Search Frictions and Asset Price Volatility

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Abstract

We examine the quantitative effect of search frictions in product markets on asset prices. We combine several features from Shi (1997) and Lagos and Wright (2002) in a model without money. Households prefer special goods and general goods. Special goods can be obtained only via a search and bargaining process in decentralized markets. General goods can be obtained via trade in centralized competitive markets and via ownership of an asset. There is only one asset in our model that yields general goods. The asset is also used as a medium of exchange in the decentralized market to obtain the special goods. The value of the asset in facilitating transactions in the decentralized market is determined endogenously. This transaction role makes the asset pricing implications of our model different from those in the standard asset pricing model. We show that a departure from the standard asset pricing model can simultaneously deliver the observed average rate of return on equity and the volatility of the asset price.

1 Introduction

LeRoy and Porter (1981) and Shiller (1981) calculated the time series for asset prices using the simple present value formula – the current price of an asset is equal to the expected discounted present value of its future dividends. Using a constant interest rate to discount the future, they showed that the variance of the observed prices for U.S. equity exceeds the variance implied by the present value formula (see figure 1). This is the *excess volatility puzzle.* Equilibrium models of asset pricing deliver a generalized version of the present value formula. In Lucas (1978), for instance, the discount factor is stochastic and depends on the intertemporal marginal rate of substitution (IMRS) of the representative consumer. There have been several attempts to explain the excess volatility puzzle. LeRoy and LaCivita (1981) and Michener (1982) examine the role of risk aversion. Flavin (1983) and Kleidon (1986) examine whether small sample bias can statistically account for violations of the variance bound. Marsh and Merton (1986) try to resolve the puzzle with different statistical assumptions on the dividend process.[1] Shiller (1984) and Ingram (1990) explore whether the existence of rule-of-thumb traders can account for the excess volatility.

The main purpose of this paper is to examine the quantitative effect of search frictions in product markets on asset prices.[2] We combine several features from Shi (1997) and Lagos and Wright (2002) in a model *without* money. Households prefer special goods and general goods. Special goods can be obtained only via trade in decentralized markets. This trading process involves search and bargaining. Similar

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[1] West (1988) develops a volatility test that circumvents the above small sample bias and dividend process criticisms and shows that the observed stock prices are indeed too volatile.

[2] Mehra and Prescott (1985) showed that for reasonable values of risk aversion the Lucas asset pricing model cannot reproduce the observed equity premium. In their concluding remarks they suggest – "Perhaps introducing some features that make certain types of intertemporal trades among agents infeasible will resolve the (equity premium) puzzle."

to Shi and Lagos-Wright, the search frictions make intertemporal trade infeasible in our model. General goods can be obtained via trade in centralized competitive markets and via ownership of an asset. There is only one asset in our model and that is similar to a Lucas tree that yields fruits that can be consumed directly. The asset is also used as a medium of exchange in the decentralized market to obtain the special goods. The value of the asset in facilitating transactions in the decentralized market is determined endogenously.[3] If we shut down the decentralized trading process (i.e., special goods and search frictions), then our model is identical to that of Lucas (1978).

With only one asset, our model cannot address the equity premium puzzle, but we provide parameters for which the model delivers the average rate of return on equity and the volatility of equity price. The price-dividend ratio implied by the model is high relative to the data while for the same risk aversion the Lucas model underpredicts the price-dividend ratio. When we calibrate the model to deliver the observed price-dividend ratio, the implied value for the medium of exchange role of the asset is on average 17.5% above the Lucas model.

An alternative approach is to calibrate our model is to a liquidity value of the asset. We show that as the liquidity value of the asset increases, the average rate of return declines and the price volatility increases.

The rest of paper is organized as follows. In the next section we set up the economic environment and derive the equilibrium asset pricing equation. In section 3, we study the quantitative implications of the model.

2 The Environment

Consider a discrete-time non-monetary economy with special goods and general goods, decentralized day markets and centralized night markets, and aggregate uncertainty. The special and general goods and the day and night markets are similar to Lagos

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[3] See Bansal and Coleman (1996) for a reduced form model of the transaction role of assets and its implications for asset returns.

and Wright (2002). There is a continuum of types of households and a continuum of households in each type. The measure of types and the measure of households in each type are both normalized to one. A type h household produces only good h but consumes all goods except h. The utility from consuming c units of the special good is $u(c)$. The utility function is increasing and strictly concave, and satisfies $u'(0) = \infty$ and $u'(\infty) = 0$. To produce q units of the special good, households incur q units of disutility. The special goods are non-storable between periods.

There is an infinitely lived asset (Lucas tree) in this economy that yields dividends (fruits) each period. Fruits are general goods and they follow an exogenous stationary stochastic process. The utility from consuming d units of fruits is $U(d)$, where $U(\cdot)$ is increasing and strictly concave. Note that there is no cost to producing the fruits. The fruits are also perishable. Each household is initially endowed with one (divisible) tree.

Special goods are exchanged in a decentralized market in daytime where agents meet in pairs, as in standard search theory. The matching technology is simple: random pairs are forme with probability α. The matching technology combined with the household preferences rules out barter in pairwise meetings. Furthermore, there is no public record of transactions to support any credit arrangements. Thus, in pairwise meetings special goods are exchanged for trees. General goods are available for trade only in the centralized market at night. The night market is frictionless and trees are exchanged for general goods at the competitive equilibrium price p.

Time is indexed by $t = 0, 1, ...$ The discount factor between periods is β. There is no discounting between day and night.

Random matching during the day will typically result in non-degenerate distributions of asset holdings. In order to maintain tractability, we use the device of large households along the lines of Shi (1997). Each household consists of a continuum of worker-shopper (or, seller-buyer) pairs. Buyers cannot produce the special good, only sellers are capable of production. We assume the fraction of buyers = fraction of sellers $= \frac{1}{2}$. Then, the probability of single coincidence meetings during the day is $\frac{1}{4}\alpha$. Each household sends its buyers to the decentralized day market with *take-it-or-leave-it* instructions (q, s) – accept q units of special goods in exchange for s trees. Each household also sends its sellers with "accept" or "reject" instructions. There is no communication between buyers and sellers of the same household during the day. After the buyers and sellers finish trading in the day, the household pools the trees and shares the special goods across its members each period. By the law of large numbers, the distribution of trees and special goods are degenerate across households. This allows us to focus on the representative household. The representative household consumption of the special good is $\frac{\alpha}{4}q$.

2.1 Timing of events in each period

- The representative household starts the period with a trees.
- It observes the aggregate state d (fruits per tree), but the fruits are not available for trade during the day.
- The household determines the take-it-or-leave-it offer (q, s). It allocates s trees to each buyer in the household and provides trading instructions to its sellers and buyers.
- The sellers and buyers from households of *all* types are randomly matched in the decentralized market. In single coincidence meetings, the sellers produce the special good in exchange for trees from the buyers.
- Each household then pools its purchases and consumes the special goods.
- Next, each household enters the centralized market at night with its new asset balance and fruits. Households trade fruits and trees in the centralized competitive asset market (much like the standard consumption based asset pricing model) at price p.

- Then, they consume the fruits and end the period with a' trees.

2.2 Optimization

We begin with the representative household's instructions to its buyers and sellers. Clearly, if a member of the household is not in a single coincidence meeting, the instruction is not to trade. The instruction to the buyers in single coincidence meetings is a the take-it-or-leave-it offer (q, s). For another household's seller to be indifferent between accepting and rejecting the buyer's offer in the random match, (q, s) has to satisfy the seller's participation constraint:

$$\Omega s - q = 0, \tag{1}$$

where Ω is the other household's valuation of the asset. The first term on the left hand side is the gain to the seller from obtaining s trees in the trade. The second term is the disutility from q units of the special good. The take-it-or-leave-it offer will leave no surplus for the seller, so the right hand side is 0. We will assume that the seller will accept the offer whenever he is indifferent. An additional restriction on the offer is that the total number of trees allocated to the buyers by the representative household cannot exceed the number of trees that the household started the period with:

$$\frac{1}{2}s \leq a. \tag{2}$$

This is because (i) the decentralized market does not support credit arrangements, so the buyer cannot short-sell the asset and (ii) the buyer is temporarily separated from other members of the household, so he cannot borrow from the other members of the household. We can eliminate s by combining the two constraints (1) and (2):

$$\frac{1}{2}\left(\frac{q}{\Omega}\right) \leq a.$$

The representative household's instruction to its sellers in single coincidence meetings are straightforward. Suppose that the buyer from the other household offers

(Q, S). The instruction is, if the surplus from (Q, S) is non-negative, accept the offer and produce Q units of the special good; otherwise, reject the offer and do not trade.

