Aggregate and Idiosyncratic Risk in a Frictional Labor Market

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November 17, 2007

Abstract

How are business cycle fluctuations in unemployment and wages affected by the extent to which workers are able to smooth their consumption through asset markets? Can incomplete markets resolve the difficulty search and matching models have in accounting for the high cyclical volatility of unemployment observed in US data? This paper examines a parsimonious equilibrium model of job search with aggregate productivity shocks, where workers face uninsured idiosyncratic unemployment risk. Risk averse entrepreneurs post optimal dynamic contracts to attract risk averse workers, providing them consumption smoothing through wages during employment. Employers cannot insure workers against the idiosyncratic risk of job loss, however. Workers face an extreme form of incomplete markets, leading to a particularly simple representation for the equilibrium as a small system of differential equations. When workers face incomplete markets, cyclical fluctuations in unemployment are amplified, and those in wages dampened. While this brings the model closer to data, the quantitative results show that market incompleteness does not resolve the volatility puzzle.
Keywords: Unemployment, Wages, Business Cycles, Search, Dynamic Contracts

Unemployment is strongly cyclical in the United States, varying by 20 percent over the business cycle. Economists developing models of unemployment are unable to explain this strong cyclicality. The cyclical variation in measured labor productivity is ten times smaller,

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This paper is based on the second essay of my dissertation at the University of Chicago. I am grateful to Fernando Alvarez, Simon Gilchrist, Lars P. Hansen, Francois Gourio, Robert King, Robert Lucas, Robert Shimer as well as seminar participants at Chicago, Federal Reserve Bank of Chicago, Washington University in St. Louis, Wharton, University of Minnesota, University of Maryland, Boston University, CREI, IIES, University of Cambridge, Collegio Carlo Alberto, Ohio State University, University of Iowa, SED2007, ESSIM2007 and University of Rochester for comments.

leading us to expect much smaller fluctuations in unemployment than observed. Hall (2005) and Shimer (2005a) discuss this in the context of the widely-used Mortensen-Pissarides search and matching model of the labor market (Pissarides 1985, Mortensen and Pissarides 1994).

Existing models generally abstract from one seemingly important feature of labor markets however – that workers cannot insure against the risk of job-loss or duration of unemployment. Nor do the unemployed generally hold much wealth for self-insurance.[1] Because the behavior of unemployed agents is a central determinant of the labor market equilibrium in a search model, it is important to consider relaxing the assumption of full insurance.

To this end, I construct an equilibrium model of job search where aggregate productivity shocks lead to business cycles and workers face uninsured idiosyncratic unemployment risk. The model is an extension of the Mortensen-Pissarides framework. The economy is populated by a continuum of risk averse workers and entrepreneurs, distinguished by their ability to access capital markets. While entrepreneurs have access to trade in a complete set of asset markets, workers are excluded from asset markets completely, consuming their income each period. This extreme form of market incompleteness delivers significant parsimony in the model. Moreover, it may be interpreted as giving an upper bound on the potential impact of incomplete markets for the dynamics of the model.

Entrepreneurs operate a production technology requiring worker labor as input. In addition to aggregate productivity shocks, the technology is subject to idiosyncratic separation shocks which lead to an employment relationship between a worker and entrepreneur becoming unproductive, and the parties separating. Workers also have access to a less productive home production technology, allowing them to survive when unemployed.

The equilibrium is a contract posting equilibrium in the spirit of the competitive search equilibrium of Moen (1997), which I extend to a dynamic environment. To attract workers, entrepreneurs pay a cost to post a vacancy. A vacancy specifies a state contingent long term wage contract, which the entrepreneur is free to choose. Unemployed workers observe all the contracts offered, and choose one to apply for. Search frictions in the labor market, modeled with a matching function, prevent unemployed workers from finding a job immediately.

Workers face uninsured idiosyncratic unemployment risk because a separation shock leads to an unemployment spell with reduced consumption for period of uncertain duration. Entrepreneurs do not face corresponding risk because they employ a measure of workers and due to a law of large numbers the shocks average out to a deterministic depreciation rate in the entrepreneur's workforce.

At first blush the equilibrium appears intractable because of a large state space of wage contracts assigned to workers upon hiring. Not only does each contract specify a large

[1] Most recently Chetty (2006) shows that in the US approximately half of unemployment benefit claimants held no liquid wealth at the time of job loss and argues that their behavior is suggestive of liquidity constraints.

amount of information about wages paid in various contingencies, but the terms of these contracts depend on the time of hiring. The equilibrium turns out to have a very simple representation, however.

First, I prove that in equilibrium there is a unique wage contract offered at each instant. This contract has the form of a risk sharing relationship between an entrepreneur and worker. It ensures that the worker's marginal rate of substitution between consumption at different states and points in time is equated to that of his employer's. With time separable constant relative risk aversion preferences, the consumption growth of a worker co-moves with his employer's in a systematic way during a contract. If the worker and employer happen to have identical preferences, then the worker's wage growth must always equal his employer's consumption growth as long as the worker remains employed. There can only be a level difference between the wage and the entrepreneur's consumption.

Second, I show that the equilibrium does not pin down the distribution of workers among entrepreneurs and that without loss of generality I can work with a representative entrepreneur. This implies that the wage growth of all workers is tied systematically to the consumption growth of the representative entrepreneur.

Finally, solving for equilibrium requires information on the total wage costs of the representative entrepreneur, not the distribution across workers. This implies that the payoff relevant state space collapses into just two endogenous state variables: employment and a measure of the total wage commitments made by the representative entrepreneur to employed workers. The equilibrium can be represented as a small system of differential equations, so the dynamics are straightforward to characterize. Examining this system I show that, in an economy without aggregate shocks, there is a unique equilibrium path converging to steady-state.

