forecast inflation. However, lower inflation volatility also makes it more difficult to settle on the best inflation forecast from among a set of competing models. Stock and Watson (1999, 2005) verify the impact of lower persistence and volatility post-1984 on inflation forecasts.[1]

The goal of this article is modest compared to the aims of Stock and Watson (2005). This article presents evidence about inflation, inflation growth, and price level dynamics that complements evidence Stock and Watson present. Estimates of AR, MA, and UC models are reported in this article on the 1967M01 – 2005M09 sample and on two samples that roll through the 1970s, 1980s, and 1990s.[2] The two rolling samples produce AR, MA, and UC model estimates that provide information about instability in

[1]Similar evidence is provided by Hansen, Lunde, and Nason (2005). They apply their metric for choosing the best forecasting models on pre- and post-1984 samples. The Hansen, Lunde, and Nason (HLN) metric finds it more difficult to distinguish between competing inflation forecasting models in the post-1984 sample. Nonetheless, HLN are able to identify several Phillips curve models that outperform a random walk model in out-of-sample inflation forecasting exercises across the two samples. This stands in contrast to results in Atkeson and Ohanian (2001) and Fisher, Liu, and Zhou (2002).

[2]Section 3 gives details of the construction of the two rolling samples.

mean inflation and the persistence and volatility of inflation and inflation growth.

Four price level measures are studied in this article. These are the monthly consumer price index (CPI) and monthly personal consumption expenditure deflator (PCED). The CPIs and PCE deflators are defined as CPI-CORE and PCED-CORE, which exclude food and energy items, and by CPI-ALL and PCED-ALL, which include the relevant universe of consumer goods. This gives four price level, inflation, and inflation growth series.

This article reports AR persistence and volatility estimates that are sensitive to the choice of sample. For example, the first rolling sample yields AR persistent estimates that exhibit little change after the 1973 – 1975 recession for the four inflation rates. When the second rolling sample drops observations from the 1970s for CPI inflation and 1970s and 1980s for PCED inflation, drift in the AR coefficients suggest instabilities in inflation persistence.

The MA and UC model estimates appear consistent with instabilities in the persistence and volatility of CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE inflation growth and levels. Especially striking are MA and UC model estimates on the second rolling sample that suggest PCED-CORE inflation is serially uncorrelated subsequent to the early 1990s recession. A result that is affirmed by the AR persistence estimates. Thus, a reasonable current forecast of PCED-CORE inflation might be its average of the last 15 years. Whether such a forecast is consistent with the Greenspan (1994) notion of price stability is outside the scope of this article.

The next section reviews the empirical models this article studies. Section 3 discusses sample construction. Results are presented in section 4. Section 5 concludes.

2. The Models

This section reviews a pth-order autoregression, AR(p), a first-order moving average MA(1), and an unobserved components-local level (UC-LL) model. These models are employed to study inflation, inflation growth, and price level dynamics. The choice of these models are guided by the literature on inflation dynamics. For example, Stock and Watson (2005) report estimates of AR(p), MA(1), and UC models on quarterly inflation and inflation growth. This article employs similar models, but engages monthly samples of CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE.

The AR(p) yields estimates of average or mean inflation, inflation persistence, and inflation volatility. The AR(p) model is in deviations from mean inflation

$$\pi_t - \pi_0 = \sum_{j=1}^{p} \gamma_j (\pi_{t-j} - \pi_0) + \varepsilon_t,$$

where inflation, π_t, is defined as the difference between the (natural) log of the month t price level, P_t, and month $t-1$ price level, $\pi_t \equiv 1200\times(\ln P_t - \ln P_{t-1})$, π_0 is mean inflation, and ε_t is the inflation forecast innovation or shock. Maximum likelihood estimates (MLEs) of the AR(p) are generated from Kalman filter iterations.[3]

Information about inflation persistence is contained in the γ_j s. One measure of inflation persistence is the sum of the γs, defined by $\gamma(\mathbf{1}) \equiv \sum_{j=1}^{p} \gamma_j$. It represents the cumulative response of inflation to its own shock ε_t. Another metric of inflation persistence is the largest AR root of the γs, Λ.[4] The largest eigenvalue Λ of the γs

[3] See the appendix for details.

[4] Computation of Λ is described in the appendix.

captures the speed at which inflation returns to its long-run average in response to ε_t.[5] Since $\gamma(\mathbf{1})$ and Λ are functions of the AR coefficients, $\gamma_1 \ldots \gamma_p$, these statistics reveal different aspects of inflation persistence. The length of time inflation takes to return half-way to its long-run mean is a function of the largest eigenvalue Λ, $\frac{\ln 0.5}{\ln \Lambda}$. Inflation persistence rises as $\gamma(\mathbf{1})$ and Λ approach one (from below).

As inflation persistence rises, it takes on a unit root and becomes non-stationary. This arises when, for example, $\gamma(\mathbf{1}) \geq 1$. Stock and Watson (2005) report estimates of $\gamma(\mathbf{1})$ larger than one that point to a unit root in quarterly U.S. inflation since 1970. A lesson they draw is that it is better to study models of inflation growth, $\Delta\pi_t = \pi_t - \pi_{t-1}$, rather than the level of inflation. One such model is the MA(1) process

$$\Delta\pi_t \quad = \quad \eta_t \ - \ \theta\eta_{t-1},$$

where θ is the MA1 coefficient of inflation growth and η_t is the MA(1) mean zero forecast innovation or shock, with homoscedastic standard deviation σ_η.[6]

Estimates of the MA1 coefficient θ contains information about inflation growth persistence. The MA(1) yields the AR(∞), $\Delta\pi_t = \sum_{j=1}^{\infty} \vartheta_j \Delta\pi_{t-j} + \eta_t$, where $\vartheta_j = \theta^j$, given $|\theta| \in (-1, 1)$. The sum $\vartheta(\mathbf{1})$ equals $\frac{-\theta}{1-\theta}$. Therefore, the long-run response of inflation growth to its shock η_t increases as $\theta \longrightarrow 1$. At $\theta = 1$, the speed of adjustment of inflation to an own shock is instantaneous.

It is interesting to explore the impact of $\theta = 1$ on the MA(1) of inflation growth. In this case, $\Delta\pi_t = \Delta\eta_t$. Since the difference operator Δ appears on either side of the

[5] Another way to describe Λ is that it is the speed of adjustment of inflation along its transition path.

[6] Nelson and Schwert (1977) and Pearce (1979) also find an itegrated MA(1) best fits U.S. inflation.

equality, the Δ operators cancel. The result is inflation collapses to the white noise process $\pi_t = \eta_t$.[7] When $\theta = 1$, inflation is unforecastable because it is driven only by the unpredictable shock η_t.

Another implication of $\theta = 1$ is that the price level is a random walk, $\ln P_t = \ln P_{t-1} + \eta_t$. A random walk forces persistence onto the price level because an increase in η_t never decays, no matter the length of the forecast horizon.[8] For example, the forecast of $\ln P_{t+j}$, $j > 1$, is $\ln P_{t-1} + \eta_t$, according to the random walk. A random walk in the price level also sets its trend to the sum of the shocks, $\ln P_t = \sum_{j=0}^{\infty} \eta_{t-j}$.

The UC-LL model imposes random walks on $\ln P_t$ and π_t. Besides placing a random walk trend in the price level, the UC-LL model endows inflation with a random walk that measures deviations from the price level trend. A convenient way to write the UC-LL model is

$$\begin{aligned}
\ln P_t &= \mu_{1,t} \\
\mu_{1,t+1} &= \mu_{1,t} + \mu_{2,t} + \delta_{t+1}, \quad \delta_{t+1} \sim \mathbf{N}(0, \sigma^2_{\delta}), \\
\mu_{2,t+1} &= \mu_{2,t} + \psi_{t+1}, \quad \psi_{t+1} \sim \mathbf{N}(0, \sigma^2_{\psi}),
\end{aligned}$$

where $\mu_{1,t}$ denotes the price level trend, δ_t is its forecast innovation, $\mu_{2,t}$ represents trend deviations from the price level, and ψ_t is its forecast innovation.[9] When δ_{t+1} rises, the impact on $\mu_{1,t+j}$ ($j \geq 1$) and $\ln P_{t+j}$ is permanent because it never decays. The same response is generated by the shock to trend deviations from the price level, ψ_{t+1}.

[7]This white noise process for inflation ignores π_0.

[8]The price level random walk ignores drift produced by π_0.

[9]Harvey (1990) and Gourieroux and Monfort (1997) sketch the UC-LL model. The appendix outlines methods to estimate it and the MA(1) model.

The UC-LL model provides estimates of expected inflation, $\mathbf{E}_t\pi_{t+1}$.[10] Recognize that $\Delta \ln P_t = \mu_{1,t} - \mu_{1,t-1} = \mu_{2,t} + \delta_{t+1}$. Next, use the expectations operator, $\mathbf{E}_t\{\cdot\}$, to find $\mathbf{E}_t\Delta \ln P_{t+1} \equiv \mathbf{E}_t\pi_{t+1} = \mu_{2,t}$. Thus, deviations from the price level trend provide estimates of expected inflation. These deviations are persistent – a random walk in fact – and have innovations, ψ_{t+1}, whose impact on $\mathbf{E}_t\pi_{t+j}$ is permanent. Given MLEs of the UC-LL model, $\mathbf{E}_t\pi_{t+1}$ can be computed using the Kalman filter or smoother.[11]

3. Data and Sample Construction

This paper studies inflation, inflation growth, and price level dynamics with the CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE.[12] The sample begin with $1967M01$ and runs to $2005M09$. This gives the entire sample $T = 465$ observations.

Another goal of this paper is to examine the impact on instability in inflation, inflation growth, and price level dynamics on the MLEs of AR(p), MA(1), and UC-LL models. Evidence of instability is explored with two samples that move or roll through the entire sample. The process starts with

(i) The first rolling sample always starts with $1967M01$ and its initial pass through the data sets its last observation to $J = 1972M08$, which covers 15 percent of the entire sample.

[10] $\mathbf{E}_t\{\cdot\}$ is the mathematical expectations operator conditional on date t information.

[11] Hamilton (1994) presents methods to compute the Kalman filter and smoother; also see the appendix.

[12] The CPI equals the ratio of the date t value of a fixed market basket of goods to the same fixed market basket valued at the base year, which is 1982 – 1984 at the moment. The PCED weights are chained to the 2000 base year. Unlike the CPI, PCED data is revised periodically. This article uses PCED data available on December 1, 2005.

