# Redistributive Shocks and Productivity Shocks

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## Abstract

We pose and estimate a bivariety shock to the production function that under competition in factor markets simultaneously accounts for movements in the Solow residual and in the factor shares of production. We show how confronting agents in a standard RBC economy with these shocks entail a much smaller response (about 40%) of hours relative to the standard modelization of the shocks that identifies the Solow residual with a univariate shock. Our findings raise a flag against the optimism embedded in the literature that states that productivity shocks are responsable for most of the cyclical behaviour of output and hours.

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# 1 Introduction

Most Business Cycle research ignores factor share fluctuations. This results from assuming a Cobb-Douglas technology with constant coefficients and maintaining the connection between factor prices and their respective marginal productivity. However, while the shares of GNP accruing to each factor do not show a secular trend, they do have sizeable high frequency movements. Over the period 1954.I-2002.IV, the labor share of income is around 30% as volatile as output, 65% as volatile as the Solow residual, and it is countercyclical (a correlation of -0.22).

Standard business cycle research that poses competition on the factor markets has abstracted from this feature of the data except perhaps Gomme and Greenwood (1995).

In this paper we posee a bivariate stochastic process that determines the Solow residual and labor shares simultaneously under the assumptions of a Cobb-Douglas production function and of competition in the factor markets.[1] We find that under this specification of shocks to productivity, the standard RBC economy entails a much smaller response (about 40%) of hours and output than it does under the standard specification that identifies the Solow residual with the shock.

# 2 The Specification of the Shocks

We start by describing the properties of the Solow residual and its structural interpretation as a shock in Section 2.1. We then describe the properties of labor share in Section 2.2. Finally, we turn to our specification of a joint process that yields both a residual and labor share as a bivariate process in Section 2.3.

## 2.1 The Standard Specification: Solow residuals as shocks

### 2.1.1 Obtaining the Solow residual from the data

The Solow residual that we denote $S_t^0$ is computed from time series of output, $Y_t$, capital, $K_t$, and labor $N_t$, and from a specification of a relative input share parameter that we denote by $\zeta$ (see Kydland and Prescott (1993) Kydland (1995) or King and Rebelo (1999)).

$$\ln S_t^0 = \ln Y_t - \zeta \ \ln K_t - (1-\zeta) \ \ln N_t \tag{1}$$

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[1] Moreover, we do it in such a way as to make deviations trend invariant to scale.

But $S_t^0$ has trend and we want a trendless object. Consider now a detrending procedure that uses the following linear regression

$$\ln X_t = \chi_x + g_x t + \widetilde{x}_t. \tag{2}$$

where $X_t$ is any economic variable, and where $\chi_0$ and $g_x$ are the regressors and $\widetilde{x}_t$ are the residuals.

Applying such detrending procedure to the Solow residual we obtain a series $\widetilde{s}_t^0$ that is the (detrended) Solow residual that we are interested in.

Alternatively, and for reasons that will be clear later, the Solow residual can be calculated in two steps.

1. Use the detrending procedure described in (2) to obtain $\{\widetilde{y}_t, \widetilde{k}_t, \widetilde{n}_t\}$.[2]
2. Then the Solow residual $s_t^0$ is defined to be

$$s_t^0 = \widetilde{y}_t - \zeta\, \widetilde{k}_t - (1-\zeta)\, \widetilde{n}_t \tag{3}$$

To see the equivalence between the two definitions note that substituting the residuals of the economics variables in (3) we get

$$\begin{aligned} s_t^0 &= (\ln Y_t - \chi_y - t\, g_y) - \zeta\, (\ln K_t - \chi_k - t\, g_k) - (1-\zeta)\, (\ln N_t - \chi_n - t\, g_n) && (4)\\ &= \ln Y_t - \zeta \ln K_t - (1-\zeta) \ln N_t - (\chi_y - \zeta\chi_k - (1-\zeta)\chi_n) - \\ &\qquad\qquad t(g_y - \zeta g_k - (1-\zeta) g_n) && (5)\\ &= \ln S_t^0 - [\chi_y - \zeta\chi_k - (1-\zeta)\chi_n] - t[g_y - \zeta g_k - (1-\zeta) g_n] && (6) \end{aligned}$$

But $s_t^0$ is a linear function of residuals so it has mean zero and no trend which implies that $[\chi_y - \zeta\chi_k - (1-\zeta)\chi_n]$- $[g_y - \zeta g_k - (1-\zeta) g_n]$ are indeed the mean and the trend of $\ln S_t$ so $s_t^0 = \widetilde{s}_t^0$.

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[2] Over the 1954-2002 period, the growth rate of capital and output are very similar, in that order, 3.1% and 3.3% annually.

### 2.1.2 Giving a structural interpretation to the Solow residual

In the standard RBC model we can also calculate a Solow residual from the model. The nice property is that with Cobb-Douglas technology and provided that we use the right share parameter, the Solow residual is the shock to productity. To see this, consider the following Cobb-Douglas technology with constant coefficients and multiplicative shocks to productivity,

$$Y_t = e^{z_t^0} A\, K_t^\theta \left[(1+\lambda)^t\, \mu\, N_t\right]^{1-\theta} = e^{z_t^0} A\, K_t^\theta \left[(1+\lambda)^t\, \mu\, (1+\eta)^t\, h_t\right]^{1-\theta} \tag{7}$$

where $z_t^0$ represents a shock that follows a univariate process, and $\lambda$ is the rate of labor-augmenting (Harrod-neutral) technological change. The labor input, $N_t$, is the product of the number of agents in the economy, $L_t$, and the fraction of time that agents devote to market activities, $0 \leq h_t \leq 1$. Population grows deterministically according to $L_t = (1+\eta)^t$. Parameters $A$ and $\mu$ just units parameters (it will be clear later why we are posing two different unit parameters).

Note that in the balanced growth path, output $Y_t$, capital $K_t$, grow at rate (approximately) $\lambda + \eta$, and that if preferences are CRRA, the model economy generates paths for capital and output that can be written as $K_t = (1+\eta)^t(1+\lambda)^t\, k_t$ and $Y_t = (1+\eta)^t(1+\lambda)^t\, y_t$ where both $k_t$ and $y_t$ are stationary. Denote by lower-case-hat log deviations of the variables, *i.e.* $\widehat{x}_t = \log(\frac{x_t}{X^*})$ and with a star the steady state value of the variable, then we obtain

$$\begin{aligned} Y_t &= (1+\eta)^t(1+\lambda)^t\, Y^*\, e^{\widehat{y}_t}, && (8)\\ K_t &= (1+\eta)^t(1+\lambda)^t\, K^*\, e^{\widehat{k}_t}, && (9)\\ N_t &= (1+\eta)^t\, h^*\, e^{\widehat{h}_t}. && (10) \end{aligned}$$

We can rewrite the production function (7) as

$$(1+\eta)^t(1+\lambda)^t\, Y^*\, e^{\widehat{y}_t} = e^{z_t^0}\, A\, [(1+\eta)^t(1+\lambda)^t\, K^*\, e^{\widehat{k}_t}]^\theta\, [(1+\eta)^t(1+\lambda)^t\, \mu\, h^*\, e^{\widehat{h}_t}]^{1-\theta}, \tag{11}$$

cancelling terms

$$Y^*\, e^{\widehat{y}_t} = e^{z_t^0}\, A\, [K^*\, e^{\widehat{k}_t}]^\theta\, [\mu\, h^*\, e^{\widehat{h}_t}]^{1-\theta} \tag{12}$$

and taking logs of (12) and rearranging yields

$$z_t^0 \;=\; \widehat{y}_t - \;\theta\widehat{k}_t - \;(1-\theta)\widehat{h}_t + \ln\frac{Y^*}{AK^{*\theta}\left(\mu h^*\right)^{\theta}} \;=\; \widehat{y}_t - \;\theta\widehat{k}_t - \;(1-\theta)\widehat{h}_t \qquad (13)$$

where the second equality follows directly from the fact that the denominator of the third term is steady-state output.

If we use model generated data variables to construct a Solow residual with share parameter $\theta$, we obtain in the first step (abstracting from sampling error) that

$$\begin{aligned}
\chi_y &= Y^* & g_y &= \lambda+\eta & \widetilde{y}_t &= \widehat{y}_t & (14)\\
\chi_k &= K^* & g_k &= \lambda+\eta & \widetilde{k}_t &= \widehat{k}_t & (15)\\
\chi_n &= h^* & g_n &= \eta & \widetilde{n}_t &= \widehat{h}_t & (16)
\end{aligned}$$

The second step yields

$$s_t = \widehat{y}_t - \theta\,\widehat{k}_t - (1-\theta)\,\widehat{h}_t \qquad (17)$$

but this expression is exactly $z_t^0$, by equation (13). Which means that we can interpret the Solow residual generated by the data as the multiplicative shock to the production function.