The representative household's problem then is described by the following dynamic program:

$$v(a,d) = \max_{q,x,a'} u\left(\frac{\alpha}{4}q\right) - \frac{\alpha}{4}Q + U(x) + \beta E_{d'|d} v(a',d') \tag{3}$$

$$\text{s. t. } \frac{1}{2}\left(\frac{q}{\Omega}\right) \leq a \tag{4}$$

$$x + pa' = \left\{a + \frac{\alpha}{4}S - \frac{\alpha}{4}\left(\frac{q}{\Omega}\right)\right\}(p+d), \tag{5}$$

where Q is the amount of the special good obtained by the buyers from other households and S is the number of trees obtained by the sellers from other households. The second constraint is the wealth constraint for the household. Note that p is the relative price a tree in terms of the fruits in the centralized night market.

Remark 1 *If $\alpha = 0$ (i.e., no search frictions or special goods), then our model is identical to that of Lucas (1978). In this case, the asset has positive value since it yields dividends. The presence of search frictions ($\alpha > 0$) implies an additional "liquidity" value to the asset.*

Uniqueness, concavity and differentiability of $v(\cdot)$ follows from theorems 9.6, 9.7, and 9.8 in Stokey, Lucas and Prescott (1989).

2.3 Equilibrium

Definition 2 *An equilibrium consists of a sequence $\{q_t, x_t, s_t, a_{t+1}\}_{t=0}^{\infty}$, given initial asset holdings, such that*

1. *Given other households' offers and valuations, each household's choices solve the dynamic program (3);*
2. *The choices and the asset valuations are the same across households;*
3. *The centralized markets clear for all t: $x_t = d_t$, $a_{t+1} = 1$.*

Let $\frac{\alpha}{2}\lambda$ be the multiplier on the constraint (4). The first order conditions for the representative household with respect to q and a' are as follows.

$$u'(\frac{\alpha}{4}q) = \frac{1}{\Omega}\left\{(p+d)\,U'(x) + \lambda\right\} \tag{6}$$

$$pU'(x) = \beta E_{d'|d}\frac{\partial v(a', d')}{\partial a'} \tag{7}$$

In these conditions, we have used the wealth constraint (5) to substitute for x. Note that if the no-short-sales constraint (4) does not bind, then $\lambda = 0$. The envelope condition for a implies that

$$\frac{\partial v(a,d)}{\partial a} = (p+d)\,U'\,(x) + \frac{\alpha}{2}\lambda \tag{8}$$

Using (6) to substitute for λ in (8), we get

$$\frac{\partial v(a,d)}{\partial a} = \left(1 - \frac{\alpha}{2}\right)(p+d)\,U'\,(x) + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q\right)\Omega.$$

We can rewrite (7) using the above expression for $\frac{\partial v}{\partial a}$:

$$pU'(x) = \beta E_{d'|d}\left\{\left(1 - \frac{\alpha}{2}\right)(p'+d')\,U'\,(x') + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q'\right)\Omega'\right\}. \tag{9}$$

We have to now impose the equilibrium conditions on (9). The valuation of the asset, Ω, by other households in the decentralized market during the day, has to equal the valuation, ω, by the representative household, in equilibrium. We can determine ω as follows. An additional unit of asset obtained in the decentralized market yields d fruits at night; the asset can also be sold for p fruits in the centralized market at night. On the margin these additional fruits are valued at $U'(x)$. In equilibrium, the general goods market clearing at night implies $x = d$. Hence,

$$\omega = \Omega = (p+d)\,U'\,(d)\,.$$

Using the equilibrium values for Ω and x, we can write (9) as

$$pU'(d) = \beta E_{d'|d}\left\{(p'+d')\,U'\,(d')\left[1 - \frac{\alpha}{2} + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q'\right)\right]\right\}.$$

Hence, the equilibrium sequence of asset prices satisfy

$$p_t U'(d_t) = \beta E_t \left\{ (p_{t+1} + d_{t+1}) \, U'(d_{t+1}) \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} u' \left(\frac{\alpha}{4} q_{t+1} \right) \right] \right\}. \tag{10}$$

Again, note that if $\alpha = 0$, then the above asset pricing equation is identical to that of Lucas (1978). In the presence of search frictions, the price in the competitive asset market accounts for the future liquidity value of the asset as well.[4]

To solve for the equilibrium sequence $\{q_t\}$, we have to account for two possible scenarios. If the constraint (4) does not bind in period t, then λ_t equals zero and $u'(\frac{\alpha}{4} q_t) = 1$. Denote the solution to this equation as q^*. Note that the solution does not depend on the aggregate state and, hence, is time-invariant. Furthermore, if $q_t = q^*$ for all t, then the search frictions are irrelevant for the asset pricing implications and the price sequence in our model is the same as in Lucas (1978). If the constraint (4) binds in period t, then

$$q_t = 2 \left(p_t + d_t \right) U' \left(d_t \right). \tag{11}$$

3 Quantitative Implications

To examine the quantitative implications of our model, we restrict the utility functions to be of the CRRA class,

$$\begin{aligned} u(c) &= \frac{c^{1-\delta}}{1-\delta} \\ U(x) &= \frac{x^{1-\delta}}{1-\delta} \end{aligned}$$

where $0 < \delta < \infty$. Hence, q^* is the unique solution to $\left(\frac{\alpha}{4} q\right)^{-\delta} = 1$.

When the no-short-sales constraint (4) binds, $q = 2(p+d)d^{-\delta}$. Thus, we can write the asset pricing equation (10) for these functional forms as

$$p_t d_t^{-\delta} = \beta E_t \left\{ (p_{t+1} + d_{t+1}) \, d_{t+1}^{-\delta} \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} \left(\frac{\alpha}{4} q_{t+1} \right)^{-\delta} \right] \right\}. \tag{12}$$

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[4] Vayanos and Wang (2002), Duffie, Garleanu and Pedersen (2003) and Weill (2003) consider search frictions in the asset market and present models of liquidity premium.

3.1 Numerical method

To compute the price sequence, we modify the version of Parameterized Expectation Approach (PEA), originally proposed by Den Haan and Marcet (1990), and combine some features of the Monte Carlo simulation method proposed by Judd (1998). In essence, the algorithm iteratively appproximates the conditional expectations that appear in the equilibrium conditions with flexible functions of a set of parameters and of the vector of state variables. Readers familiar with this computation technique may skip this subsection.

Recall the equilibrium conditions are determined by the following set of equations:

$$p_t = E_t \left\{ \beta \frac{(p_{t+1} + d_{t+1}) U'(d_{t+1}) + \frac{\alpha}{2}\lambda_{t+1}}{U'(d_t)} \right\} \tag{13}$$

$$\lambda_t = \left[u'\left(\frac{\alpha}{4} q_t \right) - 1 \right] (p_t + d_t) U'(d_t) \tag{14}$$

$$\lambda_t \left[1 - \frac{q_t}{2U'(d_t)(p_t + d_t)} \right] = 0 \tag{15}$$

The last equation is the Kuhn-Tucker condition. There is only one state variable d_t in this economy. Observe that the conditional expectation in (13) is a time-invariant function ξ of state variable d_t. Therefore, to find a solution for prices $\{p_t\}$, we want to choose a class of functions that can approximate ξ arbitrarily well. The functional form we choose is the exponential function with some polynomials

$$\xi(\theta, d) = \exp\left[\theta_0 + \theta_1 \ln d + \theta_2 (\ln d)^2 + \ldots + \theta_n (\ln d)^n \right],$$

where $\theta = [\theta_0 \ \theta_1 \ \theta_2 \ ... \ \theta_n]$ is a vector of parameters. Since the price in the model is positive, the function we choose guarantees the positive value for the expectation. We can also increase the degree of accuracy by increasing the order n of polynomials. Given θ, the price sequence is described by $p_t = \xi(\theta, d_t)$. The problem now is to find the a vector θ that is consistent with the true expectation. The detailed procedure is described as follows.

1. Assume that the log of dividends follows a trend stationary process:

$$\ln d_{t+1} = b_0 + b_1 \ln d_t + b_2 t + \eta_{t+1} \tag{16}$$

where η_{t+1} is the disturbance with mean 0.[5]

2. For each sample period t, use data in period t to simulate next period dividend level. That is, generate d_{t+1} using the coefficients in (16) and drawing the disturbances η_{t+1} from its empirical distribution. (An alternative is to draw these disturbances under the assumption that η is normally distributed.)

3. At iteration j, given $\theta = \theta^j$, use the dividend simulated in step 2 to calculate $t+1$ price using approximated function $\xi\left(\theta^j, d_{t+1}\right)$.