(ii) The second rolling sample starts where the first rolling ends – at the next observation $J + 1 = 1972M09$ – and ends with $2005M09$, which is the remaining 85 percent of the sample.

(iii) Next, the first rolling sample is extended one observation to $J = 1972M09$, which forces the second rolling sample to commence with $J + 1 = 1972M10$, but the second sample retains $2005M09$ as its final observation.

(iv) The procedure is complete when the last observation of the first rolling sample reaches $J = 1999M09$, which is 85 percent of the entire sample, and $J + 1 = 1999M10$ is the initial observation of the second rolling, which represents the other 15 percent of the sample.

Steps (i) – (iv) create two rolling samples on the four price indices from which 326 sets of MLEs of the AR(p), MA(1), and UC model parameters are taken.

4. Results

This section reports MLEs of the AR(p), MA(1), and UC-LL models on the CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE indices. The entire $1967M01$ – $2005M09$ sample and the two rolling samples are used to produce estimates. Table 1 summarizes results.

4.a AR(p) Model Estimates

The MLEs of the AR(p) model are discussed in this section. Table 2 presents MLEs of AR(p)s for CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE inflation using the entire $1967M01$ – $2005M09$ sample. Lag lengths of the AR(p)s are set by the SIC, where

$p = 1, \ldots, 18$.[13] Compared to CPI-ALL and PCED-ALL inflation, CPI-CORE and PCED-CORE inflation have smaller estimated means, $\hat{\pi}_0$, and are less volatile as measured by estimates of the standard deviation of regression residuals, $\hat{\sigma}_\varepsilon$. Also, CPI inflation is higher on average and more volatile than PCED inflation given the MLEs of $\hat{\pi}_0$ and $\hat{\sigma}_\varepsilon$.

The estimates of the AR(p)s yield evidence that CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE inflation are persistent on the $1967M01$ – $2005M09$ sample. Sums of the estimated γ_js, $\hat{\gamma}(\mathbf{1})$s, are all greater than 0.8. The estimates of the largest eigenvalue, $\hat{\Lambda}$, of the $\hat{\gamma}_j$s are no smaller than 0.94. The $\hat{\Lambda}$ translate into measures of the half-life of the response to an own shock of about 12 and 16 years for CPI-ALL and CPI-CORE inflation, respectively. The PCEDs are more persistent than the CPIs because the former's $\hat{\Lambda}$ predict between 20 and 29 years for the half-life of the response to an own shock. Also, note that CORE inflation is more persistent than ALL inflation.

Table 2 presents MLEs that suggests monthly CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE inflation are persistent. However, this is not evidence that U.S. inflation has a unit root. Rather than report unit root tests, estimates of the largest AR root of these inflation measures, along with 95 percent confidence intervals are reported next.

Andrews and Chen (1994) develop an (approximate) median-unbiased estimator of the largest AR root of a time series.[14] The largest monthly median-unbiased AR root

[13]Besides the SIC, the AIC and LR tests are computed. It is no surprise the AIC picks a longer lag length than the SIC for CPI-ALL ($p = 13$), CPI-CORE ($p = 9$), and PCED-ALL ($p = 13$), but $p = 6$ for PCED-CORE inflation. LR tests select ps that fall between SIC and AIC choices, except for PCED-CORE inflation ($p = 13$).

[14]Estimates of the largest AR root rely on AR(6)s that contain an intercept, but not a time trend.

of CPI-ALL inflation is close to but less than one at 0.9965, which fails to support the unit root hypothesis on the $1967M01$ – $2005M09$ sample. However, this estimate predicts a half-life of an own shock to CPI-ALL inflation of 16.5 years that is long-lived relative to a sample of nearly 39 years.[15] The evidence supports a unit root in the three other inflation rates because the Andrews-Chen median-unbiased estimator yields 95 percent confidence intervals of the largest AR root of CPI-CORE, PCED-ALL, and PCED-CORE inflation equal to [0.9863 1.0004], [0.9868 1.0007], and [0.9900 1.0017], respectively.

Part of the puzzle of U.S. inflation dynamics is whether it suffers from instability. For example, Cogley and Sargent (2001) argue that shifts in the structure of monetary policy alter the process generating inflation. Since such changes can force inflation to appear non-stationary, which can be confused for a unit root, it raises questions about the stability of persistence estimates and unit root tests of inflation.

Figures 1 – 3 present MLEs of π_0, $\gamma(\mathbf{1})$, and σ_ε on the two rolling samples, $\widehat{\pi}_{0,t}$, $\widehat{\gamma}(\mathbf{1})_t$, and $\widehat{\sigma}_{\varepsilon,t}$, respectively.[16] Beginning at the upper left and moving clockwise, figures 1 – 3 contains windows based on CPI-ALL, CPI-CORE, PCED-CORE, and PCED-ALL inflation.

[15] Stock (1991) and Andrews and Chen (1994) discuss that a least squares estimate of the largest AR root of a unit root process is biased downward. This explains the smaller root of CPI-ALL inflation to an own shock reported in table 2 compared to the estimate of 0.9965 of the median-unbiased estimator.

[16] The two rolling samples yield MLEs that suggest using tests, say, by Andrews (1993), of parameter instability given an unknown break date. The problem is the Hansen (1997) and Andrews (2003) critical values cannot always be used. The reason is that the two rolling samples produce MLEs of the AR(p)s, MA(1), and UC-LL models that are often on the boundary of the permissable parameter space, which implies critical values would have to be constructed on a case-by-case basis.

The blue solid (red dot-dash) plots are estimates that rely on the first (second) rolling sample.

Figure 1 plots mean inflation estimates, $\hat{\pi}_{0,t}$, constructed on the two rolling samples and four inflation measures. The first rolling sample produces $\hat{\pi}_{0,t}$ on observations that always begin with $1967M01$ and end with $J = 1972M08, \ldots, 1999M09$. For example, the first element of the blue solid plots of figure 1 is estimated on a $1967M01$ – $1972M08$ sample, the second element on a $1967M01$ – $1972M09$ sample, and so on. Figure 1 also includes plots of $\hat{\pi}_{0,t}$ estimated on the second rolling. Samples that run from $J+1 = 1972M09, \ldots, 1999M10$ to $2005M09$ sample yield the red dot-dash plots of figure 1. Thus, plots of $\hat{\pi}_{0,t}$ are obtained from the first rolling sample by adding an observation to its end at each date J, while the second rolling sample sequentially eliminates the initial observation as $J+1$ advances from $1972M09$ to $1999M10$.

The four windows of figure 1 show $\hat{\pi}_{0,t}$ fell during several of the recessions of the last forty years. The largest drop in $\hat{\pi}_{0,t}$ for the four inflation measures occurs in the 1973 – 1975 recession. However, $\hat{\pi}_{0,t}$ rises for CPI inflation in the 1980 recession on the first rolling sample, while $\hat{\pi}_{0,t}$ shows little change for PCED inflation in the same period for this sample. The recessions of 1980 and 1981 – 1982 see lower $\hat{\pi}_{0,t}$ for the four inflation measures on the second rolling sample. The first rolling sample also finds $\hat{\pi}_{0,t}$ drops in the recession of 1981 – 1982. Subsequent to this recession, $\hat{\pi}_{0,t}$ falls, which continues for the four inflation rates on the rest of the two rolling samples. By the end of the first (second) rolling sample, CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE $\hat{\pi}_{0,t}$ equals 4.8 (2.8), 4.8 (2.1), 4.1 (2.3), and 4.0 (1.7), respectively.

Figure 1 also shows that prior to mid-1974 the second rolling sample yields larger $\widehat{\pi}_{0,t}$ than the first rolling sample across all inflation measures. This is reversed subsequent to the end of the 1973 – 1975 recession. The first rolling sample also reveals $\widehat{\pi}_{0,t}$ begins to drift higher after 1975 and peaks prior to the 1980 recession at about seven percent for CPI inflation, six percent for PCED-ALL inflation, and about 5.5 percent for PCED-CORE inflation. Since the second rolling sample produces a fall in mean inflation prior to (or around) the 1980 recession, it points to possible instability in $\widehat{\pi}_{0,t}$ for the four inflation rates toward the end of the 1970s.

Figure 2 presents estimates of time-variation in inflation persistence, $\widehat{\gamma}(\mathbf{1})_t$.[17] The first rolling sample yields plots of $\widehat{\gamma}(\mathbf{1})_t$ that remain close to, but below one from just before the 1973 – 1975 recession to 1999M09. The second rolling sample generates $\widehat{\gamma}(\mathbf{1})_t$ that are close to one until the 1980 recession for the ALL inflation rates and the 1990 – 1991 recession for the CORE inflation series. Prior to the latter recession, the second rolling sample generate $\widehat{\gamma}(\mathbf{1})_t$ that drop from 0.9 to nearly 0.4 in 1996 for CPI-CORE inflation, before raising to about 0.55 in 1999. PCED-CORE inflation persistence exhibits the same behavior, except its $\widehat{\gamma}(\mathbf{1})_t$ turns negative in the mid-1990s and remains negative for the remainder of the second rolling sample, which suggests either a negatively serially correlated or a serially uncorrelated process once observations from the 1970s and 1980s are dropped. Thus, eliminating observations from the 1970s and 1980s leads to smaller persistence estimates for the four inflation series.

[17]Plots of Λ_t yield the same qualitative evidence about persistence for CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE inflation using the two rolling samples. These plots are available on request.

Sargent (1999) provides an interpretation of the first and second rolling sample $\hat{y}(\mathbf{1})_t$s found in figure 2. In this analysis, a key element is the interaction of beliefs about monetary policy and the discount applied to past observations, say, on inflation. For example, discounting past observations can lead to less inflation persistence. According to Sargent, the reason is that discounting is a reasonable response by monetary policy if inflation dynamics are suspected of being unstable. Whether this explains the last 40 years of U.S inflation and monetary policy is not addressed by this paper, but Cogley and Sargent (2005) and Sargent, Williams and Zha (2006) provide useful analyses.

Figure 3 contains plots of the volatility of the four inflation series measured by $\hat{\sigma}_{\varepsilon}$. Given the first rolling sample, plots of $\hat{\sigma}_{\varepsilon,t}$ support the view that CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE inflation volatility began to increase around the first oil price shock of the mid-1970s, which continued until the end of the 1981 – 1982 recession, before leveling off or declining in the early to mid-1980s.