The Cobb-Douglas technology so defined implies under competitive factor markets that factor shares are constant at all frequencies.[3] But are they? We now turn to explore this issue.

## 2.2 The behavior of labor share

The ratio of all payments to labor relative to output is labor share. Its exact value depends on the details of the definition of output and its partition into payments to labor and payments to capital. Perhaps, the more standard definition of labor share, which is the one that we take as the benchmark, is that proposed by Cooley and Prescott (1995) that poses that the ratio of ambiguous labor income to ambiguous income is the same as the ratio of unambiguous labor income to unambiguous income. Another definitions that we explore expand the capital stock and capital services to include durables and government respectively, while a fourth definition sets labor share equal to the ratio of compensation of employees to Gross National Product, which

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[3] See that $\frac{R\,K}{Y} = \frac{\frac{\partial F}{\partial K}K}{Y} = \theta$.

Figure 1: The Labor Share, U.S. 1954.I-2002.IV

renders all ambiguous income to capital.[4]

The baseline definition of labor share for the period 1954.I-2002.IV is plotted in Figure 1. It oscillates between a minimum value of 0.66 and a maximum above 0.71. with no discernible trend. The other definitions, while differing on their average have very similar properties as can be seen in Figure 2 that plots their deviations with respect to the mean.

### 2.2.1 The cyclical behavior of labor share

From the point of view of the study of business cycles, what matters is not whether labor share moves but whether it does so in any systematic way with respect to the main macroeconomic aggregates. Table 1 displays some business cycle statistics (all variables are hp-filtered): the standard deviations of output, the Solow residual and labor share as well as the correlations of the Solow residual and labor share with output. We see that labor share's volatility is a little bit more than half of that of the Solow residual, it is quite persistent, and, perhaps more importantly it is negatively corraleted with output albeit not much.

Perhaps more important is the phase shift of these variables reported in Table 2. There is a clear pattern. Before the peak of an expansion, labor share is below average with the negative

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[4] A detailed analysis on the construction of these labor share of income data series is given in the Appendix.

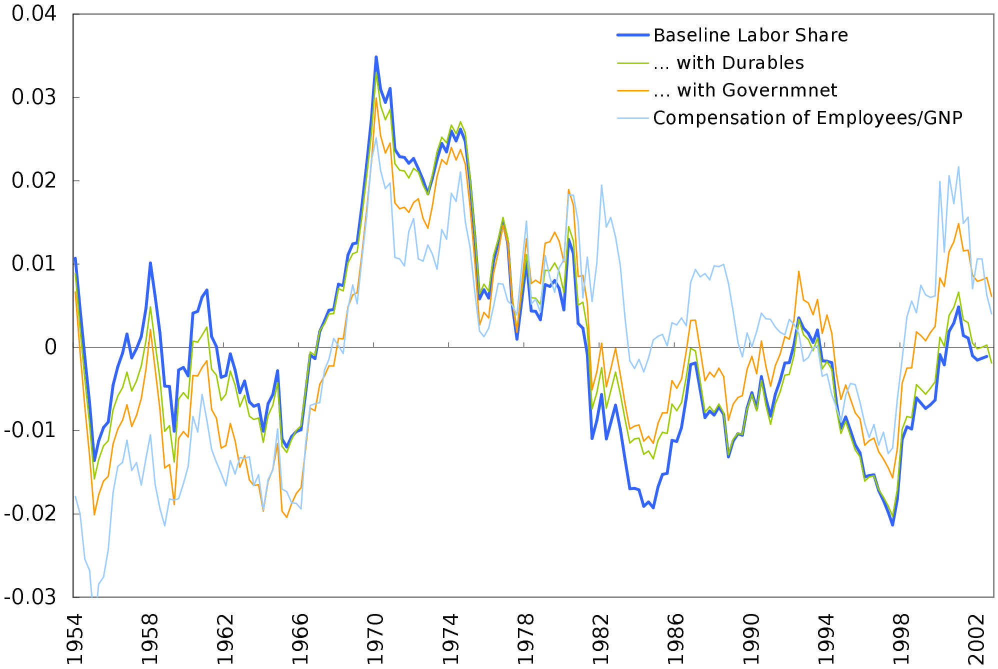

Figure 2: Demeaned Labor Share, U.S. 1954.I-2002.IV

| | $\sigma_x$ | $\sigma_x/\sigma_{GNP}$ | $\rho(x, GNP)$ | $\rho(x_t, x_{t-1})$ |
|---|---|---|---|---|
| $GNP$ | 1.61 | 1.00 | 1.00 | .85 |
| Solow Residual: $z^0$ | .84 | .52 | .75 | .71 |
| Labor Share | .47 | .29 | -.22 | .78 |
| ... with Durables | .45 | .28 | -.19 | .77 |
| ... with Government | .49 | .30 | -.25 | .77 |
| CE/GNP | .47 | .29 | -.10 | .71 |

Table 1: Standard deviation and correlation with output of Labor Share, U.S. 1954.I-2002.IV.

correlation being largest two period s before the peak of output. Subsequently, labor share starts to increase quite above its mean with its maximum value peaking one year after output peaked. In fewer words, labor share lags output by one year or so.

To explore the issue of whether this behavior of labor share has any implication for our understanding of business cycles, we specify a very simply real business cycle model that has a moving labor share. In such a model labor share is posed to be exogenous and stochastic. Given its specific cyclical properties, the process for productivity and for labor share cannot be independent. We now turn to describe how such a model can allow us to give structural interpretations as shocks to objects that can be constructed directly from the data in a very similar fashion to that specified in the previous section.

|  | Cross-correlation of $GNP_t$ with |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  | $x_{t-5}$ | $x_{t-4}$ | $x_{t-3}$ | $x_{t-2}$ | $x_{t-1}$ | $x_t$ | $x_{t+1}$ | $x_{t+2}$ | $x_{t+3}$ | $x_{t+4}$ | $x_{t+5}$ |
| Labor Share | -.20 | -.25 | -.31 | -.33 | -.31 | -.22 | .05 | .26 | .40 | .46 | .43 |
| ... with Durables | -.21 | -.25 | -.30 | -.31 | -.29 | -.19 | .08 | .29 | .41 | .46 | .41 |
| ... with Government | -.18 | -.23 | -.29 | -.32 | -.31 | -.25 | .05 | .27 | .41 | .46 | .41 |
| CE/GNP | -.25 | -.29 | -.33 | -.37 | -.29 | -.21 | .11 | .33 | .47 | .48 | .44 |

Table 2: Phase-Shift of the Labor Share, U.S. 1954.I - 2002.IV

### 2.3 A bivariate process that determines factor shares and the Solow residual

We want to pose a stochastic process that simultaneously yields the movements in the labor share and in the Solow residual. Using time series of output $Y_t$, capital $K_t$, and labor $L_t$, and a measure of the labor income $W_tN_t$, we can construct a labor share data series $\ell_t$, and in turn, a residual that is as closely related as possible to the Solow residual. As discussed above, the data definition of the labor share is a measure of the labor income divided by output, $\ell_t = \frac{W_t N_t}{Y_t}$, and the deviations of the labor share from its mean are

$$\widetilde{\ell}_t = \ell_t - \ell \tag{18}$$

with $\ell = \sum_t \frac{\ell_t}{T}$.

We now compute a residual as we did in Section 2.1, with one difference: that we use now the time-varying relative input share $\ell_t$ instead of a constant share parameter. We define the residual $s_t^1$ as

$$s_t^1(\ell_t) = \widetilde{y}_t - \ell_t\, \widetilde{k}_t - (1-\ell_t)\, \widetilde{n}_t \tag{19}$$

where as before $g_y$, $g_k$ and $g_h$ are the slopes of a fitted linear trend to the **logged** original series of output, capital and labor and $\widetilde{y}_t$, $\widetilde{k}_t$ and $\widetilde{n}_t$ are the corresponding residuals.