4. Set $q_{t+1} = q^*$ under the assumption that $\lambda_{t+1} = 0$. Then test whether $q^* \leq 2U'\left(d_{t+1}\right)\left(\xi\left(\theta^j, d_{t+1}\right) + d_{t+1}\right)$. If the inequality is satisfied, set $\lambda_{t+1} = 0$, otherwise set

$$\begin{aligned} q_{t+1} &= 2U'\left(d_{t+1}\right)\left(\xi\left(\theta^j, d_{t+1}\right) + d_{t+1}\right), \\ \lambda_{t+1} &= \left[u'\left(\frac{\alpha}{4}q_{t+1}\right) - 1\right]\left(\xi\left(\theta^j, d_{t+1}\right) + d_{t+1}\right)U'\left(d_{t+1}\right). \end{aligned}$$

5. Calculate the price at period t

$$\widetilde{p}_t = \frac{\beta}{U'\left(d_t\right)}\left[U'\left(d_{t+1}\right)\left(\xi\left(\theta^j, d_{t+1}\right) + d_{t+1}\right) + \frac{\alpha}{2}\lambda_{t+1}\right].$$

6. Repeat steps 2 to 5 many times. The number of replications we use is 3000. The average value of these 3000 calculations of $\widetilde{p}_t$ is p_t.

7. Repeat steps 2 to 6 for periods $t+1$, $t+2, \ldots$, until the end of sample period. Thus, we have a sequence of prices $\{p_t\}$ over the whole sample period.

8. Using the *observed* dividend sequence, run the regression

$$\ln p = \theta_0 + \theta_1 \ln d + \theta_2 \left(\ln d\right)^2 + \ldots + \theta_n \left(\ln d\right)^n$$

and obtain the OLS estimate $\widehat{\theta}$. Update the parameters used in the next iteration by $\theta^{j+1} = \gamma\widehat{\theta} + (1-\gamma)\theta^j$, where $\gamma \in [0,1]$ is the parameter to control the smoothness of convergence.

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[5] See DeJong and Whiteman (1991) for evidence on trend stationarity.

9. If $\left\|\theta^{j+1}-\theta^{j}\right\|$ is less than some tolerance value then stop, otherwise go back to step 2. We use a tolerance value of 10^{-6}.

Using the time series of p calculated from above steps, we can compute the rate of return sequence $\{R_{t+1}\}$ for the whole sample period $R_{t+1}=\frac{(p_{t+1}+d_{t+1})}{p_t}$ and the unconditional moments of prices and returns. One of key problems in implementing PEA algorithm is to select initial conditions for θ. In our calculation, we first solve the standard Lucas asset pricing model using Judd's algorithm and get the coefficients θ. Then we use the coefficients in Lucas model as our initial value. It turns out that the covergence is fast. One advantage of our algorithm is that it can handle the binding constraint pretty easily, as we can see in step 4.

3.2 Data and Parameters

The data are all in real terms and obtained from Shiller's website. The sample period is 1871-1995. We measure the asset prices and dividends by the S&P 500 prices and per capita dividends. We measure the volatility of a variable by the standard deviation of the detrended time series of the variable. The average rate of return on equity in this sample is 8% and the standard deviation of the equity price is 81. The mean growth rate of dividend is 1.91% and the standard deviation of detrended dividend is 1.61.

Other than the coefficients in the trend stationary process, we have two preference parameters, δ and β, and one parameter α that describes the matching friction. The estimates of the coefficients are

$$\begin{aligned} b_0 &= 0.308 \\ b_1 &= 0.802 \\ b_2 &= 0.002 \end{aligned}$$

and the variance of η is 0.0136.

We set $\beta = 0.96$. We searched for α and δ to match the observed average rate of return on equity and standard deviation of the asset price.

Table 1. Benchmark Parameters

β	α	δ
0.96	0.42	1.8

For the benchmark parameters in the table below the average rate of return on the asset is 8% and the standard deviation of the asset price is 80. Recall that the mean return in the data is 8% and the standard deviation of the asset price is 81.

3.3 Results

In figure 3, we illustrate the equilibrium price sequence implied by the model. In figure 4 we illustrate the price-dividend ratio. The mean price-dividend ratio in the data is 22.75 while the model implies a mean of 27.

In figures 5 and 6, we plot the price sequence implied by the model as we vary the parameters δ and α. (The other parameter β is fixed at its benchmark value 0.96.) Changes in δ affect the curvature of the utility function. As δ increases, the asset price volatility increases. As we decrease the search frictions (increase in α), the asset price volatility decreases. The price-dividend ratio exhibits a similar pattern. Figures 7, 8 and 9 illustrate the effects of δ and α on the average rate of return, the volatility and the price-dividend ratio. The table below present a summary of the comparative dynamics associated with changes in δ and α.

Table 2. Comparative dynamics ($\beta = 0.96$)

Average rate of return (%)					
	δ				
α	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.05	5.15	5.37	4.8	5.8	7.8
0.2	5.15	6.16	7.02	6.77	8.24
0.4	5.15	6.16	7.24	8	8.78
0.6	5.15	6.16	7.24	8.44	9.32
0.75	5.15	6.17	7.24	8.50	9.66

Std. dev. of the asset price					
	δ				
α	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.05	29.5	47.7	173.4	798.5	3647
0.2	29.5	38.8	66.4	274.4	1174
0.4	29.5	38.8	49.1	147.8	624.8
0.6	29.5	38.8	49.1	92.3	394.5
0.75	29.5	38.8	49.1	80.1	343.9

The standard asset pricing model ($\alpha = 0$) delivers the observed average rate of return on equity for risk aversion $\delta = 1.85$. In figure 10, we plot the price sequence for this case. The standard deviation of the asset price, however, is 56 while the observed volatility is 81. Changes in δ affect the price sequence as shown in figure 11. As α approaches 1 in our model, the search frictions become smaller and the average rate of return and the volatility in our model approach the values in the Lucas model.

To compute the "liquidity value" of the asset, we calculate the asset prices for a model with $\alpha = 0$ and β and δ set at their benchmark values. This is, of course, the standard asset pricing model. Since the standard model does not assign any medium of exchange role to the asset, the difference between the prices implied by the two models would be the liquidity value of the asset. In figure 12, we illustrate the liquidity value as a fraction of the price implied by the standard model i.e., liquidity value $= \frac{P_{\text{model}} - P_{\text{Lucas}}}{P_{\text{Lucas}}}$. The mean liquidity value implied by the model with search frictions is 17.5%.

Table 3. Liquidity Value ($\beta = 0.96$)

Average liquidity value (%)					
	δ				
α	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.05	0.2	23	203	844	2849
0.2	0.2	0.1	18.18	226.2	850.45
0.4	0.2	0.07	0.1	75	405
0.6	0.2	0.04	0.08	25	243
0.75	0.2	0.02	0.06	9.2	177

3.3.1 The Hansen-Jagannathan Bound

In this section we examine whether the IMRS in our model satisfies the Hansen and Jagannathan (1991) bound. Hansen and Jagannathan proposed a test that generalizes the variance bounds developed by LeRoy and Porter (1981) and Shiller (1981). They used asset return data to derive a lower bound on the volatility of a representative household's IMRS. An asset pricing model is said to be consistent with the data if the volatility of the IMRS implied by the model is greater than the HJ bound. To derive the bound, Hansen and Jagannathan projected the model IMRS onto a space of contemporaneous asset returns and utilized only a necessary condition associated with dynamic models, namely the intertemporal Euler equation. For instance, in the Lucas model, the unconditional version of the Euler equation can be written as

$$ER_{t+1}m_{t+1} = 1,$$

$$\text{where } R_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t} \text{ and } m_{t+1} = \beta \left(\frac{d_{t+1}}{d_t} \right)^{-\delta}.$$

To compute the HJ bound for the case of 1 risky asset, consider the least squares projection of the IMRS onto the linear space spanned by a constant and contemporaneous returns. The projection is of the form

$$m = Em + (R - ER)\theta + \nu,$$

where Em is the mean of the model IMRS and ER is the mean asset return. The projection error ν is orthogonal to the constant as well as contemporaneous returns, so $ER\nu = 0$, and $E\nu = 0$. Hence,

$$\begin{aligned} \text{var}(m) &= \theta^2 var(R) + \text{var}(\nu) \\ &\geq \theta^2 \text{var}(R). \end{aligned}$$

(The notation var(x) refers to variance of x.) The projection coefficient $\theta = \frac{Cov(R,m)}{var(R)}$, where the numerator is the contemporaneous covariance between R and m. We can rewrite $\theta = \frac{ERm - EmER}{var(R)}$. The Euler equation then implies $\theta = \frac{1-EmER}{\text{var}(R)}$. Satisfying the HJ bound amounts to verifying whether

$$\begin{aligned} \text{var}(m) &\geq \frac{(1-EmER)^2}{\text{var}(R)}, \text{ or} \\ \text{std}(m) &\geq \frac{1-EmER}{\text{std}(R)} \end{aligned}$$

for the chosen preference parameters and observed dividend data.