The second rolling sample generates $\hat{\sigma}_{\varepsilon,t}$ plots that give a different view of inflation volatility. Figure 3 shows that volatility in the four inflation measures begins to drop off subsequent either to the second oil price or to the 1981 -1982 recession based on the second rolling sample. CPI-CORE inflation has the largest fall in $\hat{\sigma}_{\varepsilon,t}$, while the other three inflation measures decline less.

There is no consistent pattern to inflation volatility instability according to the AR(p) model estimates. For CPI-ALL inflation, instability in $\hat{\sigma}_{\varepsilon,t}$ possibly exists between the 1980 recession and the late 1990s. The 1980 recession begins a period of instability in $\hat{\sigma}_{\varepsilon,t}$ for CPI-CORE inflation. The beginning and end of the two rolling samples most likely suggest when instability in $\hat{\sigma}_{\varepsilon,t}$ of PCED-ALL inflation could be found. For PCED-CORE inflation, instability in $\hat{\sigma}_{\varepsilon,t}$ appears to occur from 1972M08 to the 1980 recession. Thus, there seems no consistent pattern of instability in $\hat{\sigma}_{\varepsilon,t}$ for the four inflation rates.

4.b MA(1) Model Estimates

This section explores MLEs of the MA(1) of inflation growth. Table 3 reports estimates of the MA1 coefficient, $\hat{\theta}$, and the standard deviation of the MA(1) residual, $\hat{\sigma}_{\eta}$ on the 1967M01 – 2005M09 sample. Estimates of $\hat{\theta}$ are similar across the four inflation growth measures. The point estimates range from 0.72 to 0.78, which predict that within one month inflation growth loses about three-fourths of the increase caused by an own unit shock. However, inflation growth is lower by about 2.5 to 3.5 percent in the long-run given such a shock, as measured by $\hat{\vartheta}(\mathbf{1})$. This indicates inflation growth is subject to large low frequency fluctuations. Not unexpectedly, $\hat{\sigma}_{\eta}$ shows that CPI-ALL (PCED-ALL) inflation growth is more volatile than CPI-CORE (PCED-CORE) inflation growth. PCED-ALL and PCED-CORE inflation growth are also less volatile than their CPI counterparts.

Figure 4 suggests instability in $\hat{\theta}_t$ across the four inflation growth measures and the two rolling samples. Instability in $\hat{\theta}_t$ appears to arise between the recessions of 1980 and 1981 – 1982 for CPI-ALL inflation growth and the first oil price shock for CPI-CORE, PCED-ALL, and PCED-CORE inflation growth. Also, the first rolling sample generates $\hat{\theta}_t$ that are close to 0.70 and stable subsequent to the 1981 – 1982 recession. The plots of $\hat{\theta}_t$ drift toward one for the four inflation growth measures on the second rolling sample between the recessions either of 1980 and 1981 – 1982 or of 1973 – 1975.

A prominent feature of the bottom right window of figure 4 is that $\hat{\theta}_t = 1$ for

PCED-CORE inflation growth on the second rolling sample from $1992M04$ to $1999M10$. The impact on inflation dynamics is that the MA(1) collapses to $\pi_t = \eta_t$ (ignoring mean inflation), when $\theta = 1$. Thus, the second rolling sample yields $\hat{\theta}_t$ that predict PCED-CORE inflation is driven only by white noise shocks from the recovery of the early 1990s to $1999M10$. This matches the small AR persistence estimates reported for PCED-CORE inflation in section 4.a and evidence reported by Stock and Watson (2005).

Figure 5 contains plots of $\hat{\sigma}_{\eta,t}$ for the four inflation growth series. The top left window of figure 5 shows that CPI-ALL inflation growth and the first rolling sample yield $\hat{\sigma}_{\eta,t}$ that are below those of the second rolling sample, except around the first oil price shock. CPI-CORE inflation growth and the first rolling sample produce $\hat{\sigma}_{\eta,t}$ that are always above those of the second rolling sample, as seen in the top right window of figure 6. The second oil price shock matters for CPI-CORE inflation growth volatility because around this time $\hat{\sigma}_{\eta,t}$ falls by 45 and 70 percent on its first and second rolling sample.

The bottom row of windows of figure 5 include plots of $\hat{\sigma}_{\eta,t}$ that qualitatively resemble plots of $\hat{\sigma}_{\varepsilon,t}$ for PCED-ALL and PCED-CORE inflation at the bottom of figure 3. Thus, the MA(1) and AR(p) models produce PCED inflation growth and inflation volatility estimates that are similar.

4.c UC-LL Model Estimates

This section reports estimates of the UC-LL model. Table 4 contains estimates of the standard deviations of innovations to the price level trend, $\hat{\sigma}_\delta$, and to price level trend deviations, $\hat{\sigma}_\psi$. The largest estimates of $\hat{\sigma}_\delta$, and $\hat{\sigma}_\psi$ are obtained from the CPI indices. The last row of table 4 shows that shocks to the price level trend dominate fluctuations in the four price indices because $\hat{\sigma}_\delta$ is always larger than $\hat{\sigma}_\psi$ by about a factor of three to four. Also, note that the largest estimated ratio of $\hat{\sigma}_\delta$ to $\hat{\sigma}_\psi$ is for PCED-CORE.

The UC-LL model predicts that inflationary expectations equal trend price level deviations, $\mathbf{E}_t \pi_{t+1} = \mu_{2,t}$. Figure 6 contains smoothed (blue solid plot) and filtered (red dot-dash plot) estimates of $\hat{\mu}_{2,t}$ computed from MLEs of the UC-LL model for the four price indices on the 1967M01 – 2005M09 sample. Although the filtered $\hat{\mu}_{2,t}$ are more volatile and 'choppier' than smoothed $\hat{\mu}_{2,t}$, the latter have earlier turning points. The reason is that smoothing employs information in the entire 1967M01 – 2005M09 sample. Only observations from 1967M01 to date t are available to compute filtered $\hat{\mu}_{2,t}$.[18]

Estimates of $\mathbf{E}_t \pi_{t+1}$ reveal that the relatively small $\hat{\sigma}_\psi$ of table 3 generate economically important fluctuations in inflationary expectations. For example, CPI estimates of $\mathbf{E}_t \pi_{t+1}$ peak during every recession during the 1967M01 – 2005M09 sample, except the 2001 recession, as found in the top row of windows of figure 6. These plots show peaks in CPI-ALL and CPI-CORE filtered (smoothed) expected annual inflation rates of 12.4 and 12.7 (10.9 and 11.6) percent at 1974M08 and 1974M07 (1974M06 and 1974M06) and 14.8 and 14.2 (13.4 and 12.8) percent at 1980M03 and (1979M12). Subsequently, filtered (smoothed) $\mathbf{E}_t \pi_{t+1}$ falls to -0.9 (0.7) percent by 1986M04 (1986M02) for CPI-ALL and to 3.4 (3.6) percent by 1986M05 (1986M04) for CPI-CORE. At 1990M09 and 1990M07 (1990M06 and 1990M05), filtered (smoothed) CPI-ALL and CPI-CORE $\mathbf{E}_t \pi_{t+1}$ peak at about

[18] Filtered and smoothed $\hat{\mu}_{2,t}$ are generated with the Kalman filter, as discussed by Hamilton (1994). Smoothed $\hat{\mu}_{2,t}$ involves a two-sided (in-sample) forecast (*i.e.*, uses the full data set), while filtered $\hat{\mu}_{2,t}$ is a one-sided forecast given observations $1, \ldots, t$. Filtered $\hat{\mu}_{2,t}$ is initialized with $\mathbf{E}_0 \pi_{1967M01} = 0$.

seven and six (six and five) percent. From 1992 to 2004, filtered (smoothed) CPI-ALL and CPI-CORE $\mathbf{E}_t\pi_{t+1}$ are no higher than 3.5 and 3.7 percent, no smaller than −0.02 and 0.08 percent, before reaching 6.7 and 1.5 percent by 2005M09.

PCED-ALL and PCED-CORE $\mathbf{E}_t\pi_{t+1}$ appear in the bottom two rows of figure 6. These measures of inflation are qualitatively similar to those for the CPI indices in the top row of windows of figure 6, but estimates of $\mathbf{E}_t\pi_{t+1}$ peak only during the 1973 – 1975, 1980, and 1990 – 1991 recessions. Peaks in PCED-ALL and PCED-CORE $\mathbf{E}_t\pi_{t+1}$ are successively lower at each recession, irrespective of the filtered or smooth estimates. These estimates are between nine and 10.5 percent during the 1973 – 1975 recession and 1974M06, 9.5 and 12 percent at the 1980 recession, and 4.5 to five percent for the 1990 – 1991 recession. From 1992 through 2004, PCED-ALL and PCED-CORE $\mathbf{E}_t\pi_{t+1}$ range from 0.5 to three percent. By 2005M09, PCED-ALL and PCED-CORE $\mathbf{E}_t\pi_{t+1}$ equal 5.2 and 1.8 percent, respectively.

Information about parameter instability in the MLEs of σ_δ and σ_ψ appear in figures 7 and 8. Parameter instability in the UC-LL model garners information about changing CPI-ALL, CPI-CORE, PCED-ALL, and PCI-CORE price dynamics. This information is useful to understand if the declines in $\mathbf{E}_t\pi_{t+1}$ subsequent to the recession of the early 1980s that appear in figure 6 are related to small shock realizations to trend deviations from the price level, ψ_t, or to instability in the volatility of this shock, σ_ψ.

Figure 7 contains four windows that plot the first and second rolling sample estimates of the standard deviation of the price level trend shock innovation, $\widehat{\sigma}_{\delta,t}$. These estimates suggest instability in σ_δ. The instability in $\widehat{\sigma}_{\delta,t}$ appears to arise in the late

1990s for the ALL price indices. Evidence of a break in $\hat{\sigma}_{\delta,t}$ for CPI-CORE is suggested by its drop in the second rolling sample at the end of the 1980 recession. For PCED-CORE, the instability in $\hat{\sigma}_{\delta,t}$ possibly occurs during the first oil price shock.

Another feature of figure 7 is that the second rolling sample generates $\hat{\sigma}_{\delta,t}$ with little movement until the early 1990s when it begins to rise steadily for CPI-ALL, PCED-ALL, and PCED-CORE. CPI-CORE is the exception because on the second rolling sample $\hat{\sigma}_{\delta,t}$ falls from around 1.75 in mid-1979 to slightly greater than one from late 1983 to late 1999. Also note that $\hat{\sigma}_{\delta,t}$ rises – for the most part – from 1972M07 to 1999M10 for PCED-ALL and PCED-CORE on the two rolling samples.