We now pose a production function with stochastic factor shares but is otherwise a Cobb-Douglas production function

$$Y_t = e^{z_t^1}\, A\, K_t^{\theta - z_t^2}\, \left[\mu\, (1+\lambda)^t\, (1+\eta)^t\, h_t\right]^{1-\theta+z_t^2} \tag{20}$$

where $z_t^1$ and $z_t^2$ are the two elements of a bivariate stochastic process and we refer to them as the productivity and the redistributive shock respectfully. We use again parameters $A$ and $\mu$ to determine the units of effective labor and to normalize output to one. However, unlike in the previous specification, $\mu$ plays now an important role.

Under competitive markets, labor share of income in the model is given by

$$\frac{W_t N_t}{Y_t} = \frac{\frac{\partial Y_t}{\partial N_t} N_t}{Y_t} = (1-\theta) + z_t^2 \tag{21}$$

But this implies that the deviation from mean labor share is the redistributive shock: $\widetilde{\ell}_t = z_t^2$.

We now turn to the residual. First, divide both sides of (20) by $(1+\lambda)^t(1+\eta)^t$

((and use the transformation of the variables defined in (2), which yields

$$Y^* \, e^{\widetilde{y}_t} = e^{z_t^1} \, A \, \left(K^* \, e^{\widetilde{k}_t}\right)^{\theta - z_t^2} \, \left(\mu \, h^* \, e^{\widetilde{h}_t}\right)^{1-\theta+z_t^2}, \tag{22}$$

taking logs we have

$$z_t^1 = \widetilde{y}_t - (\theta - z_t^2)\widetilde{k}_t - (1-\theta+z_t^2)\widetilde{h}_t + z_t^2 \ln\left(\frac{K^*}{\mu h^*}\right) \tag{23}$$

))

[[which yields

$$Y^* \, e^{\widehat{y}_t} = e^{z_t^1} \, A \, \left(K^* \, e^{\widehat{k}_t}\right)^{\theta - z_t^2} \, \left(\mu \, h^* \, e^{\widehat{h}_t}\right)^{1-\theta+z_t^2}, \tag{24}$$

and taking logs we have

$$z_t^1 = \widehat{y}_t - (\theta - z_t^2)\widehat{k}_t - (1-\theta+z_t^2)\widehat{h}_t + z_t^2 \ln\left(\frac{K^*}{\mu h^*}\right) \tag{25}$$

]]

where we have used $Y^* = AK^{*\theta}\left(\mu h^*\right)^{\theta}$.

((Note now that))

[[Using the equivalences as in (14), note now that]]

$$z_t^1 = s_t^1 + z_t^2 \ln\left(\frac{K^*}{\mu h^*}\right) \tag{26}$$

which means that the units matter: If the units in the model are chosen so that the ratio of capital to effective labor is one then the residual $s_t^1$ coincides with the shock. This is what we do.

Another way of seeing the role of the choice of units is that if $K^* \neq \mu h^*$ then shocks to factor shares also have implications for productivity. We want to distinguish pure redistributive shocks, that we associate to $z_t^2$ from productivity shocks that we associate to $z_t^1$ and the suitable choice of units allows us to do so.

In addition, it turns out that the two residuals that we compute, $s_t^0$ and $s_t^1$ are extremely similar as can be in Figure 3. This can also be seen by noting that we can write an expression that links the two residuals $s_t^0$ and $s_t^1$ as follows,

$$s_t^1 \;=\; s_t^0 + \widehat{\ell}_t(\widehat{k}_t - \widehat{h}_t)$$

and that the last term, $\widehat{\ell}_t(\widehat{k}_t - \widehat{h}_t)$, is very small.

We now turn to estimate a parameterization to represent the univariate process $z_t^0$ and another one for the bivariate process $\{z_t^1, z_t^2\}$.

# 3 Estimation of a process for the shocks

We start discussing a univariate process for the Solow residual in Section 3.1 and then we move to a bivariate process for the Solow residual and labor share in Section 3.2.

## 3.1 A univariate process for the Solow residual

While a univariate representation of the Solow residual $z_t^0$ is one of the most widely used processes, there are very few actual estimations of it, and most authors just use Prescott (1986) calculations. We assume the Solow residual follows an AR(1) process with normally distributed innovations.

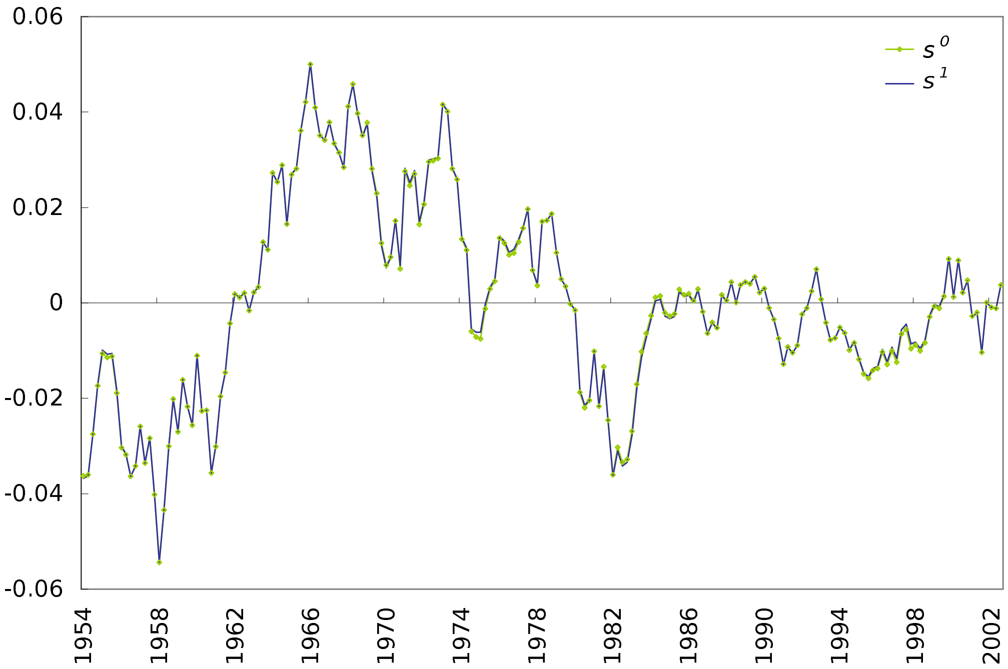

Figure 3: The two sets of productivity residuals $s_t^0$ and $s_t^1$, U.S. 1954.I-2002.IV

For the whole sample 1954.I-2002.IV the *full* maximum-likelihood estimation delivers,[5]

$$z_t^0 = \underset{(.019)}{.949}\ z_{t-1}^0 + \epsilon_t^0, \qquad \epsilon_t^0 \sim N\left(0, \underset{(.000)}{.00645}\right)$$

Notice that the volatility of the innovations is lower than the value of .00763 originally estimated in Prescott (1986) or the value of .007 used in Cooley and Prescott (1995). This is due to the sample period. There has been a reduction in volatility recently.[6]

## 3.2 A bivariate process for the Solow residual and labor share

We now pose a statistical model to find an underlying stochastic process that generates the joint distribution of $z_t^1$ and $z_t^2$ described in Section 2 using the residuals obtained. In particular, we aim at capturing the volatility and persistence of each series and their observed contemporaneous correlation. We assume the processes to be weakly covariance stationary so that classical estimation and inference procedures apply.

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[5] The OLS estimation yields a (biased) regressor coefficient of .938 and a standard deviation of .00641. Despite the high persistence of the process we do not find substantial differences between these estimates and the full maximum likelihood estimates in terms of fluctuations in $Y_t$, $\sigma_y$.

[6] For instance, using a similar sample (1955.III-2003.II), Andres Arias and Ohanian (2005) obtains an autocorrelation coefficient of 0.95 and a volatility of the innovations of .0065.

For estimation purposes we specify a vector autoregression model or VAR($n$). Thus, we express each variable $z_t^1$ and $z_t^2$ as a linear combination of $n$-lags of itself and $n$-lags of the other variable.

| *Lags* | Akaike's | Schwartz's Bayesian | Hannan and Quinn |
|---|---|---|---|
| 1 | -16.306* | -16.2019* | -16.2638* |
| 2 | -16.2957 | -16.1223 | -16.2254 |
| 3 | -16.2911 | -16.0483 | -16.1927 |
| 4 | -16.2803 | -15.9681 | -16.1538 |
| 5 | -16.267 | -15.8855 | -16.1124 |

Table 3: Lag Selection Order Criteria

Information criteria (Akaike's, Schwartz's Bayesian and Hannan and Quinn, reported in Table 3) suggest that the correct specification is a VAR(1), which we write compactly as

$$z_t = \Gamma \; z_{t-1} \; + \; \epsilon_t, \qquad \epsilon_t \sim N\left(0, \Sigma\right) \tag{27}$$

where $z_t = (z_t^1, z_t^2)'$ and $\Gamma$ is a 2-by-2 square matrix with generic element $\gamma_{ij}$. The innovations $\epsilon_t = (\epsilon_t^1, \epsilon_t^2)'$ are serially uncorrelated and follow a bivariate Gaussian distribution with unconditional mean zero and a symmetric positive definite variance-covariance matrix $\Sigma$. Thus, this especification has seven parameters: the four coefficient regressors in $\Gamma$, and the variances and covariance in $\Sigma$.