He and Modest (1995) and Luttmer (1996) showed that the presence of frictions alters the HJ bound. The unconditional version of the Euler equation could be, for instance,

$$ER_{t+1}m_{t+1} = \psi < 1.$$

In this case, the lower bound on the volatility of the IMRS is $\frac{\psi - EmER}{\text{std}(R)}$. They then choose the value of ψ that minimizes the volatility bound. Clearly, such a strategy assumes that ψ does not depend on the model parameters. The environment described in section 2 suggests a different approach. Suppose that we can measure the medium of exchange transactions q. The asset pricing equation (10) can be written as

$$E\left\{\left(\frac{p_{t+1}+d_{t+1}}{p_t}\right)\beta\frac{U'(d_{t+1})}{U'(d_t)}\left[1-\frac{\alpha}{2}+\frac{\alpha}{2}u'\left(\frac{\alpha}{4}q_{t+1}\right)\right]\right\} = 1.$$

Rewrite this equation in the familiar form

$$ER_{t+1}m_{t+1} = 1,$$

$$
\begin{aligned}
\text{where } R_{t+1} &= \frac{p_{t+1}+d_{t+1}}{p_t} \text{ and} \\
m_{t+1} &= \beta \frac{U'\left(d_{t+1}\right)}{U'(d_t)} \left[1-\frac{\alpha}{2}+\frac{\alpha}{2}u'\left(\frac{\alpha}{4}q_{t+1}\right)\right].
\end{aligned}
$$

The HJ bound then is $\frac{1-EmER}{\mathrm{std}(R)}$, exactly the same as in the case without frictions. However, the IMRS is very different.

4 Conclusion

In this paper, we consider an environment with search frictions in the goods market. The asset in our model is used to facilitate trading in the goods market. This transaction role makes the asset pricing implications of our model different from those in the standard asset pricing model. We show that a departure from the standard asset pricing model can simultaneously deliver the observed average rate of return on equity and the volatility of the asset price.

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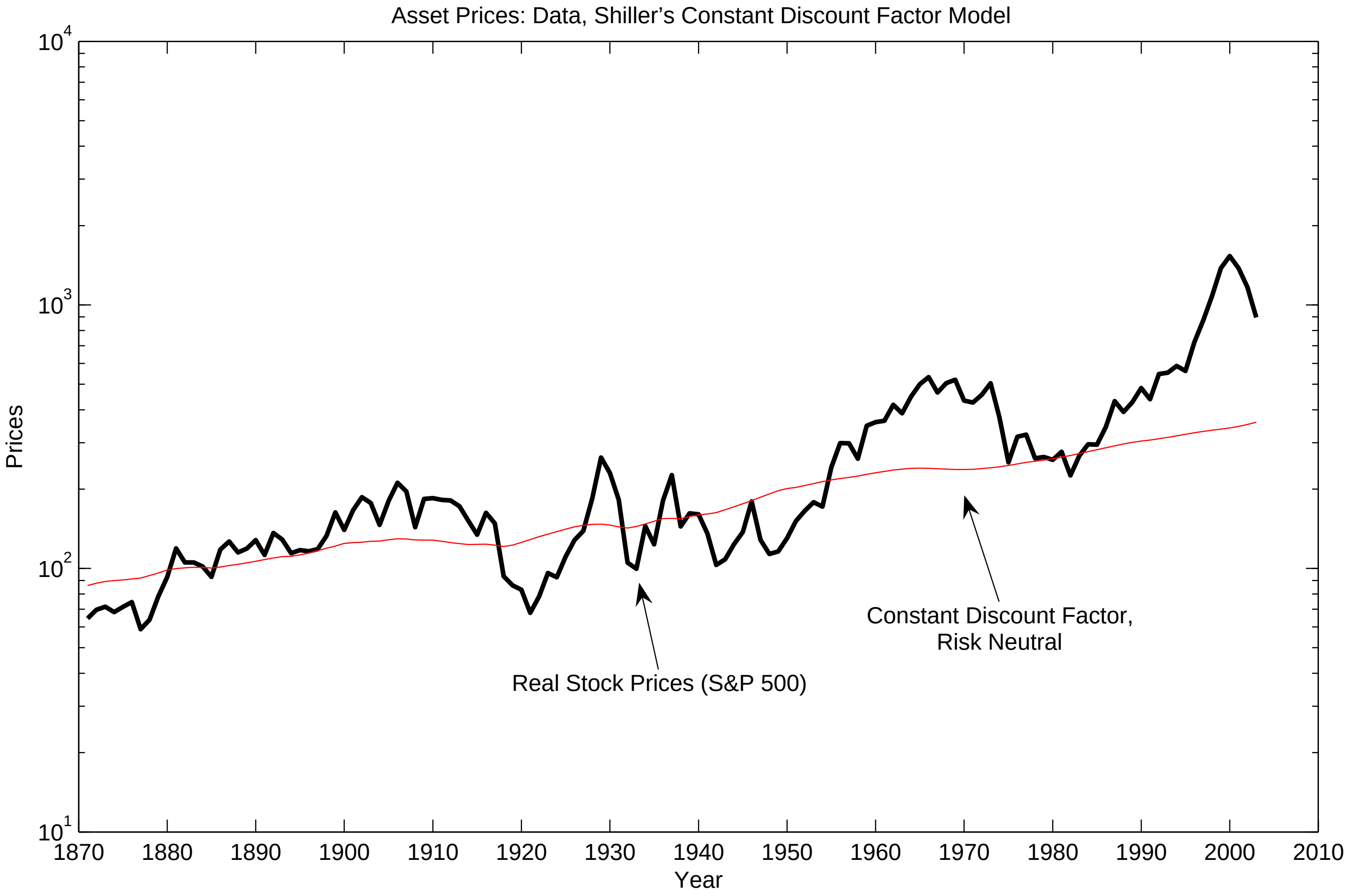