Figure 8 replicates figure 7, except $\hat{\sigma}_{\psi,t}$ replaces $\hat{\sigma}_{\delta,t}$.[19] Instability in $\hat{\sigma}_{\psi,t}$ is possible for CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE according to figure 8. The CPIs and the two rolling samples suggest instability begins at the 1980 recession. For PCED-ALL and PCED-CORE, instability seems to start with the 1973 – 1975 recession.

The first rolling sample yields $\hat{\sigma}_{\delta,t}$ that range from about 0.4 to 0.8 for CPI-ALL, PCED-ALL, and PCED-CORE and between 0.5 and 0.9 for CPI-CORE. Thus, adding observations to the first rolling sample produces $\hat{\sigma}_{\psi,t}$ that do not fall by much. These estimates suggest that smaller realizations of $\hat{\psi}_t$ and smaller $\hat{\sigma}_{\psi}$ are responsible for the fall in $\mathbf{E}_t\pi_{t+1}$ subsequent to the 1980 recession, as plotted in figure 6.

[19]Figures 7 and 8 contain plots of $\hat{\sigma}_{\psi,t}$ and $\hat{\sigma}_{\delta,t}$ that appear to be step functions on the first rolling sample. The mapping from the MA(1) coefficients θ and σ_η to σ_ψ and σ_δ is one explanation for this observation. The recursive mapping is $(1+\theta^2)\sigma_\eta^2 = 2\sigma_\delta^2 + \sigma_\psi^2$ and $-\theta\sigma_\eta^2 = -\sigma_\delta^2$. Watson (1986) and Morley, Nelson, and Zivot (2003) review the link between UC and ARMA models.

The lower right window of figure 8 reveals that PCED-CORE and the second rolling sample drive $\hat{\sigma}_{\psi,t}$ to zero by $1992M08$, where it remains through $1999M10$. If $\sigma_{\psi} = 0$, the UC-LL model predicts the price level is a random walk driven by δ_t. An implication is that PCED-CORE inflation resembles a white noise process when observations from the 1970s, 1980s and early 1990s are eliminated from the second rolling sample. A similar result is reported in sections 4.a and 4.b for the AR(p) and MA(1) models and by Stock and Watson (2005).

5. Conclusions

This article studies U.S. inflation, inflation growth, and price level dynamics with the CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE on a sample that runs from $1967M01$ to $2005M09$. Two rolling samples are constructed to uncover evidence about instability in inflation, inflation growth, and price level dynamics.

Autoregressive models produce persistence and volatility estimates that vary with different combinations of the two rolling samples and four price indices. For example, inflation and inflation growth persistence estimates differ across CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE and are sensitive to observations from the 1970s, 1980s, and early 1990s. For example, inflation persistence appears to be large and stable if these observations are included in the sample. However, instabilities appear to arise when the observations are discounted. Equally striking is that PCED-CORE inflation approximates serially uncorrelated white noise when observations up to and including the recession of 1990 – 1991 are eliminated.

Inflation, inflation growth, and price level volatility estimates behave similarly across CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE and the two rolling samples. An important example is that the volatility of shocks to expected inflation have fallen substantially for the four price indices either prior to or during the 1980 recession. Especially striking is the lack of volatility in these shocks for PCED-CORE subsequent to the 1990 – 1991 recession. Along with the AR persistence estimates, it suggests that at the moment a sensible forecast for PCED-CORE inflation is its mean on the 1992 – 2005 sample.

Another way to summarize the empirical results of this article is that instability in the persistence and volatility of CPI-ALL, CPI-CORE, PCED-ALL, and PCED-CORE inflation, inflation growth, and levels coincide with different economic events. An unresolved question is whether such changes are one-time events or can be expected to be repeated systematically in the future. For example, it is open to debate whether the decline in PCED-CORE inflation persistence around the end of the 1990 – 1991 recession was caused by changes in beliefs about the systematic engineering of monetary policy or reflected technology innovations, changes in market structure, or changes in the composition of the economy (*i.e.*, away from manufacturing to the service sector). Such questions pose a challenge to economic research, forecasting, and monetary policy. Any response should find the tools Sargent (1999), Cogley and Sargent (2005), Sims and Zha (2006), Sargent, Williams, and Zha (2006), and Brock, Durlauf, and West (2006) develop to be useful.

References

Andrews, Donald W.K. 2003. Tests for Parameter Instability and Structural Change with Unknown Change Point: A Corrigendum. *Econometrica* 71, no. 1:395 – 397.

Andrews, Donald W.K. 1993. Tests for Parameter Instability and Structural Change with Unknown Change Point. *Econometrica* 61, no. 4:821 – 856.

Andrews, Donald W.K., and Hong-Yuan Chen. 1993. Approximately Median-Unbiased Estimation of Autoregressive Models. *Journal of Business and Economic Statistics* 12, no. 2:187 – 204.

Atkeson, Andrew, and Lee E. Ohanian. 2001. Are Phillips Curves Useful for Forecasting Inflation? Federal Reserve Bank of Minneapolis *Quarterly Review* 25 no. 1:2 – 11.

Brock, William A., 1974, Money and Growth: The Case of Long Run Perfect Foresight. *International Economic Review* 15 no. 3:750 – 777.

Brock, William A., Steven N. Durlauf, and Kenneth D. West. 2006. Model Uncertainty and Policy Evaluation: Some Empirics and Theory. *Journal of Econometrics*, forthcoming.

Cogley, Timothy, and Thomas J. Sargent. 2005. The Conquest of U.S. Inflation: Learning and Robustness to Model Uncertainty. *Review of Economic Dynamics*, 8 no. 2:528 – 563.

Cogley, Timothy, and Thomas J. Sargent. 2001. Evolving Post-World War II U.S. Inflation Dynamics. In *NBER Macroeconomic Annual, 2001*, edited by Benjamin S. Bernanke. Cambridge, MA: MIT Press.

Fisher, Jonas D.M., Chin Te Liu, and Ruilin Zhou. 2002. When Can We Forecast Inflation? Federal Reserve Bank of Chicago *Economic Perspectives* 26 no. 1:30 – 42.

Gourieroux, Christian, and Alain Monfort. 1997. *Time Series and Dynamic Models*, New York, NY:Cambridge University Press.

Greenspan, Alan. 1994. Statement before the Subcommittee on Economic Growth and Credit Formulation of the Committee on Banking, Finance, and Urban Affairs, U.S. House of Representatives, February 22.

Hamilton, James D. 1994. *Time Series Analysis*, Princeton, NJ:Princeton University Press.

Harvey, Andrew C. 1990. *Forecasting, Structural Time Series Models and the Kalman Filter*, New York, NY:Cambridge University Press.

Hansen, Bruce E. 1997. Approximate Asymptotic *P* Values for Structural-Change. *Journal of Business and Economic Statistics* 15, no. 1:60 – 67.

Hansen, Lars P., and Thomas J. Sargent. 1980. Formulating and Estimating Dynamic Linear Rational Expectations Models. *Journal of Economic Dynamics and Control* 2, no. 1:7 – 46.

Hansen, Peter R., Asger Lunde, and James M. Nason. 2005. Model Confidence Sets for Forecasting Models. Working Paper no. 2005 – 7, Federal Reserve Bank of Atlanta.

Koopman, Siem Jan. 1997. Exact Initial Kalman Filtering and Smoothing for Nonstationary Time Series Models. *Journal of the American Statistical Association* 92, no. 440:1630 – 1638.

Morley, James C., Charles R. Nelson, and Eric Zivot. 2003. Why are Beveridge-Nelson and unobserved component decompositions of GDP so different?", *Review of Economics and Statistics* 85, no. 2:235 – 234.

Nelson, Charles R., and G. William Schwert. 1977. Short-Term Interest Rates as Predictors of Inflation: On Testing the Hypothesis that the Real Rate of Interest Is Constant. *American Economic Review* 67 no. 3:478-486.

Pearce, Douglas K. 1979. Comparing survey and rational measures of expected inflation, *Journal of Money, Credit, and Banking* 11, no. 4:447 – 456.

Sargent, Thomas J. 1999. *The Conquest of American Inflation*, Princeton, NJ:Princeton University Press.

Sargent, Thomas J., Noah Williams, and Tao Zha. 2006. Shocks and Government Beliefs: The Rise and Fall of American Inflation. *American Economic Review*, forthcoming.

Sims, Christopher A., and Tao Zha. 2006. Were There Regime Switches in U.S. Monetary Policy. *American Economic Review*, forthcoming.

Stock, James H. 1991. Confidence intervals for the largest autoregressive root in U.S. macroeconomic time series, *Journal of Monetary Economics* 28, no. 3:435 – 459.

Stock, James H., and Mark W. Watson. 2005. Has Inflation Become Harder to Forecast? manuscript, Department of Economics, Princeton University.

Stock, James H., and Mark W. Watson. 1999. Forecasting Inflation, *Journal of Monetary Economics* 44, no. 2:293 — 335.

Watson, Mark W. 1986. Univariate Detrending Methods with Stochastic Trends, *Journal of Monetary Economics* 18, no. 1:49 — 75.

Table 1. Summary of Empirical Results

	CPI-ALL	CPI-CORE	PCED-ALL	PCED-CORE
Mean π				
1st R.S.[†]	Falls at end of 2nd Oil P.S.	same	same	same
2nd R.S.	Falls at end of 1st Oil P.S.	same	same	same
Persistence				
1st R.S.	Near one after 1973–75 Recession	same	same	same
2nd R.S.	Falls during 1980 Recession	same as CPI-ALL	Falls at 1990–91 Recession	same as PCED-ALL
Volatility				
1st R.S.	Rises at 1st Oil Price Shock	same	same	same
2nd R.S.	Rises at 1990-91 Recession	Falls at 1980 Recession	same as CPI-ALL	Falls at 1981-82 Recession

[†]The mnemonic R.S. denotes rolling sample.