The endogenous variables $z_t^1$ and $z_t^2$ share the same set of regressors. Thus, we can separately apply the OLS method to each VAR equation and yield consistent and efficient estimates. Also, with normally distributed innovations, the OLS estimates are equivalent to the *conditional* maximum likelihood estimates. Using the whole quarterly 1954.I-2002.IV sample, the estimated parameters associated with the baseline labor share are

$$\widehat{\Gamma} = \left( \begin{array}{cc} .938 & .003 \\ .055 & .932 \end{array} \right), \qquad \widehat{\Sigma} = \left( \begin{array}{cc} (.00643)^2 & -.1043E-04 \\ -.1043E-04 & (.00304)^2 \end{array} \right)$$

The estimated parameters under alternative definitions of the labor share are very similar (see Table 4). Notice that all parameters except $\gamma_{12}$ are statistically significant. If we restrict the model with $\gamma_{12} = 0$, we obtain a set of constrained estimates similar to those originally unconstrained because the original estimate $\gamma_{12}$ is already close to zero. We will use the unrestricted statistical

|  | $\gamma_{11}$ | $\gamma_{12}$ | $\gamma_{21}$ | $\gamma_{22}$ | $\sigma_1$ | $\sigma_2$ | $\sigma_{12}$ |
|---|---|---|---|---|---|---|---|
| Baseline Labor Share | .938 | .003 | .055 | .932 | .00643 | .00304 | -.1043E-04 |
|  | (.024) | (.040) | (.011) | (.019) |  |  |  |
| ... with Durables | .937 | -.017 | .057 | .935 | .00646 | .00289 | -.1016E-04 |
|  | (.024) | (.041) | (.010) | (.018) |  |  |  |
| ... and Government | .919 | -.052 | .063 | .956 | .00706 | .00315 | -.1405E-04 |
|  | (.026) | (.044) | (.011) | (.019) |  |  |  |
| CE/GNP | .941 | -.029 | .057 | .936 | .00667 | .00341 | -.166E-04 |
|  | (.023) | (.039) | (.012) | (.020) |  |  |  |

Table 4: Unconstrained Estimation

model to feed our economic model. The Appendix explores the behavior of the model economy when we use the constrained estimates. The findings reported in Section 4 remain unchanged.

To get a better idea of dynamics of the VAR system we use impulse response functions and forecast error variance decompositions. First, we check that the estimated VAR is stable with eigenvalues .949 and .921 so that we can have a moving average representation of it. Second, since our innovations $\epsilon_t$ are contemporaneously correlated, we transform $\epsilon_t$ to a set of uncorrelated components $u_t$ according to $\epsilon_t = \Omega\ u_t$, where $\Omega$ is an invertible square matrix with generic element $\omega_{ij}$, such that

$$\widehat{\Sigma} = \frac{1}{n}\sum_t \epsilon_t \epsilon_t^{'} = \Omega \left( \frac{1}{n}\sum_t u_t u_t^{'} \right) \Omega^{'} = \Omega\Omega' \tag{28}$$

and we have normalized $u_t$ to have unit variance. Notice that while $\widehat{\Sigma}$ has three parameters, the matrix $\Omega$ has four: there are many such matrices. We further impose the constraint that $u_t^2$ to have a contemporaneous effect on $z_t^2$ but not on $z_t^1$, that is, we set $\Omega$ to be a lower triangular matrix[7]. This choice follows from the fact that we aim to treat $z_t^2$ as purely redistributive shocks

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[7] Because $\widehat{\Sigma}$ is positive definite symmetric, it has a unique representation of the form $\widehat{\Sigma} = ADA'$ where A is a lower triangular matrix with diagonal elements equal to one and D is a diagonal matrix. A particularization of this is to set $\Omega = AD^{1/2}$, as we do, which is the Cholesky factorization.

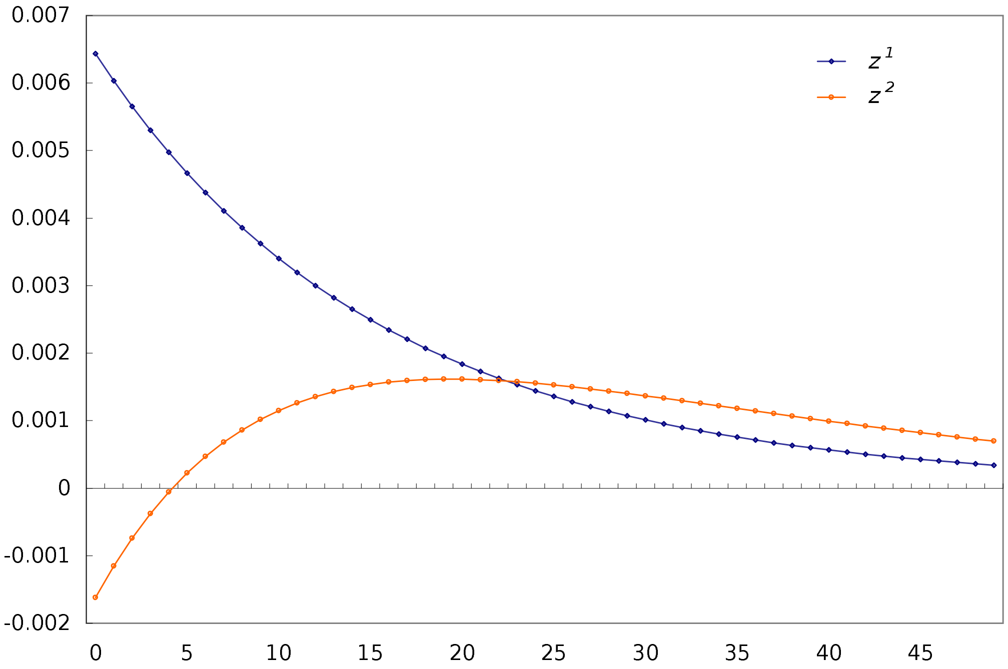

Figure 4: Impulse response functions to Orthogonalized Productivity Innovations $\epsilon^1$.

with no influence on productivity. Our factorization of $\widehat{\Sigma}$ results in

$$\begin{pmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{pmatrix} = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} = \begin{pmatrix} .00645 & .0 \\ -.00162 & .00257 \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix}$$

where $\omega_{11} = \sigma_{\epsilon^1}$, $\omega_{21} = E[\epsilon_t^2 | \epsilon_t^1]$, and $\omega_{22}$ is the standard error of the regression of $\epsilon_t^2$ on $\epsilon_t^1$.

In Figure 4 we observe the consequences for $z_t^1$ and $z_t^2$ if $u_t^1$ were to increase by one at $t = 0$ and be set to zero afterwards . We find that $z_t^1$ reacts promptly and positively to this perturbation in its own innovations and that it dies slowly out afterwards, very similarly (if not exactly) as the univariate process $z_t^0$ does in response to a one-time one-standard-deviation of $\epsilon_t^0$. We find that the labor share of income immediately drops at $t = 0$, from where it raises to be above average after the fifth quarter, reaching a maximum in the twentieth quarter and approaching monotonically to its unconditional mean afterwards.

We learn the time-path of $z_t^1$ and $z_t^2$ derived from a one-time shock $u_0^2 = 1$ in Figure 5. This perturbation results in a labor share above average that monotonically decreases from a maximum attained at $t = 0$. The assumptions made on the purely redistributive nature of $z_t^2$ and $u_t^2$ make the response of $z_t^1$ to redistributive innovations negligible.