Figure 1: Data and Shiller's Constant Discount Factor Model

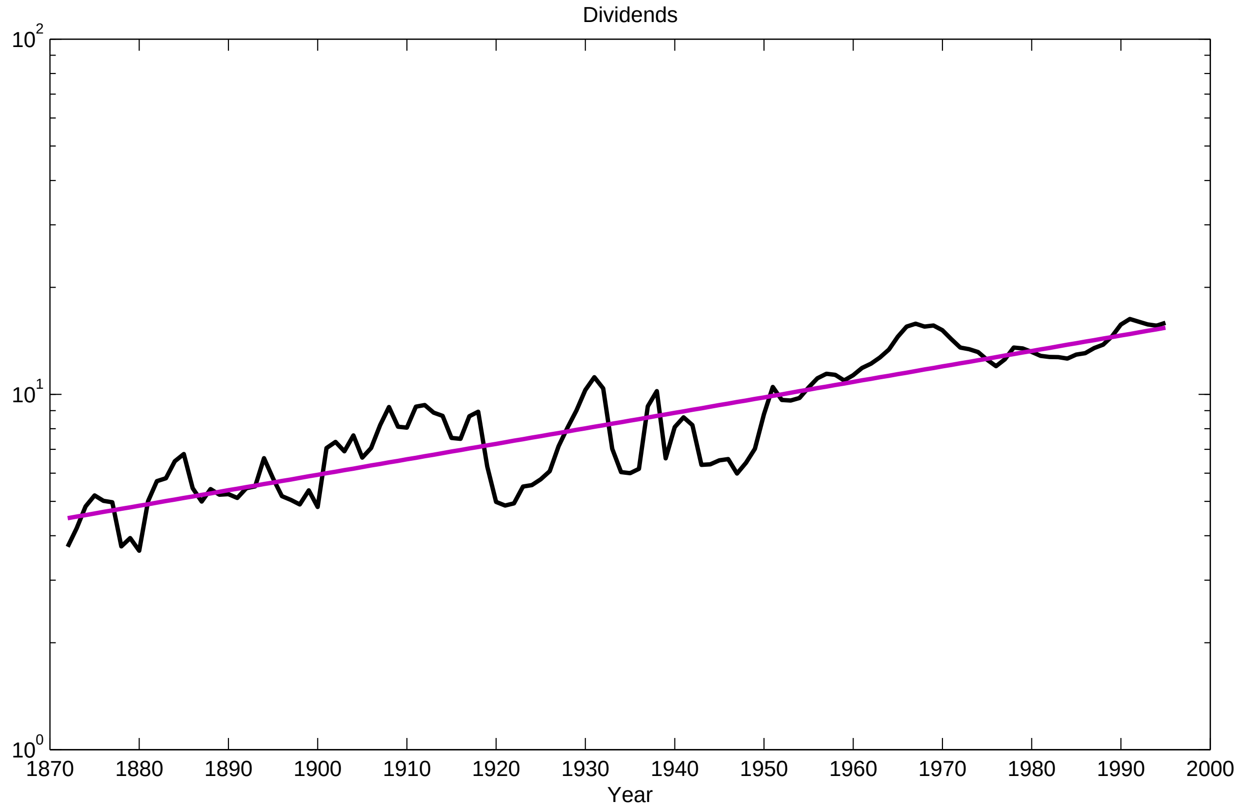

Figure 2: Dividends

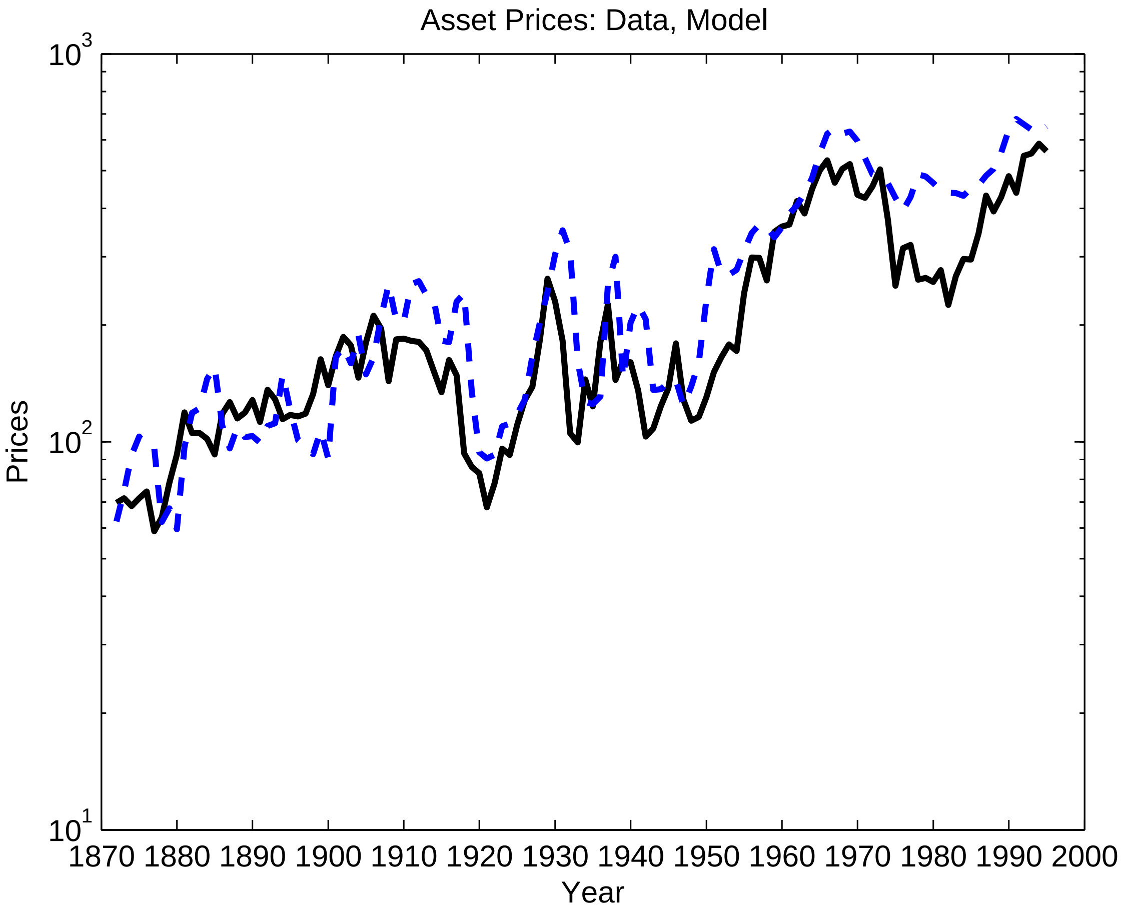

Figure 3: Data and Model

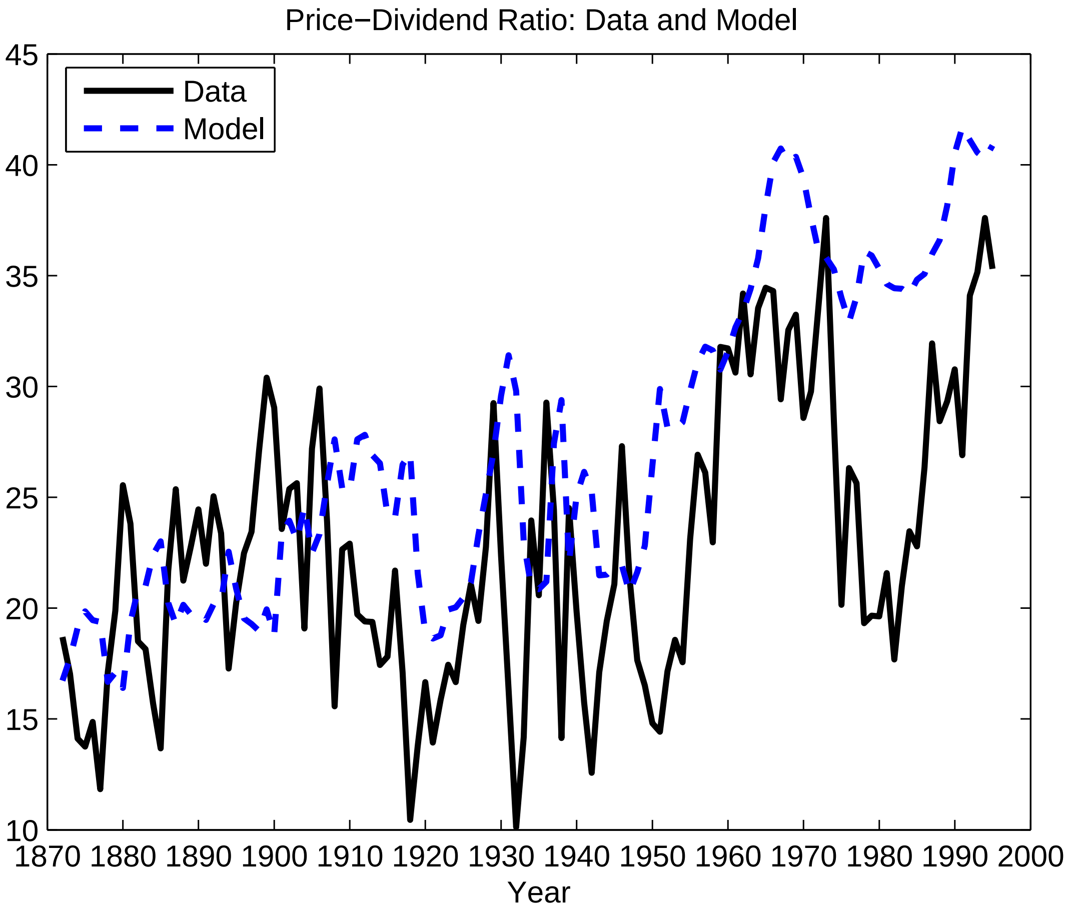

Figure 4: Price-Dividend Ratio - Data and Model