Table 2: Estimates of AR(p)s of Inflation

Sample: 1967M01 – 2005M09

	CPI-ALL	CPI-CORE	PCED-ALL	PCED-CORE
$\hat{\pi}_0$	4.64	4.36	4.10	3.97
	(0.71)	(0.73)	(0.87)	(0.92)
$\hat{\gamma}_1$	0.40	0.31	0.41	0.29
	(0.07)	(0.05)	(0.06)	(0.08)
$\hat{\gamma}_2$	0.14	0.31	0.04	0.07
	(0.06)	(0.07)	(0.06)	(0.05)
$\hat{\gamma}_3$	0.06	0.05	0.09	0.18
	(0.05)	(0.06)	(0.05)	(0.05)
$\hat{\gamma}_4$	0.08	−0.06	0.06	0.10
	(0.05)	(0.06)	(0.05)	(0.05)
$\hat{\gamma}_5$	0.15	0.10	0.15	0.07
	(0.06)	(0.05)	(0.06)	(0.05)
$\hat{\gamma}_6$	–	0.17	0.16	0.22
		(0.06)	(0.05)	(0.05)
$\hat{\gamma}(\mathbf{1})$	0.83	0.89	0.90	0.92
	(0.05)	(0.06)	(0.04)	(0.04)
$\hat{\Lambda}$	0.94	0.96	0.97	0.98
	[11.7]	[16.3]	[20.5]	[29.4]
$\hat{\sigma}_\varepsilon$	2.65	2.03	1.89	1.50
	(0.15)	(0.13)	(0.07)	(0.09)

Heteroskedastic consistent asymptotic standard errors appear in parentheses. The persistence measures are $\hat{\gamma}(\mathbf{1}) = \sum_{j=1}^{p} \hat{\gamma}_j$ and $\hat{\Lambda}$ which is the largest eigenvalue of the estimated AR coefficients, $\hat{\gamma}_j$, $j = 1, \ldots, p$. The brackets contain an estimate of the half-life (in years) of the response of inflation to an own shock, according to the persistence measure $\ln 0.5 / \ln \hat{\Lambda}$.

Table 3: Estimates of Inflation Growth MA(1)

SAMPLE: 1967M01 – 2005M09

	CPI-ALL	CPI-CORE	PCED-ALL	PCED-CORE
$\hat{\theta}$	0.75	0.72	0.75	0.78
	(0.05)	(0.06)	(0.05)	(0.05)
$\hat{\sigma}_\eta$	2.67	2.07	1.94	1.53
	(0.15)	(0.15)	(0.08)	(0.09)
$\hat{\vartheta}(\mathbf{1})$	−2.97	−2.56	−2.93	−3.45
	(0.39)	(0.34)	(0.37)	(0.49)

Heteroskedastic consistent asymptotic standard errors appear in parentheses.

Table 4: Estimates of the UC-LL Model

SAMPLE: 1967M01 – 2005M09

	CPI-ALL	CPI-CORE	PCED-ALL	PCED-CORE
$\hat{\sigma}_\delta$	2.31	1.74	1.67	1.34
	(0.16)	(0.15)	(0.08)	(0.05)
$\hat{\sigma}_\psi$	0.68	0.61	0.51	0.36
	(0.14)	(0.14)	(0.10)	(0.07)
$\hat{\sigma}_\delta/\hat{\sigma}_\psi$	3.38	2.85	3.29	3.77
	(0.76)	(0.73)	(0.74)	(0.86)

Heteroskedastic consistent asymptotic standard errors appear in parentheses.

Appendix: The Models

This appendix reviews the univariate models of inflation, inflation growth, and the price level. The models are a pth-order autoregression, AR(p), a first-order moving average MA(1), and an unobserved components (UC) structure. These models are employed to study inflation, inflation growth, and price level dynamics. The choice of these models is guided by the literature on inflation dynamics. For example, Stock and Watson (2005) report estimates of AR(p), MA(1), and UC models on quarterly GDP deflator inflation, PCED-ALL, and PCED-CORE inflation and inflation growth. This article employs similar models, but estimates on monthly samples of PCED-ALL, and PCED-CORE, as well as the CPI-ALL, CPI-CORE.

The univariate AR(p) yields estimates of mean inflation, inflation persistence, and inflation volatility. In deviations from mean inflation, π_0, the AR(p) model is

$$\text{(A.1)} \qquad \pi_t - \pi_0 = \sum_{j=1}^{p} \gamma_j (\pi_{t-j} - \pi_0) + \varepsilon_t,$$

where inflation, π_t, is defined by the difference between the (natural) log of the month t price level, P_t, and month $t-1$ price level, $\pi_t \equiv 1200 \times (1 - \mathbf{L}) \ln P_t$, the lag operator $\mathbf{L}$ produces $\mathbf{L} \ln P_t \equiv \ln P_{t-1}$, and ε_t is the Gaussian inflation forecast innovation with standard deviation σ_ε. The article reports maximum likelihood estimates (MLEs) estimates of π_0, $\gamma_1, \ldots, \gamma_p$, and σ_ε from Kalman filter iterations of the state space model

$$\text{(A.2)} \qquad \begin{aligned} \pi_t &= \pi_0 + \mathbf{e}_1 \Xi_t \\ \Xi_{t+1} &= \Gamma\, \Xi_t + \mathcal{E}_{t+1}, \end{aligned}$$

where the first equation is the observer equation, the second equation is the state equation, $\Xi_t = [\xi_{1,t} \ \ldots \ \xi_{p,t}]'$ is the $p \times 1$ state vector, the row vector $\mathbf{e}_1 = [1 \ \ \mathbf{0}_p]$, $\mathbf{0}_p$ is a

$1 \times (p-1)$ vector of zeros, $\mathcal{E}_t = [\varepsilon_t \;\; \mathbf{0}_p]'$, and Γ is the companion matrix of the γ_j s

$$\Gamma = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_{p-1} & \gamma_p \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

The state vector Ξ_t is initialized with a vector of zeros because under the null of the AR(p) the eigenvalues of Γ are inside the unit circle. At date $t = 0$, the mean square error of Ξ_t is set to $[\mathbf{I}_{p^2} - \Gamma \otimes \Gamma]^{-1} vec(\mathbf{E}\{\mathcal{E}_t \mathcal{E}_t'\})$, where $vec(\cdot)$ denotes placing the second column below the first, the third column below the previous two, and so on. Hamilton (1994) discusses the Kalman filter approach to MLE of ARMA models in detail.

Information about inflation persistence is contained in the γ_j s. One measure of inflation persistence is denoted with the sum $\gamma(\mathbf{1}) \equiv \sum_{j=1}^{p} \gamma_j$. Another is the largest AR root of inflation, which can be found by computing the largest eigenvalue of Γ, $\Lambda(\Gamma)$. Since $\gamma(\mathbf{1})$ and $\Lambda(\Gamma)$ are functions of the AR coefficients, $\gamma_1 \ldots \gamma_p$, these statistics reveal different aspects of inflation persistence. A metric of the cumulative response of inflation to an own shock is $\gamma(\mathbf{1})$. The largest eigenvalue of the companion matrix Γ measures the speed of adjustment of inflation to an own shock along the transition path. The speed of adjustment is translated into the length of time inflation takes to return half-way to its (long-run) mean with $\frac{\ln 0.5}{\ln \Lambda(\Gamma)}$. Inflation persistence rises as $\gamma(\mathbf{1})$ and $\Lambda(\Gamma)$ approach one (from below).[A.1]

[A.1] A priori there is no restriction that $\gamma(\mathbf{1}) \leq 1$ or $0 \leq \gamma(\mathbf{1})$, but (in modulus) $\Lambda(\Gamma) \in [0,\ 1]$.

As $\gamma(\mathbf{1}) \longrightarrow 1$, inflation persistence increases. In the limit, inflation takes on a unit root and becomes non-stationary. Stock and Watson (2005) report estimates of $\gamma(\mathbf{1}) \geq 1$ that point to a unit root in quarterly inflation since 1970. A lesson they draw is that it is better to work with inflation growth, instead of the level of inflation. This leads Stock and Watson to advocate a model in which inflation growth is decomposed into unobserved permanent (*i.e.*, a unit root or random walk) and transitory components

$$\begin{aligned} \pi_t &= \mu_{\pi,t} + \upsilon_t, \quad \upsilon_t \sim \mathbf{N}(0, \sigma_\upsilon^2) \\ \mu_{\pi,t+1} &= \mu_{\pi,t} + \tau_{t+1}, \quad \tau_{t+1} \sim \mathbf{N}(0, \sigma_\tau^2), \end{aligned}$$

where $\mu_{\pi,t}$ is the permanent component of inflation, its forecast innovation is τ_t, and υ_t denotes the transitory shock innovation.[A.2] Also assume $\mathbf{E}\{\upsilon_{t+q}\tau_{t+j}\} = 0, \ \forall\, q,\, j$.

The reduced-form of the Stock and Watson (SW-)UC model is a first-order MA. The reduced-form MA(1) is constructed by passing the first difference operator, $1-\mathbf{L}$, through $\pi_t = \mu_{\pi,t} + \upsilon_t$ and substituting for $(1-\mathbf{L})\mu_{\pi,t} = \tau_t$ to find $(1-\mathbf{L})\pi_t = \tau_t + \upsilon_t - \upsilon_{t-1}$, with the one-step ahead forecast error $\eta_t \equiv \tau_t + \upsilon_t + \mu_{\pi,t} - \mu_{\pi,t-1|t-1}$, where $\mu_{\pi,t-1|t-1}$ is conditional on observations through date $t-1$. The first-order moving average dynamics of inflation growth motivates studying it with the fixed-coefficient MA(1)

$$(1 - \mathbf{L})\pi_t = (1 - \theta\mathbf{L})\eta_t, \tag{A.3}$$

to obtain evidence of changes in inflation growth persistence, as measured by θ. In this case, time-variation in σ_υ and σ_τ drive changes in inflation growth persistence and volatility. The map between the SW-UC model and the MA(1) of equation (A.3) consists of $(1+\theta^2)\sigma_\eta^2 = 2\sigma_\upsilon^2 + \sigma_\tau^2$ and $-\theta\sigma_\eta^2 = -\sigma_\upsilon^2$.

[A.2] The SW-UC model implies the mean of inflation growth is zero.

MLEs of θ and σ_η are obtained from iterating the Kalman filter. It is initialized following the procedure outlined for the AR(p) of equation (A.1). The article reports estimates of θ and σ_η on the two rolling samples corrected for Blaschke factors when necessary. Hansen and Sargent (1980) and Hamilton (1994) show how to extract Blaschke factors of non-invertiable moving average processes to adjust MLEs of θ and σ_η.