Finally, we decompose the variance of $z_t^1$ and $z_t^2$ and find that the fluctuations in $z_t^1$ are 100%

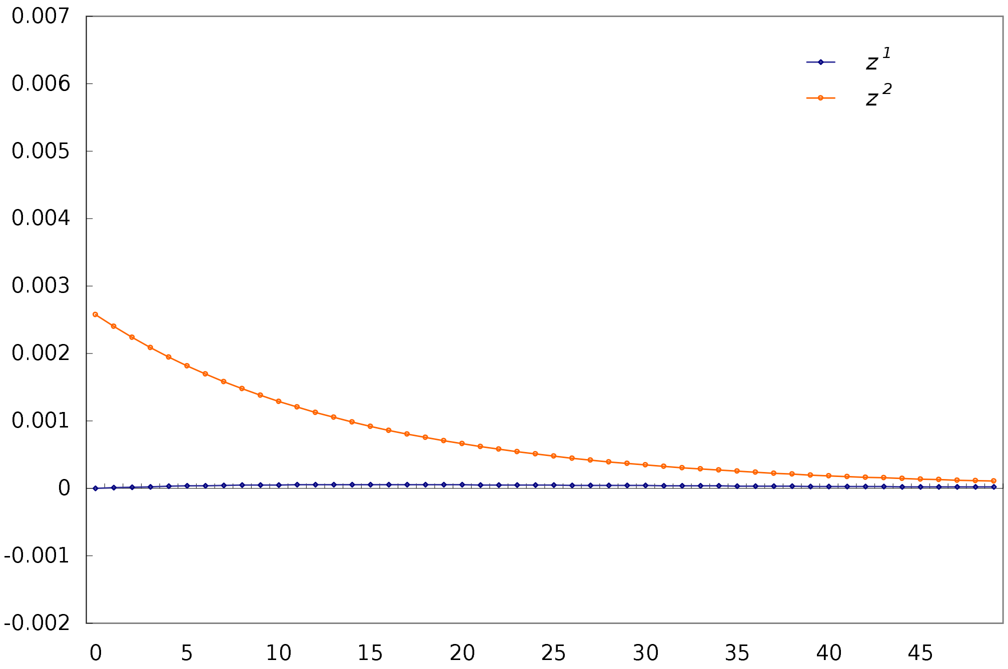

Figure 5: Impulse response functions to Orthogonalized Distributive Innovations $\epsilon^2$.

due to its own innovations, $u_t^1$, while 60% of the variation in $z_t^2$ is due to innovations in $u_t^1$ and 40% to its own innovations $u_t^2$.

# 4 The implications of the specification of the shocks for output and employment fluctuations

In this section we explore the implications of the two alternative specifications of shocks to the production function for the behavior of standard RBC models. Since it is well known that the answer to how important are productivity shocks in generating business cycle fluctuations depends on the labor elasticity, we explore two different sets of preferences with different values for this elasticity. We start specifying the model economies in Section 4.1

## 4.1 The Model Economies

The economy is populated by a large number of identical infinitely-lived households with the following preferences

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \; L_t \; u(c_t, 1 - h_t) \right\} \tag{29}$$

where $c_t$ is per capita consumption and $h_t$ denotes the proportion of time devoted to work. We choose standard momentary utility functions $u(.,.)$ that imply balanced growth paths. One parametrization that fulfills this requirement is the log-log utility function used in Cooley and Prescott (1995).

$$U(c_t, 1-h_t) = (1-\alpha)\log(c_t) + \alpha\ \log(1-h_t) \tag{30}$$

This specification has a unit labor elasticity.[8]

The other utility function that we use is the Rogerson (1988) log-linear utility function popularized by Hansen (1985) where the linearity in leisure arises from nondivisibilities and the use of lotteries and it generates a very high aggregate labor elasticity (in fact its labor elasticity as well as its Frisch labor elasticity are both infinity).

$$U(c_t, 1-h_t) = \log(c_t) + \kappa\ (1-h_t) \tag{31}$$

Population grows at rate $\eta$, $L_t = L_0(1+\eta)^t$. Output, $Y_t$, is used either for consumption or for investment, $I_t$, and the aggregate stock of capital $K_t$ evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + Y_t - C_t \tag{32}$$

where $\delta$ is the geometric depreciation rate.

The production function is as described in Section 2 Cobb-Douglas with labor augmenting technical progress where we consider model economies with univariate shocks $z_t^0$ and model economies with bivariate shocks $z_t^1$ and $z_t^2$. The specification that we posed to obtain the Solow residual and pose it as a univariate process with both productivity and population growth was

$$Y_t = e^{z_t^0} A\ K_t^{\theta} \left[(1+\lambda)^t\ (1+\eta)^t\ \mu\ h_t\right]^{1-\theta} \tag{33}$$

In this model economy the units are irrelevant. Still for consistency across models we choose the so that steady state output is one and the ratio of steady state capital $K^*$ to steady state efficient labor $\mu h^*$ is also set to one.

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[8] Its Frisch elasticity is 2.

The production that we posed to model the bivariate process with productivity and redistributive shocks is

$$Y_t = e^{z_t^1} A K_t^{\theta - z_t^2} \left[(1+\lambda)^t (1+\eta)^t \, \mu \, h_t\right]^{1-\theta+z_t^2} \tag{34}$$

As we saw in Section 2.3 the units matter for this specification. We set again $A$ and $\mu$ so that both steady state output and the capital to efficient labor ratio are one. In this fashion, $z_t^2$ do not have implications for productivity as they are pure redistributive shocks.

We can stationarize the model economies by taking into account population and technological growth. As before, we use small case letters to denote detrended variables and we use small-case hat variables to denote detrended log deviations. With log-log utiliy, in the transformed economy the planner's problem is to solve[9]

$$\max_{\{c_t, k_{t+1}, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \, (1+\eta)^t \left[(1-\alpha)\log(c_t) + \alpha \log(1-h_t)\right] \tag{35}$$

subject to

$$c_t + k_{t+1}(1+\eta)(1+\lambda) \;=\; y_t + (1-\delta)k_t \tag{36}$$

and either

$$y_t \;=\; e^{z_t^0} \, A \, k_t^{\theta} \, h_t^{1-\theta} \tag{37}$$

or

$$y_t \;=\; e^{z_t^1} \, A \, k_t^{\theta - z_t^2} \, (\mu h_t)^{1-\theta+z_t^2} \tag{38}$$

The aggregate shocks, either $z_t^0$ or $\{z_t^1, z_t^2\}$ follow the processes described in Section 3.

## 4.2 Calibration

Calibration is very simple in this model since there are only four parameters, $\alpha$, $\beta$, $\theta$ and $\delta$, in addition to the population growth rate $\eta$ and the productivity growth rate $\gamma$, that we choose

---

[9] In our economies the welfare theorems hold so we can use the planner's problem in lieu of solving for the competitive equilibrium.

**Raul di aqui cuales escoges y confirma que son los regresores $g_x$ del primer paso de la construccion del solow residual.** Denoting again with by $x^*$ the steady state value of $x$ (with the shocks set to zero–their unconditional mean) we have a system of four equations that when solved yield the value of the four parameters for four targets of the steady state values.

$$\begin{aligned}(1-\theta)\frac{y^*}{c^*} &= \frac{\alpha}{1-\alpha}\frac{h^*}{1-h^*} \quad (39)\\ (1+\gamma) &= \beta\left[\left(1-\delta+\theta\frac{y_*}{k_*}\right)\right] \quad (40)\\ \delta &= \frac{i^*}{k^*}-(1+\eta)(1+\gamma)+1 \quad (41)\\ 1-\theta &= \text{Labor Share}^* \quad (42)\end{aligned}$$

The targets that we choose are

1. The fraction of time devoted to market activities: $h^* = 0.30$.
2. The steady-state consumption-output ratio: $c^*/y^* = 0.75$.
3. The capital-output ratio in yearly terms $K^*/y^* = 2.31$.[10]
4. Labor share =0.68.[11]

For the Hansen-Rogerson version of the model (with indivisible labor), the only equilibrium condition that changes is (39) that is substitued with

$$(1-\theta)\frac{y^*}{c^*} = \kappa h^* \quad (43)$$

The implied value of the parameters is reported in Table 5. We report both the discount rate and the depreciation rates in quarterly terms and we report for the sake of completion the values of $A$ and $\mu$ and the values used in the original sources.

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[10] This is the target only for the benchmark model economy; it only includes fixed private capital. When we extend measured output with durables this ratio goes to 2.33, and adding government capital we get 2.82.