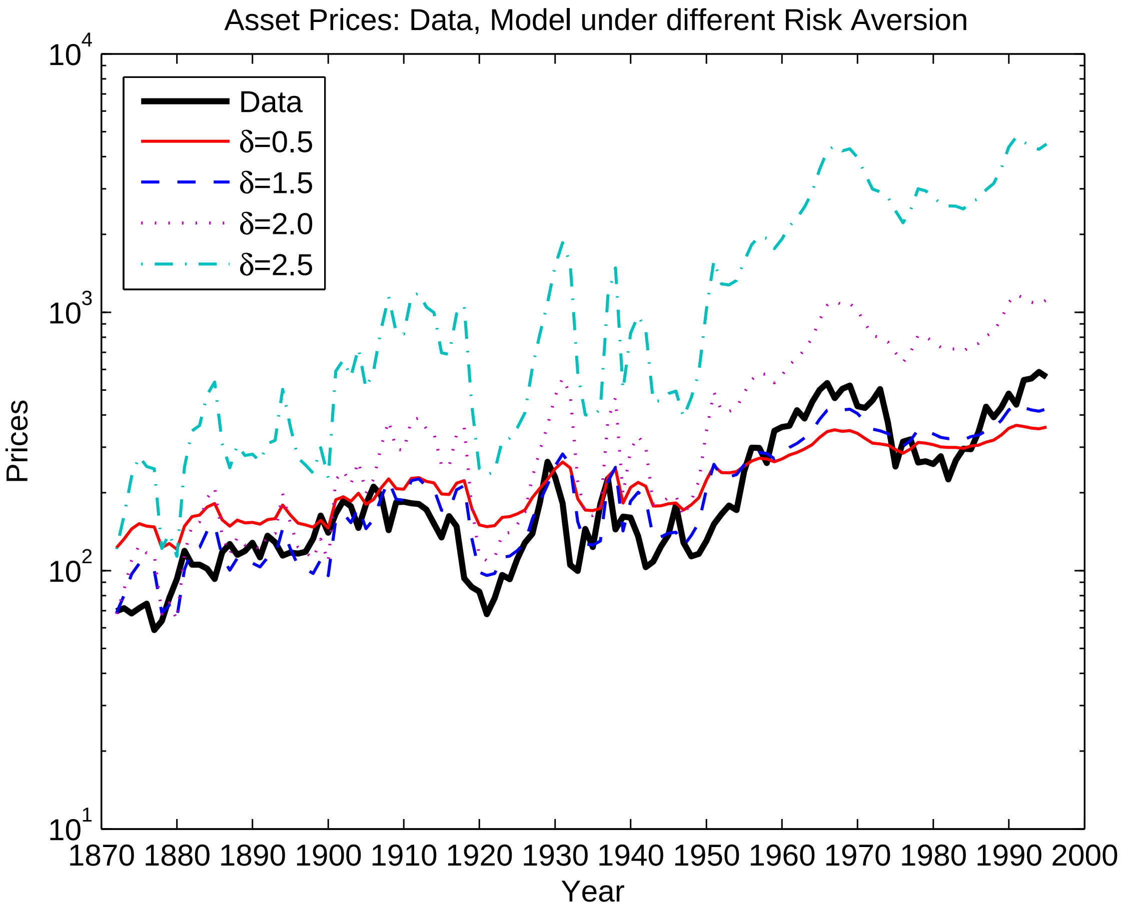

Figure 5: Model under different δ

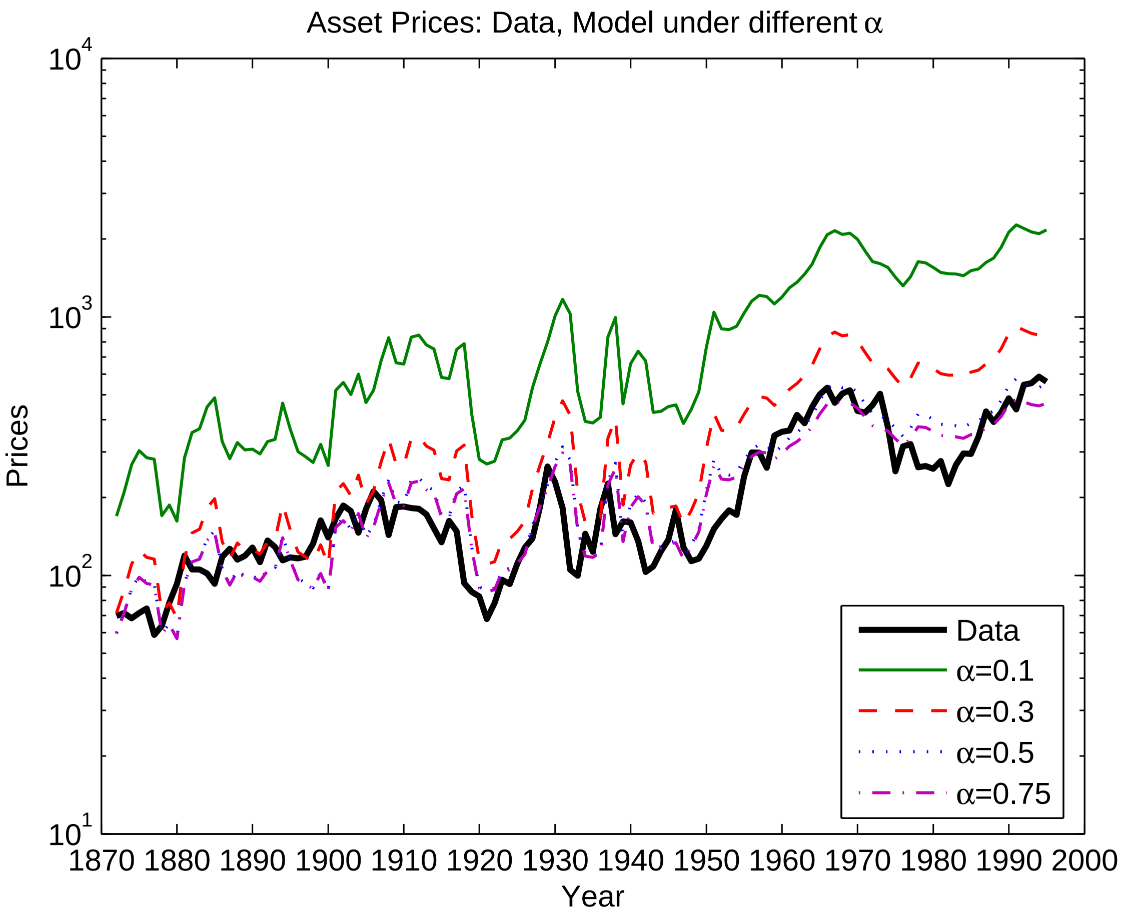

Figure 6: Model under different α

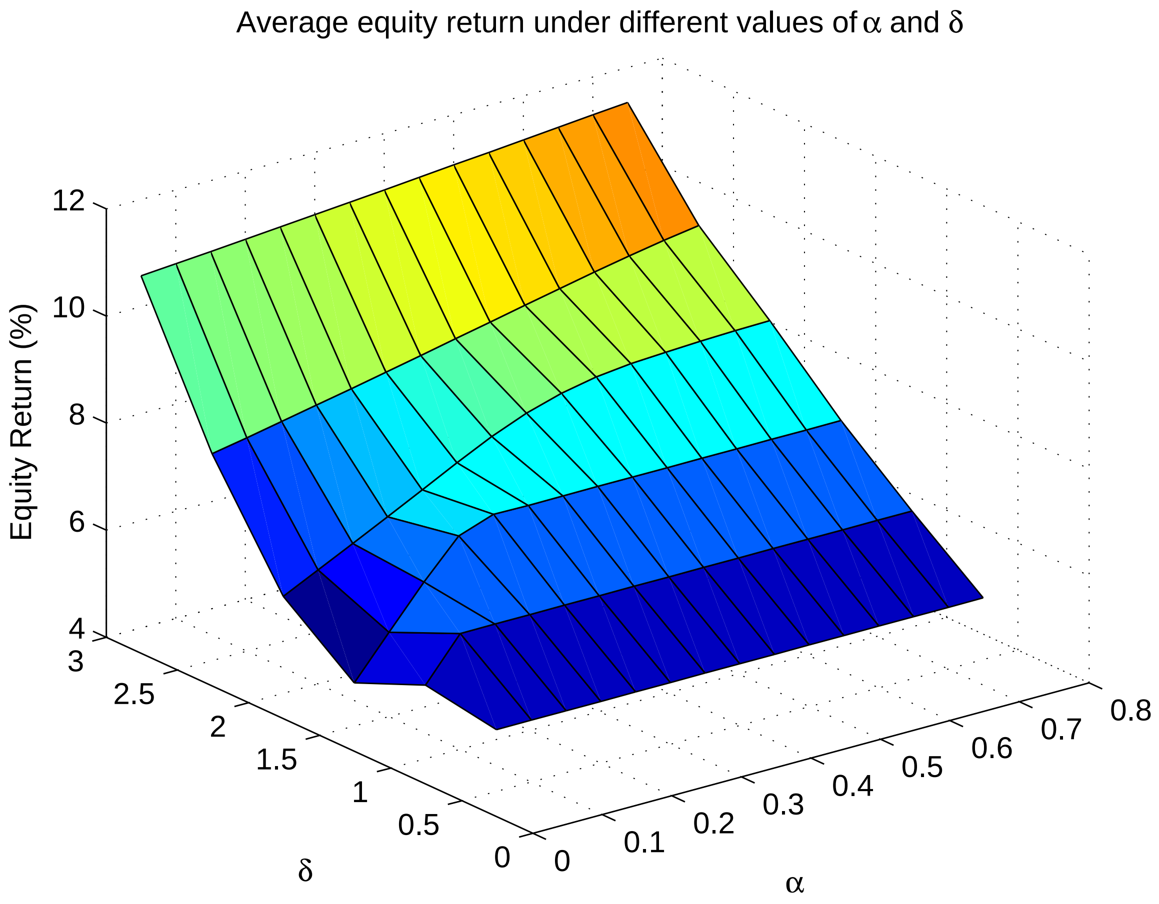

Figure 7: Average Equity Return under different values of α and δ

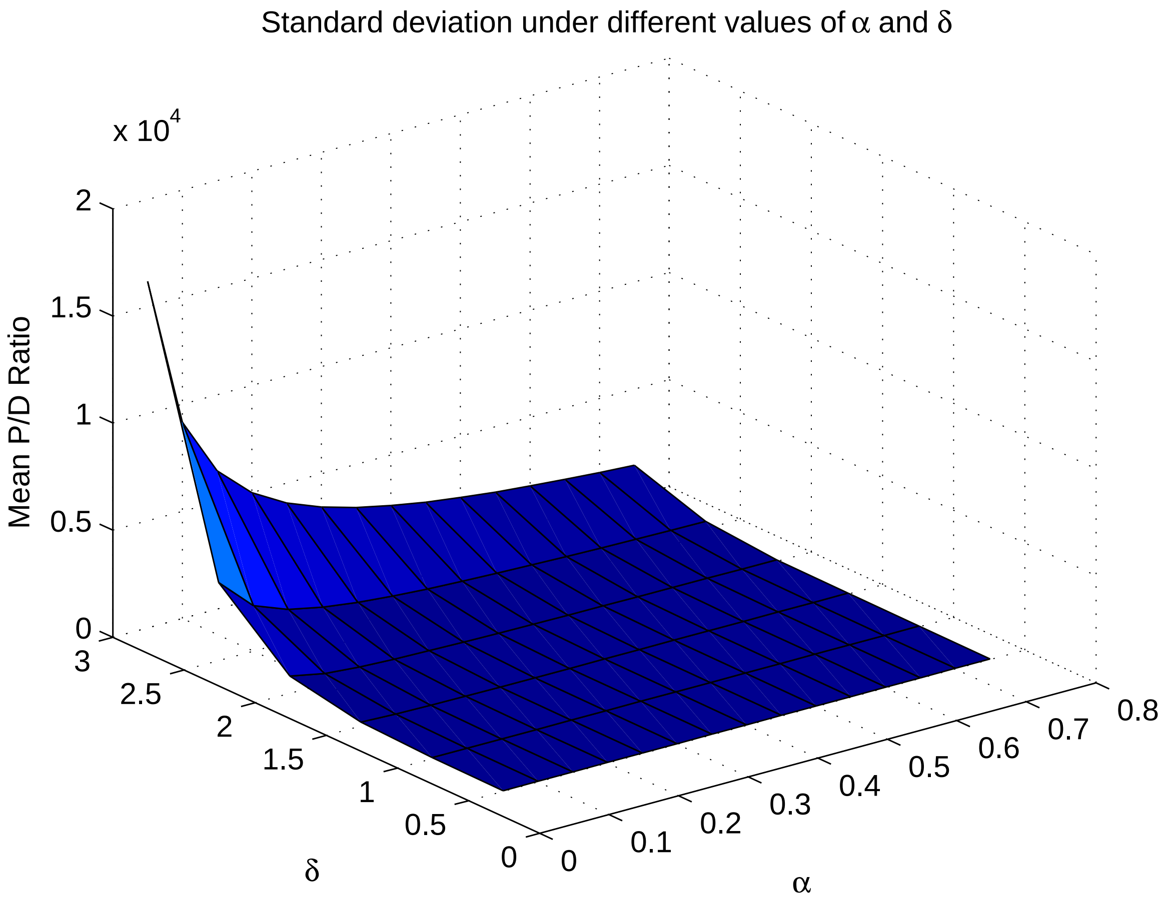