Estimates of the MA1 coefficient θ of equation A.3 contains information about inflation growth persistence. The MA(1) of equation (A.3) yields the AR(∞), $(1-\mathbf{L})\pi_t = \sum_{j=1}^{\infty} \vartheta_j (1-\mathbf{L})\pi_{t-j} + \eta_t$, where $\vartheta_j = \theta^j$, given $|\theta| \in (-1, 1)$. The sum $\vartheta(\mathbf{1})$ equals $\frac{-\theta}{1-\theta}$. Therefore, the long-run response of inflation growth to an own shock increases as $\theta \longrightarrow 1$. At $\theta = 1$, the speed of adjustment of inflation to an own shock is instantaneous.

Price level dynamics are not directly studied by the SW-UC model. Rather than define the observer equation with inflation, expressing it in (the log of) the price level $\ln P_t$ gives the UC-local level (LL) model. The UC-LL model is

$$\begin{aligned} \ln P_t &= \mu_{1,t} \\ \mu_{1,t+1} &= \mu_{1,t} + \mu_{2,t} + \delta_{t+1}, \quad \delta_{t+1} \sim \mathbf{N}(0, \sigma^2_\delta) \\ \mu_{2,t+1} &= \mu_{2,t} + \psi_{t+1}, \quad \psi_{t+1} \sim \mathbf{N}(0, \sigma^2_\psi), \end{aligned} \tag{A.4}$$

where $\mu_{1,t}$ denotes the price level trend, δ_t is its forecast innovation, $\mu_{2,t}$ represents price level trend deviations, and ψ_t is its forecast innovation. When δ_{t+1} rises, the impact on $\mu_{1,t+j}$ $(j \geq 1)$ and $\ln P_{t+j}$ is permanent because it never decays. The same response is generated by the shock to price level trend deviations, ψ_{t+1}. Details about UC models are found in Harvey (1990) and Gourieroux and Monfort (1997). Harvey suggests that mean square error estimates of the state vector distinguish the SW-UC and UC-LL models.

The reduced-form MA(1) of the UC-LL model is $(1-\mathbf{L})\pi_t = (1-\mathbf{L})\delta_t + \psi_t$. Since the reduced form of the UC-LL model is a first-order moving average, the results discussed above about the connection between the SW-UC model and the MA(1) of equation (A.3) are applicable. For the UC-LL model, the mapping is $(1+\theta^2)\sigma_\eta^2 = 2\sigma_\delta^2 + \sigma_\psi^2$ and $-\theta\sigma_\eta^2 = -\sigma_\delta^2$. The UC-LL also draws out implications for inflation of price level trend shocks, δ_t. This aspect of the UC-LL model ties inflation persistence and volatility, in part, to price level shocks as predicted by the monetary growth model Brock (1974) analyzes.

Harvey (1990) and Gourieroux and Monfort (1997) show how to obtain MLEs of the UC-LL model from the Kalman filter. An issue is that the Kalman filter cannot be initialized using standard approaches because the state space includes unit root processes. Instead, an algorithm Koopman (1997) proposes is employed to initialize the non-stationary components of the state vector. These procedures impose a diffuse prior on the initial state vector to compute an exact initialization of the Kalman filter.

The UC-LL model (A.4) and Kalman filter yield estimates of expected inflation. Let $\mathbf{E}_t\pi_{t+1}$ denote expected inflation, where $\mathbf{E}_t\{\cdot\}$ is the mathematical expectations operator conditional on date t information. Pass the first difference operator $1-\mathbf{L}$ through the first equation of (A.4) followed by the expectations operator, $\mathbf{E}_t\{\cdot\}$, to obtain $\mathbf{E}_t(1-\mathbf{L})\ln P_{t+1} \equiv \mathbf{E}_t\pi_{t+1} = \mu_{2,t}$. Thus, expected inflation equals deviations from the price level trend. These deviations are persistent – a random walk in fact – and have innovations, ψ_{t+1}, whose impact on $\mathbf{E}_t\pi_{t+1+j}$ is permanent. Given MLEs of the UC-LL model, $\mathbf{E}_t\pi_{t+1}$ is computed using the Kalman filter or smoother. Hamilton, (1994) describes these procedures. The initialization of the filtered estimates of $\mathbf{E}_t\pi_{t+1}$ follows Koopman (1997).