[11] This is the target only for the benchmark model economy. When we extend measured output with durables this ratio goes to 0.66.. It is 0.64 when we consider government and 0.it is 57 when we use the narrowest definition of labor share that only includes Compensation to Employees in Labor share.

|  | $\theta$ | $\delta$ | $\beta$ | $\alpha$ | $\mu$ | $A$ | $\kappa$ |
|---|---|---|---|---|---|---|---|
| Benchmark Model | .32 | .019 | .989 | .68 | 30.8 | .108 | 3.02 |
| ... with Durables | .37 | .019 | .983 | .66 | 30.9 | .107 | 2.78 |
| ... with Government | .42 | .014 | .982 | .64 | 37.5 | .089 | 2.57 |
| CE/GNP | .43 | .014 | .981 | .64 | 37.5 | .089 | 2.53 |
| Cooley-Prescott (1995) | .40 | .012 | .987 | .64 | - | - | - |
| Hansen (1985) | .36 | .025 | .990 | - | - | - | 2.84 |

Table 5: Calibrated Parameters

### 4.3 Findings

We now turn to discuss the main finding of the paper, that posing the productivity shocks as a bivariate process that affects factor shares implies a striking reduction in the volatility of the cycle: Aggregate hours worked are less volatile by a factor of 3.

We start by looking at the business cycle properties of the the U.S. and of the standard and the Hansen-Rogerson preferences RBC economies with both specifications of the shocks in Section 4.3.1. Next, we discuss the reasons for the small cyclical fluctuations of aggregate hours in the bivariate shocks economies in Section 4.3.2.

#### 4.3.1 Business Cycle Properties of the Model Economies

Table 6 reports the business cycle statistics for the main economic variables and factor prices 1954.I-2002.IV in the U.S. and in the model economies with standard log-log preferences. The first thing to note is that in the univariate model economy, productivity shocks account for 79.5% of the standard deviation (63.2% of the variance) of output in the data. In the bivariate model economy shocks account for 54.7% (29.9% of the variance.)

However, the most important statistic to measure the ability of the model to generate fluctuations is the standard deviation of hours since output moves both because of hours and because of the shocks. In this respect, the univariate model accounts for 42.7% of the standard deviation of the data (17.4% of the variance). The striking finding is that the bivariate model accounts for 12.8% of the standard deviation of hours in the data (1.64% of the variance). The differential behavior of hours in the bivariate economy also shows up in the correlation between hours and output. While it is very high in the data (0.88) and in the univariate shock economy (0.98), it is much lower in the bivariate shock economy (0.30).

| | U.S. Data | | | Univariate $\{z^0\}$ | | | Bivariate $\{z^1, z^2\}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\sigma_x$ | $\rho(y,x)$ | $\rho(x_{t-1},x_t)$ | $\sigma_x$ | $\rho(y,x)$ | $\rho(x_{t-1},x_t)$ | $\sigma_x$ | $\rho(y,x)$ | $\rho(x_{t-1},x_t)$ |
| $y$ | **1.61** | 1.00 | 0.84 | **1.28** | 1.00 | 0.72 | **0.88** | 1.00 | 0.72 |
| $h$ | **1.56** | 0.88 | 0.89 | **0.65** | 0.98 | 0.70 | **0.20** | 0.30 | 0.73 |
| $c$ | 1.27 | 0.87 | 0.85 | **0.42** | 0.89 | 0.81 | **0.68** | 0.88 | 0.77 |
| $i$ | 7.27 | 0.91 | 0.79 | **4.05** | 0.99 | 0.71 | **1.95** | 0.87 | 0.69 |
| $r$ | 0.09 | 0.74 | 0.76 | 0.05 | 0.96 | 0.71 | 0.06 | 0.68 | 0.70 |
| $w$ | 0.74 | 0.10 | 0.73 | 0.66 | 0.98 | 0.75 | 0.74 | 0.84 | 0.77 |
| $s^0, s^1$ | 0.84 | 0.75 | 0.71 | 0.84 | 0.99 | 0.71 | 0.83 | 0.98 | 0.70 |
| $s^2$ | 0.47 | -0.22 | 0.75 | - | - | - | 0.42 | -0.29 | 0.72 |

Table 6: Cyclical Behavior of the Data, U.S. 1954.I-2002.IV and log-log Utility RBC Models with Univariate and Bivariate Shocks

With respect to the other aggregate variables the behavior of consumption is quite surprising: in the economy with bivariate shocks its standard deviation is higher than in the economy with univariate shocks despite having a lower standard deviation of output, a feature that we discuss below. Consequently, the univariate shock economy displays much higher volatility of investment than the bivariate shock economy. Both factor prices are strongly correlated with output in the univariate model economy and less so in the bivariate model economy. Finally, the behavior of both residuals is very similar and they are ere very correlated with output. (recall that the residuals are virtually identical across economies, but output is not). While the univariate model economy does not display movements in labor share, the bivariate economy does and like in the data they are negatively correlated with output.

Table 7 shows the phase-shift of the variables in both economies. The behavior of hours is quite different between the two economies: While in the univariate economy hours are very procyclical and the have a slight lead in the cycle, in the bivariate economy hours are quite flat and they lag the cycle. In both economies, consumption lags the cycle and investment leads it, although not by much.

The behavior of rates of return is also quite different. In the univariate economy they are quite strongly correlated with output, they lead the cycle and they do not become negative until a year after output peaks. In the bivariate economy they are less correlated, the lead the cycle and they become negative tree quarters after output peaks. Wages are very correlated with output in the univariate economy, and they lagged somewhat while in the bivariate economy they are less correlated with output and they slightly lag the cycle. Overall, though the behavior of wages is

|  | Cross-correlation of $y_t$ with |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  | $x_{t-5}$ | $x_{t-4}$ | $x_{t-3}$ | $x_{t-2}$ | $x_{t-1}$ | $x_t$ | $x_{t+1}$ | $x_{t+2}$ | $x_{t+3}$ | $x_{t+4}$ | $x_{t+5}$ |
|  | Univariate Model |  |  |  |  |  |  |  |  |  |  |
| $y$ | -.02 | .11 | .27 | .47 | .71 | 1.00 | .71 | .47 | .27 | .11 | -.02 |
| $h$ | .07 | .19 | .34 | .52 | .74 | .98 | .63 | .36 | .14 | -.03 | -.15 |
| $c$ | -.23 | -.11 | .06 | .27 | .55 | .89 | .77 | .64 | .51 | .38 | .27 |
| $i$ | .04 | .16 | .32 | .50 | .73 | .99 | .66 | .39 | .18 | .01 | -.11 |
| $r$ | .11 | .23 | .37 | .54 | .74 | .96 | .59 | .30 | .08 | -.09 | -.21 |
| $w$ | -.11 | .01 | .18 | .39 | .66 | .98 | .75 | .55 | .38 | .23 | .10 |
| $s^0$ | .00 | .12 | .28 | .48 | .72 | .99 | .69 | .44 | .24 | .07 | -.04 |
|  | Bivariate Model |  |  |  |  |  |  |  |  |  |  |
| $y$ | -.02 | .11 | .27 | .47 | .71 | 1.00 | .71 | .47 | .27 | .11 | -.02 |
| $h$ | -.11 | -.08 | -.02 | .06 | .16 | 0.30 | .29 | .26 | .24 | .20 | .16 |
| $c$ | -.13 | -.01 | .14 | .34 | .58 | 0.88 | .72 | .57 | .42 | .29 | .17 |
| $i$ | .10 | .21 | .34 | .49 | .67 | 0.87 | .52 | .24 | .04 | -.11 | -.21 |
| $r$ | .15 | .24 | .33 | .43 | .54 | 0.68 | .34 | .09 | -.08 | -.20 | -.28 |
| $w$ | -.13 | -.02 | .12 | .31 | .55 | 0.84 | .70 | .55 | .42 | .29 | .18 |
| $s^1$ | .02 | .15 | .30 | .49 | .71 | 0.98 | .66 | .40 | .19 | .03 | -.08 |
| $s^2$ | -.18 | -.21 | -.24 | -.26 | -.27 | -0.29 | -.06 | .09 | .20 | .27 | .30 |

Table 7: Phase-Shift of the Model Economies

more similar across the two economies than that of rates of return.