Figure 8: Standard Deviation of Asset Price under different values of α and δ

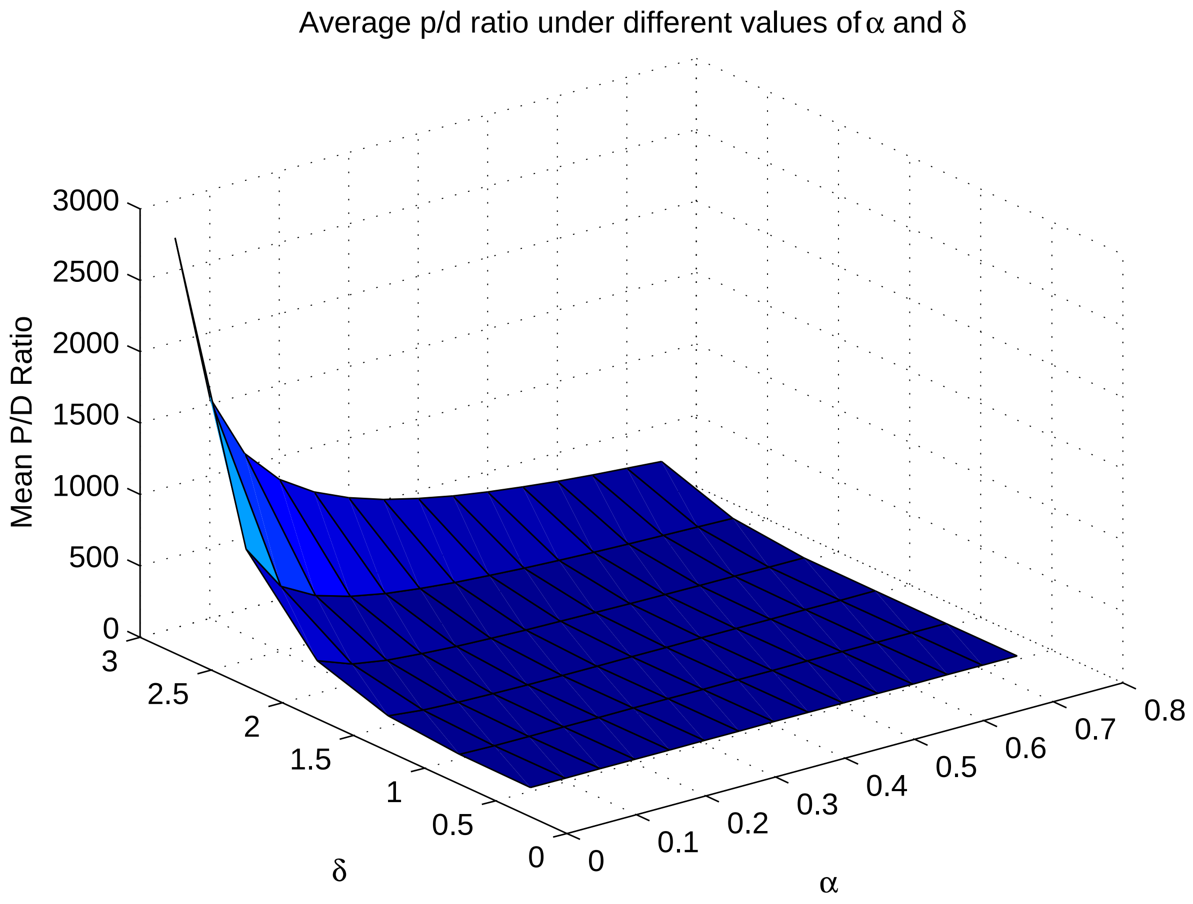

Figure 9: Mean Price-Dividend Ratio under different values of α and δ

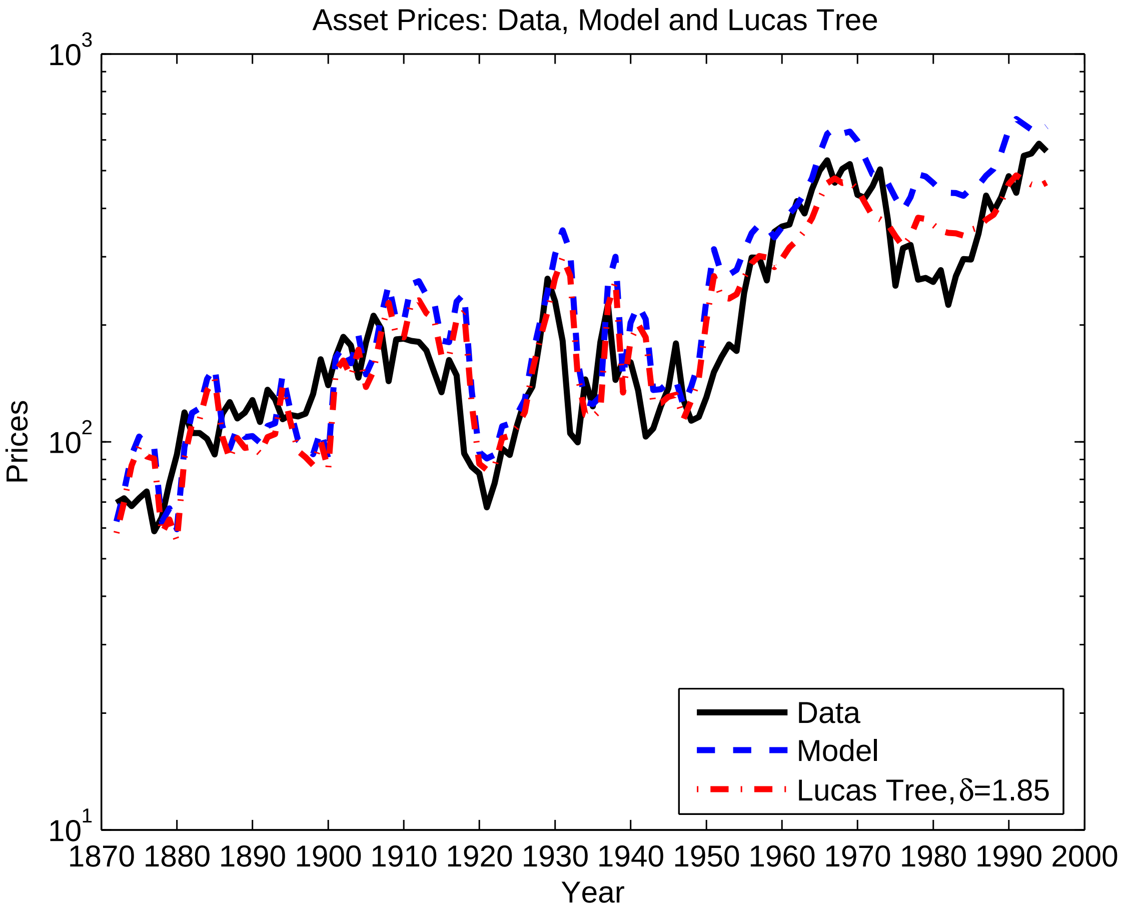

Figure 10: Asset Prices in the Lucas Tree Model that matches the Average Equity Return