Figure 1: Time-Variation in Mean Inflation

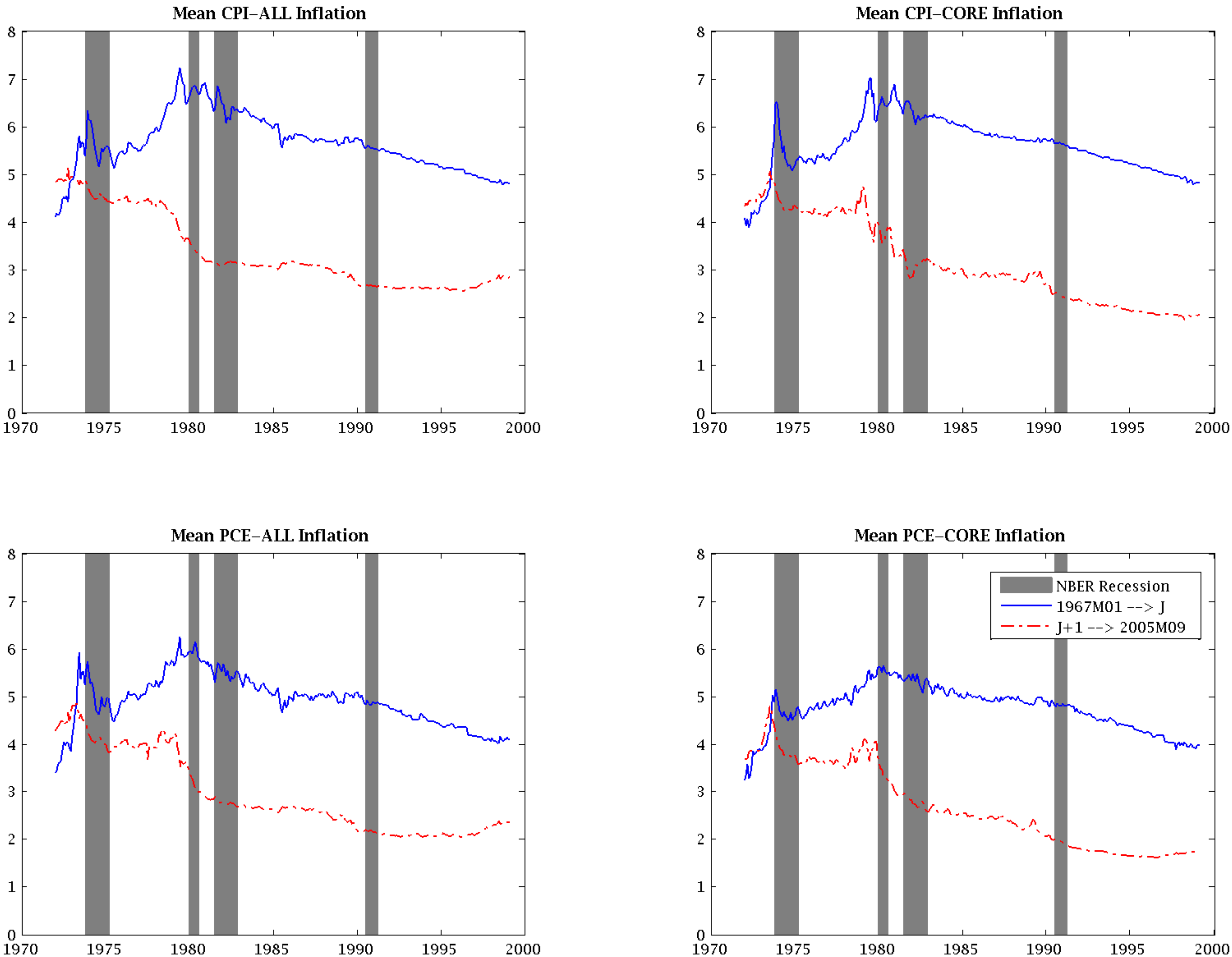

Figure 2: Time-Variation in AR Coefficient Sums

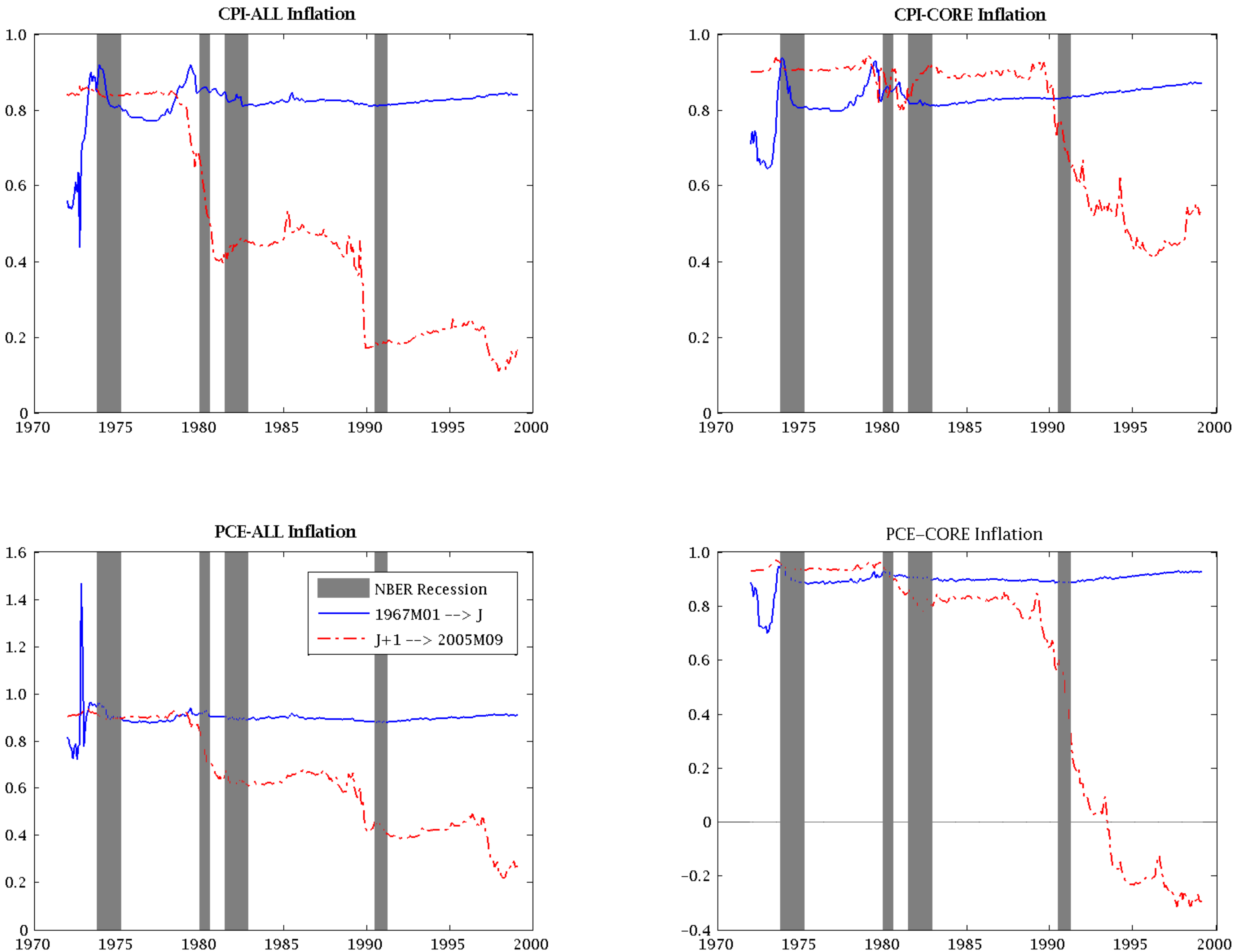

Figure 3: Time-Variation in Standard Deviation of Inflation AR Residuals

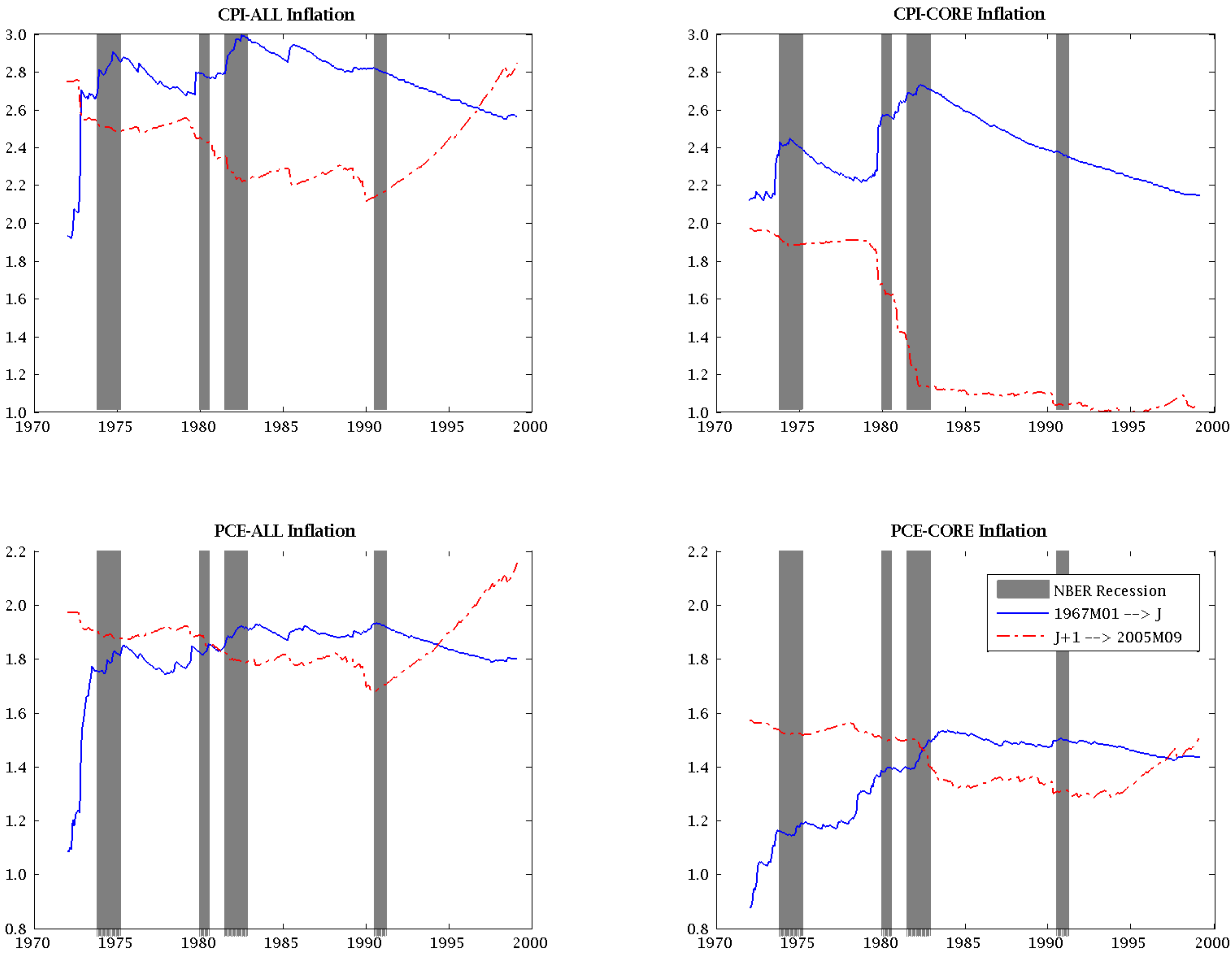

Figure 4: Time−Variation in MA1 Coefficient of Inflation Growth

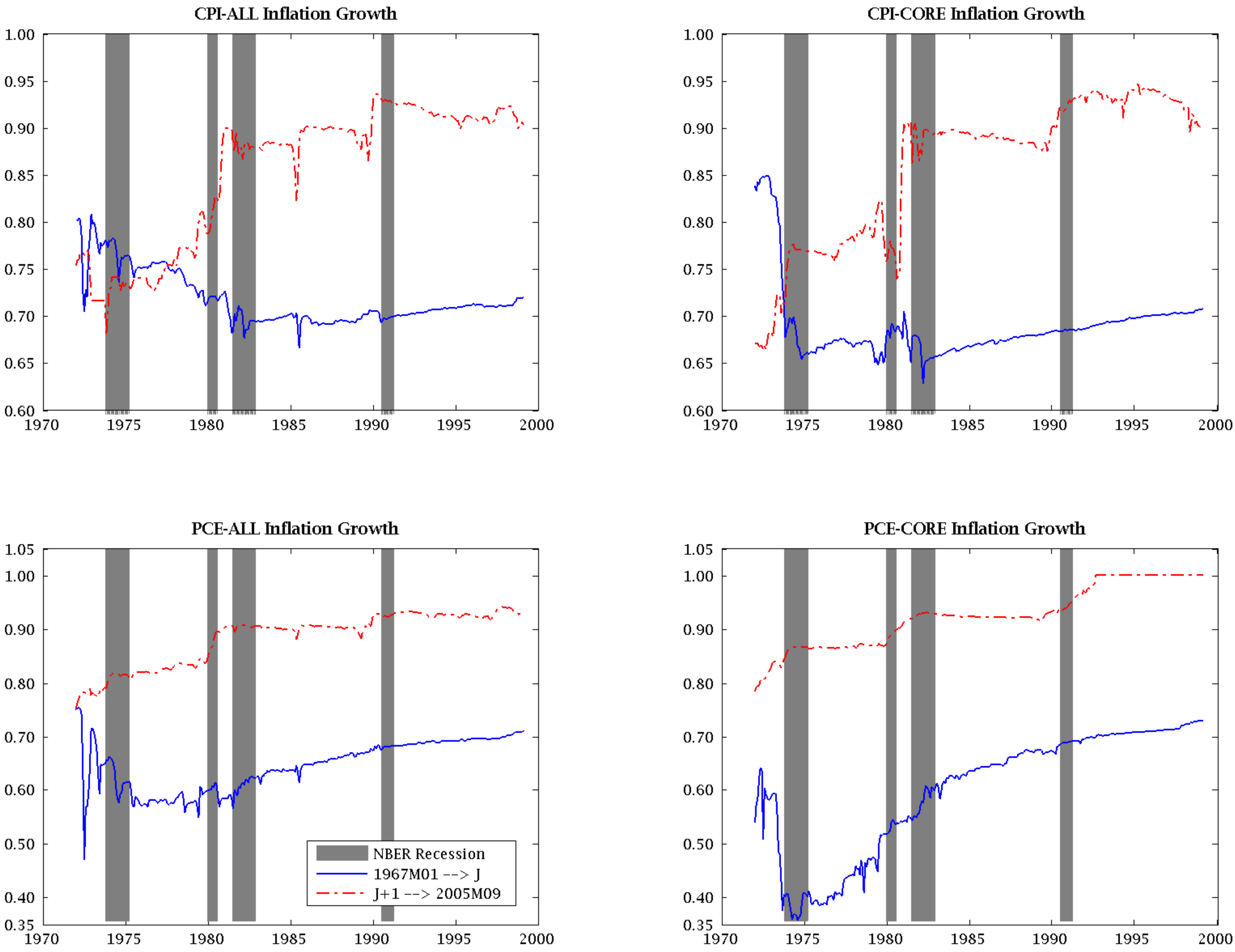

Figure 5: Time-Variation in Standard Deviation of Inflation Growth MA(1) Residuals