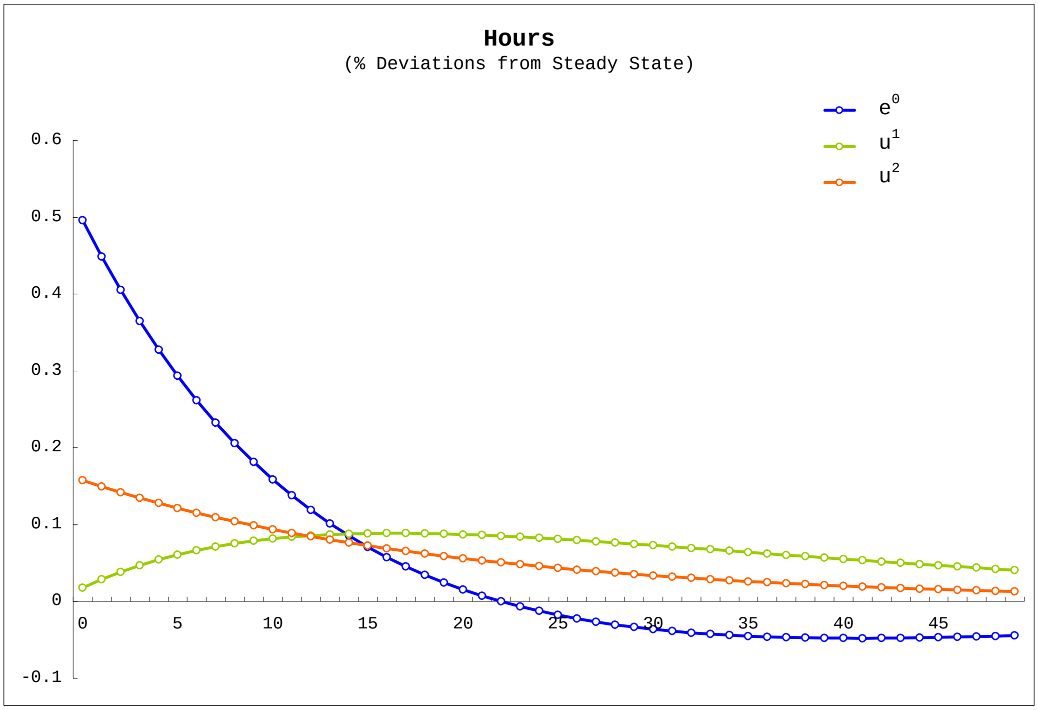

Figure 6: Hours impulse response functions to innovations to all shocks.

Figure 6 shows the impulse response of hours to innovations to all three shocks. We see that a one standard deviation innovation to the only shock in the univariate model increase hours by one half of one percent. In addition, the response of hours dies out pretty rapidly. In the bivariate shock economy the situation is quite different. There is barely any immediate response of hours to a current innovation in the productivity shock, $u^1$, and the response is delayed dramatically as it increases for about 15 quarters (still not to a very high level) before coming up down. A redistributive shock $u^2$ towards labor increases hours (albeit by about a third of that of the level of a productivity shock in the univariate economy), and it dies out quite slowly.

Table 8 displays a variance decomposition of the main variables by the source of the innovation. We see that while for most variables most of the variance is due to the innovation to productivity, the variance of hours is due in equal measure to both innovations. Innovations to the redistributive shock also have important effects on interests rates and less so to wages. In addition, note that given the ortogonalization of the innovations that we chose, only 40% of the variance of the

|  | $y$ | $c$ | $i$ | $h$ | $r$ | $w$ | $z^1$ | $z^2$ |
|---|---|---|---|---|---|---|---|---|
| $\epsilon^1$ | 98 | 94 | 93 | 51 | 71 | 91 | 100 | 60 |
| $\epsilon^2$ | 2 | 6 | 7 | 49 | 29 | 9 | 0 | 40 |

Table 8: Forecast Error Variance Decomposition (%)

redistributive shock itself is due to the redistributive innovation.

**The Rogerson-Hansen Economies** Table 9 reports the business cycle statistics for data and the Hansen-Rogerson log-linear preferences with univariate and bivariate shocks. As it is well-known, the higher elasticity of hours of this model generates a larger response to the shocks. The economy with univariate shocks displays 82.7% of the standard deviation of hours and 106.2% of output (68.4% and 112.8% of the variance respectively). However when we turn to the cyclicality of the bivariate model economy, the reduction is spectacular. The standard deviation of hours is now 25.6% of that in the data (6.6% of the variance). The bivariate process generates a 31.0% of the standard deviation of the univariate process (9.6% of the variance). As in the log-log economy, consumption is more volatile than investment in the bivariate shock than in the univariate shocks.

|  | U.S. Data |  |  | Hansen RBC $\{z^0\}$ |  |  | Hansen RBC $\{z^1, z^2\}$ |  |  |
|---|---|---|---|---|---|---|---|---|---|
|  | $\sigma_x$ | $\rho(y,x)$ | $\rho(x_{t-1},x_t)$ | $\sigma_x$ | $\rho(y,x)$ | $\rho(x_{t-1},x_t)$ | $\sigma_x$ | $\rho(y,x)$ | $\rho(x_{t-1},x_t)$ |
| $y$ | **1.61** | 1.00 | 0.84 | **1.71** | 1.00 | 0.71 | **0.89** | 1.00 | 0.73 |
| $h$ | **1.56** | 0.88 | 0.89 | **1.29** | 0.98 | 0.70 | **0.40** | 0.33 | 0.73 |
| $c$ | 1.27 | 0.87 | 0.85 | **0.51** | 0.89 | 0.82 | **0.71** | 0.93 | 0.77 |
| $i$ | 7.27 | 0.91 | 0.79 | 5.59 | 0.99 | 0.70 | 1.77 | 0.90 | 0.69 |
| $r$ | 0.09 | 0.74 | 0.76 | 0.06 | 0.96 | 0.70 | 0.06 | 0.62 | 0.70 |
| $w$ | 0.74 | 0.10 | 0.73 | 0.51 | 0.98 | 0.82 | 0.70 | 0.93 | 0.77 |
| $s^0, s^1$ | 0.84 | 0.75 | 0.71 | 0.84 | 0.99 | 0.71 | 0.84 | 0.94 | 0.71 |
| $s^2$ | 0.47 | -0.22 | 0.75 | - | - | - | 0.42 | -0.16 | 0.72 |

Table 9: Cyclical Behavior of the U.S. Data and of the Hansen-Rogerson RBC Model

We avoid the cumbersome reporting of all the features of the Hansen-Rogerson economy, but the picture is clear. As it is well known, the higher elasticity of hours of these preferences translate in a much higher volatility of hours worked. However, posing the productivity shocks in the bivariate way that we are exploring in this paper dramatically dampens the volatility of hours

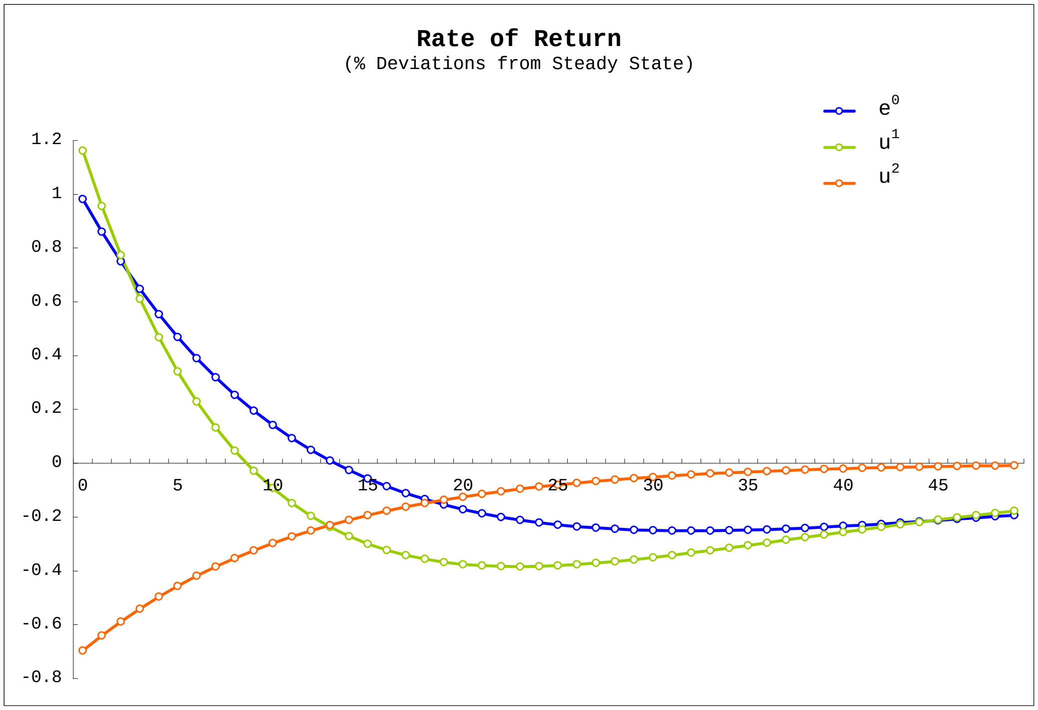

Figure 7: Interest rate impulse response functions to innovations to all shocks.

worked. It does so in a similar or more dramatic fashion than it does to the economy with a lower elasticity of hours worked (the standard deviation of hours is less than a third than that of the univariate shock) and for similar reasons that we will explore next.