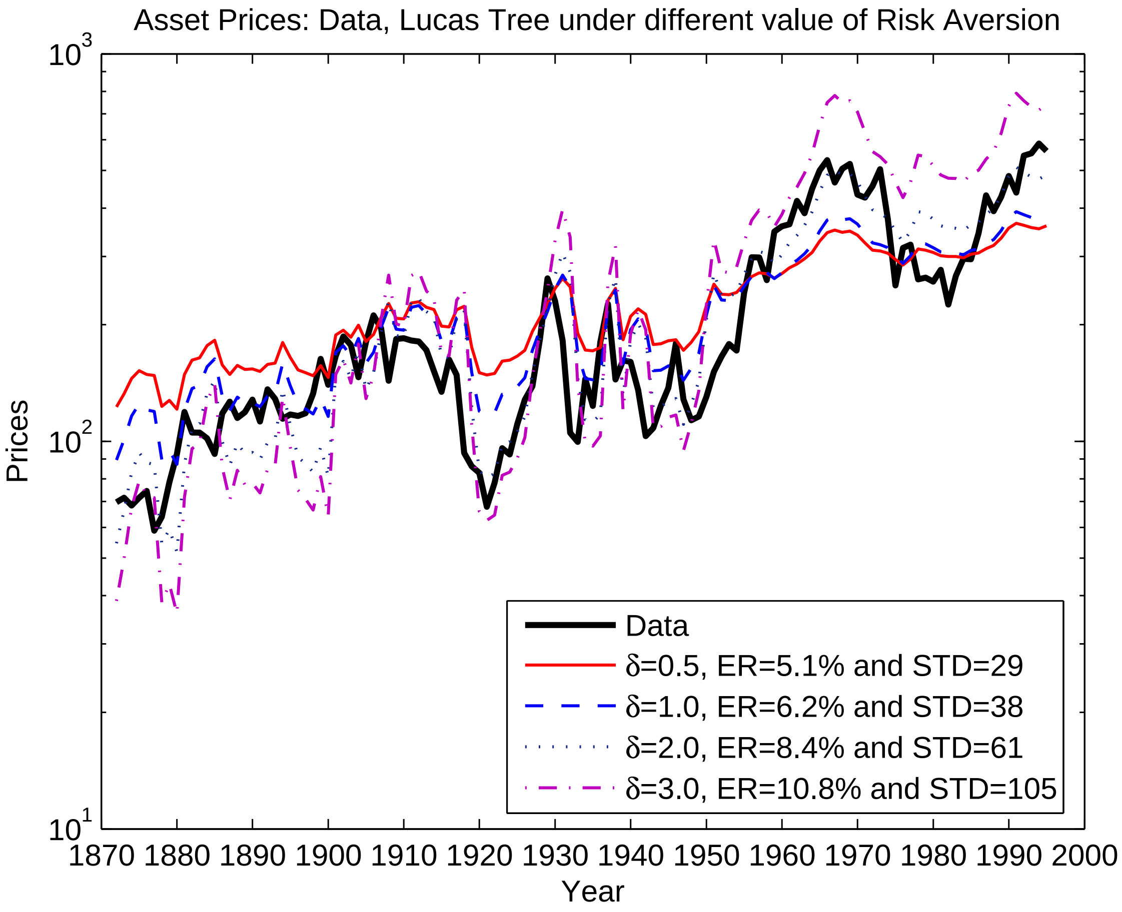

Figure 11: Asset Prices in the Lucas Tree Model under different values of risk aversion

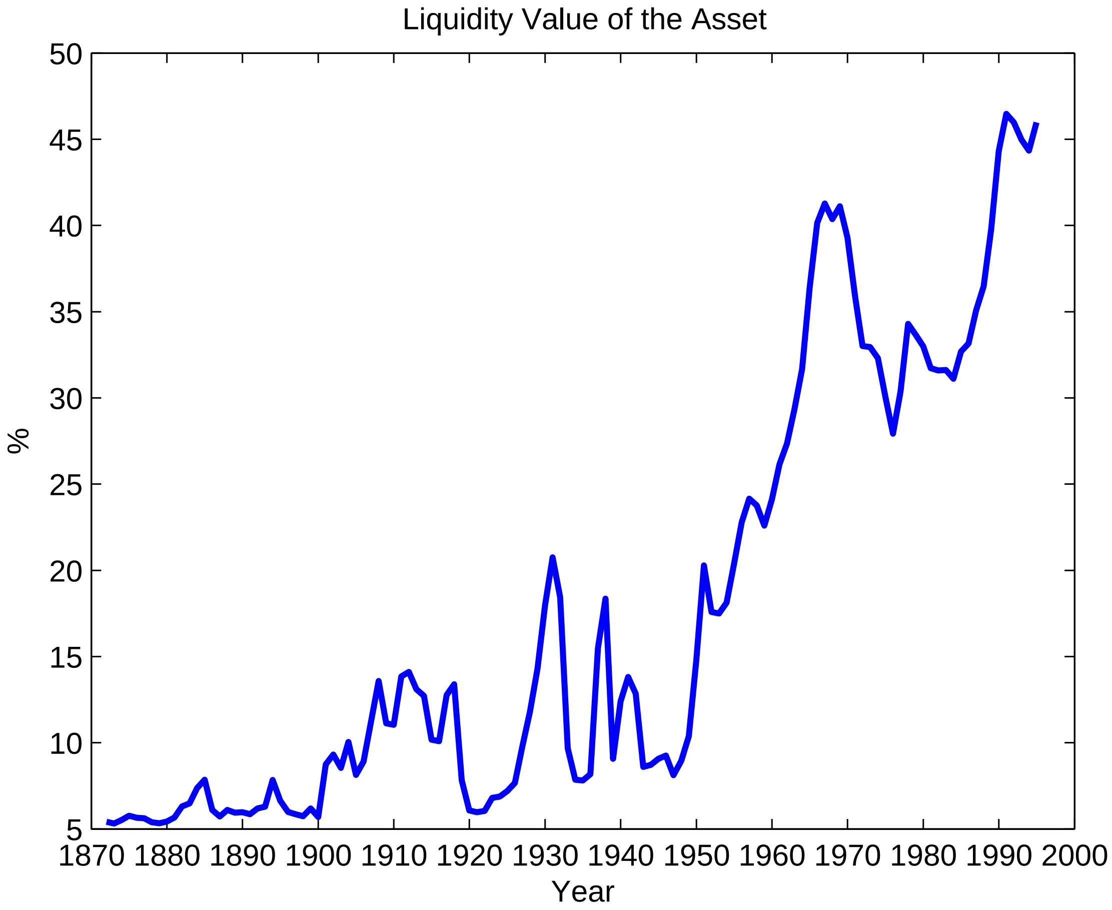

Figure 12: Liquidity Value of the Asset: $(P_{model} - P_{Lucas})/P_{Lucas}$