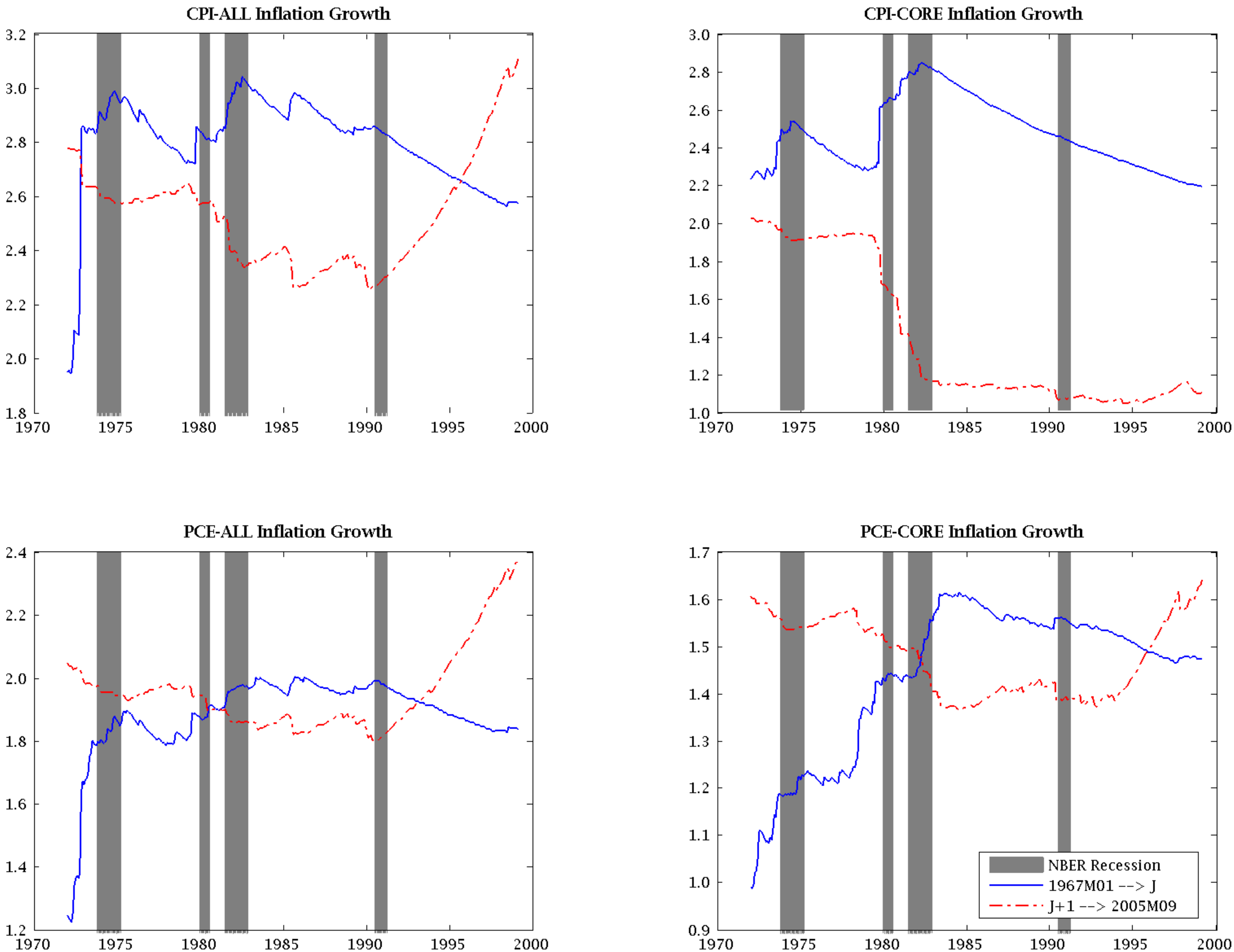

Figure 6: UC-LL Estimates of Expected Inflation

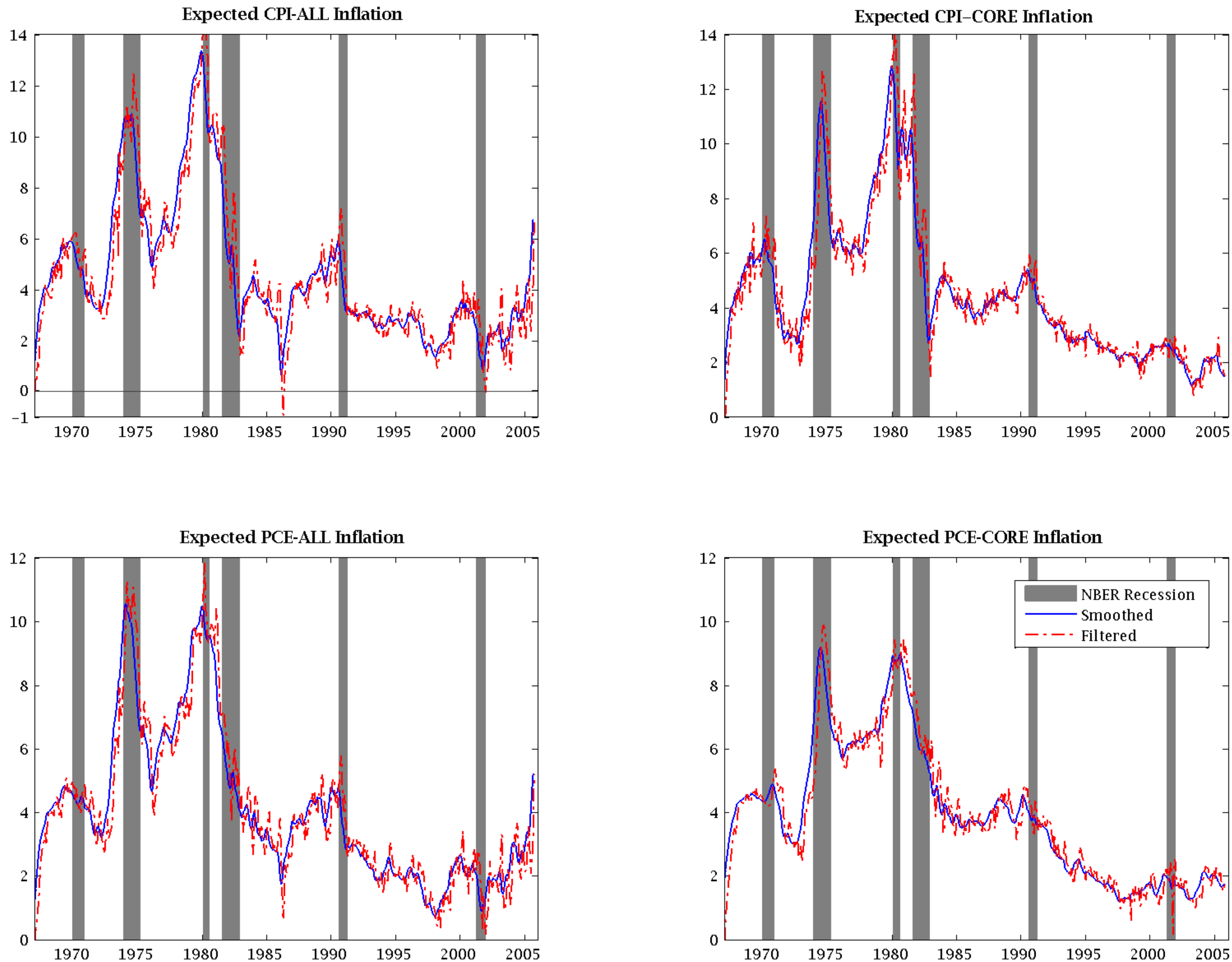

Figure 7: Time-Variation in Standard Deviation of Price Level Trend

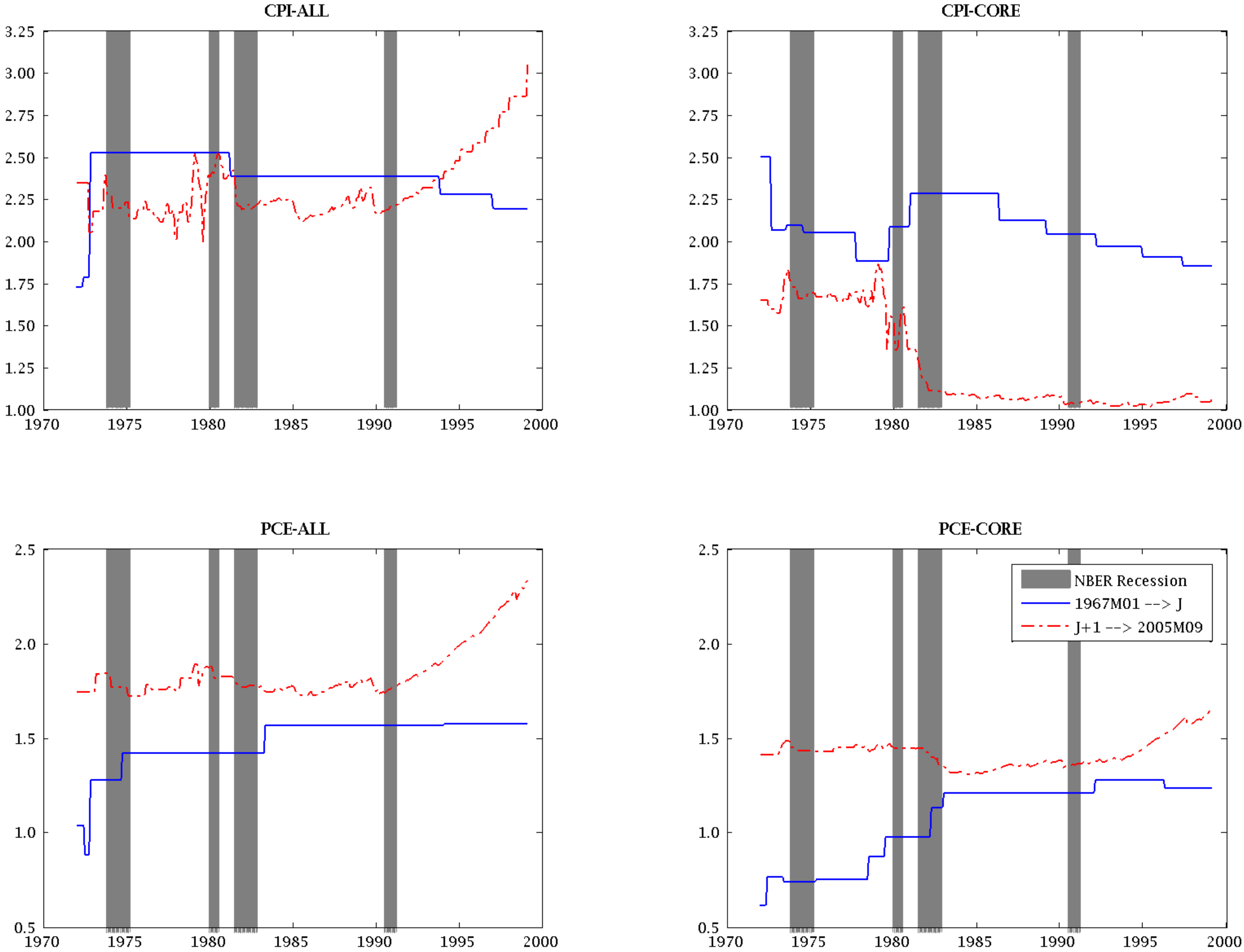

Figure 8: Time-Variation in Standard Deviation of Price Level Trend Deviations