### 4.3.2 Why do hours move so little in the bivariate economies?

The key question now is why does such a seemingly small departure from the standard model generate such a large change in the behavior of aggregate hours.

We find it useful to decompose the exploration of what happens into two parts, first is how the different behavior of wages and interest rates in both economies vary and how they imply different allocations, and second, how the two sets of shocks yield different paths for wages and interest rates.

**Hours and wages in the univariate and bivariate economies** Agents have the same preferences in both the univariate and bivariate economies which means that if they do different things it is due to the fact that they face different wages and interest rates.

Figures 7 and 8 respectively plot the impulse response functions of the interest rate (actually, tomorrow's rate of return) and real wages to productivity innovations.

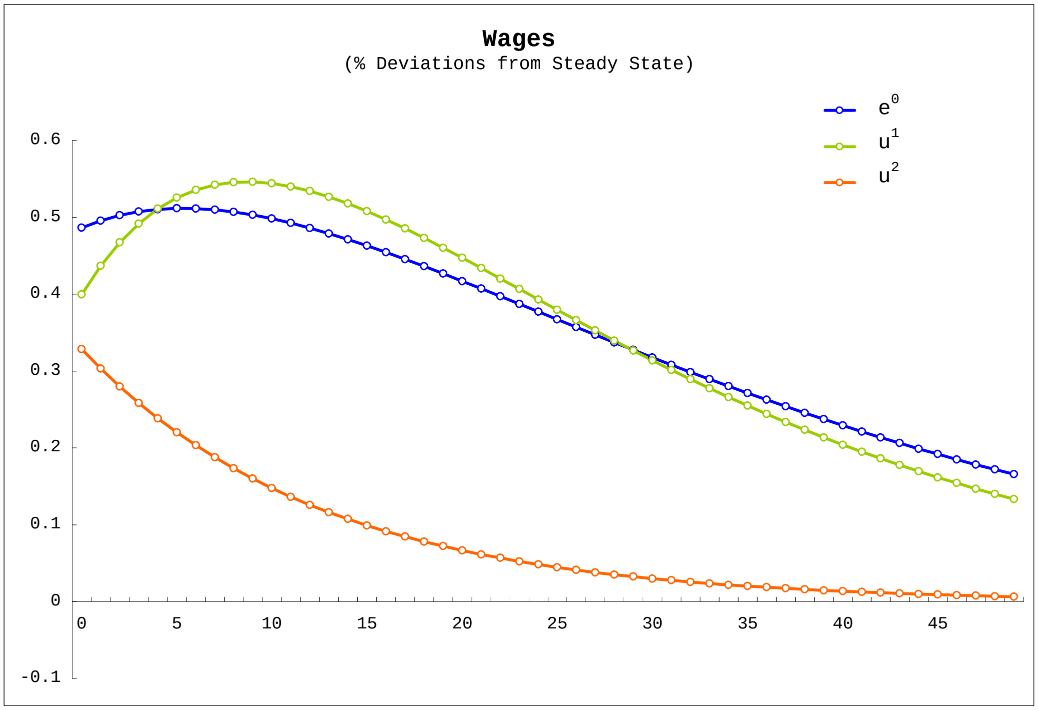

Figure 8: Wage impulse response functions to innovations to all shocks.

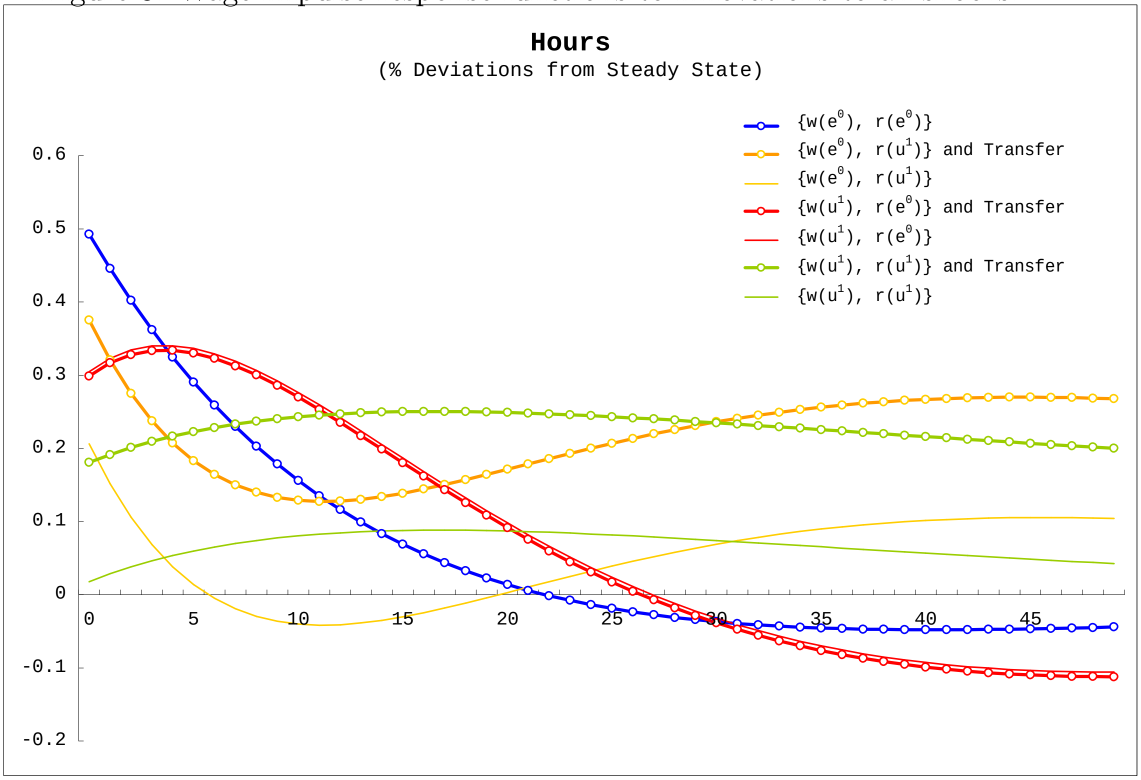

Figure 9: Hours choices for different combinations of wages and interest rates

In the Appendix, we report in some detail the results for alternative calibrations of labor share and its movements as well as some additional information about the economy with Hansen-Rogerson preferences. These results confirm the findings already discussed.

## 5 Conclusion

We pose and estimate a bivariety shock to the production function that under competition in factor markets simultaneously accounts for movements in the Solow residual and in the factor shares of production. We show how confronting agents in a standard RBC economy with these shocks entail a much smaller response (about 40%) of hours relative to the standard modelization of the shocks that identifies the Solow residual with a univariate shock. Our findings raise a flag against the optimism embedded in the literature that states that productivity shocks are responsable for most of the cyclical behaviour of output and hours.

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## Appendix

| | $\gamma_{11}$ | $\gamma_{12}$ | $\gamma_{21}$ | $\gamma_{22}$ | $\sigma_1$ | $\sigma_2$ | $\sigma_{12}$ |
|---|---|---|---|---|---|---|---|
| Labor Share | .938 | - | .055 | .933 | .00641 | .00303 | -.104e-04 |
| | (.022) | | (.011) | (.016) | | | |
| ... with Durables | .935 | - | .058 | .931 | .00644 | .00288 | -.101e-04 |
| | (.023) | | (.010) | (.015) | | | |
| ... with Government | .919 | - | .063 | .941 | .00706 | .00314 | -.141e-04 |
| | (.026) | | (.011) | (.015) | | | |
| CE/GNP | .939 | - | .058 | .925 | .00666 | .00341 | -.166e-04 |
| | (.023) | | (.012) | (.013) | | | |

Table 10: Constrained Estimation under $\gamma_{12} = 0$