The equilibrium captures a traditional idea that the wages of employed workers affect the labor market for new hires, which is absent in the Mortensen-Pissarides model. The second state variable represents wage commitments made to employed workers and it affects the conditions in the market for new hires: their wages as well as how much hiring takes place.[2] The second state variable has slow dynamics beyond the business cycle time frame, because wage commitments change only through turnover in the labor force. This introduces more persistence into the dynamics of employment than in the Mortensen-Pissarides model. In a calibrated model these dynamics turn out to be overshadowed in magnitude, however.

I find that when workers cannot smooth consumption privately, cyclical fluctuations in unemployment are amplified, and those in wages dampened. This helps bring the model closer

[2]That the equilibrium features a connection between the wages of existing and new workers is interesting because of arguments that wage rigidity is an important missing factor in the Mortensen-Pissarides model, underlying its inability to produce sufficient volatility (Hall 2005). Here the connection between the wages of existing and new workers arises endogenously as an equilibrium outcome. It is not quite the kind of rigidity sought after by Hall, however, because despite the link between the wages of existing and new workers, wages turn out to be relatively pro-cyclical in this model.

to data in terms of the size of fluctuations in both unemployment and wages. Nevertheless, quantitatively this mechanism has limited ability to explain the large difference between model and data. Given the extreme form of market incompleteness in the model, this suggests that incomplete markets cannot resolve the unemployment volatility puzzle.

More specifically, I show that decreasing the willingness of workers to substitute consumption across time and states in the incomplete markets model leads to the wage level of new hires responding less to changes in productivity. This leads to more cyclical profits from new employment relationships and hence more cyclical vacancy creation. In relating these findings to the Mortensen-Pissarides model with linear preferences, the changes in dynamics can be viewed as a combination of two effects: a response to concavity in preferences *per se*, and a response to incomplete markets. I show that the first effect is relatively small in magnitude, leading to amplified responses of both the vacancy-unemployment ratio and the wage level of new hires to a productivity shock. This has to do with a slightly countercyclical interest rate affecting the profitability of investment. More generally the direction of this effect is ambiguous however.

The second effect reflects the entrepreneurs' response to the workers' preference for consumption smoothing. It dampens the response of the starting wage level to shocks and amplifies the response of the vacancy-unemployment ratio. A useful way to think about this effect is to consider the steady-state of the economy (without aggregate shocks), and how the steady-state wage level responds to changes in the steady-state productivity. In this steady-state workers periodically transition between low consumption during unemployment and high consumption during employment. An increase in productivity increases both wages and vacancy creation. When workers dislike substituting consumption across time and states, the increase in the wage level becomes muted as the workers' gain from a high level of consumption during employment diminishes. This leads to the productivity increase having a stronger effect on increasing vacancy creation.

In the simulations these two effects of concavity in preferences and incomplete markets work in opposite directions for wages and the same direction for the vacancy-unemployment ratio. I find that net effect is an amplified response of the vacancy-unemployment ratio to shocks together with a muted response of starting wages.

I begin with the assumption that workers and entrepreneurs have identical preferences, but this assumption is straightforward relax to allow for differences between the two groups. I show that the amplifying effect of incomplete markets on the vacancy-unemployment ratio hinges on the workers' willingness to substitute consumption across time and states, while the entrepreneurs' willingness to substitute makes little difference. I also verify that the results do not change significantly if workers are less patient than entrepreneurs, an assumption which could justify the distribution of wealth holdings in the economy.

The paper proceeds as follows: Section 1 presents the model and defines a contract posting equilibrium. Section 2 derives properties of equilibrium, including a useful aggregation result. Section 3 analyzes an economy without aggregate shocks, showing how the state space of the economy collapses to two variables, discussing the existence of a unique equilibrium as well as demonstrating the novel dynamic properties of the model. Section 4 returns to the environment with aggregate shocks and explores quantitative impact of market incompleteness. Section 5 discusses the empirical evidence and related literature on incomplete markets. Section 6 concludes.

1 Model

Preferences Consider a continuous time economy populated by two types of infinitely lived agents: workers and entrepreneurs. I normalize the measure of each type of agent to one.[3] Assume that both types have identical preferences $E_0 \int_0^\infty e^{-\rho t} u(c(t))dt$, where u is a constant relative risk aversion utility function: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with risk aversion $\gamma > 0$, and c is consumption of the one good in the economy. I will relax the assumption of identical preferences later.

Production technologies Entrepreneurs have access to a linear technology producing $z(t)$ units of output per worker and unit of time, where $z(t)$ is an aggregate productivity level. Each entrepreneur can employ any measure of workers. The technology is subject to match-specific separation shocks, which drop the productivity of a worker with his current employer to zero permanently. These shocks arrive at Poisson rate δ. When not employed, workers can produce at home b units of output per unit of time, where $0 < b < z(t)$.

Matching technology The organization of workers and entrepreneurs into production is hampered by search frictions in the labor market. To find a worker, an entrepreneur must post a vacancy, specifying a wage contract (described shortly). Posting a vacancy costs an entrepreneur κ units of output per unit of time. Unemployed workers observe all contracts offered and choose one to apply for. In this way the labor market segments into contract specific sub-markets, and in each sub-market a matching function governs the rate at which workers find jobs. The workers' choice of sub-market depends both on the value of the wage contract, and how quickly the worker expects to be hired. This rate is a function of the ratio of vacancies to job seekers in the sub-market, denoted θ. With a Cobb-Douglas matching function, the job-finding rate is $\mu(\theta) = k\theta^{1-\alpha}$, with $k > 0, \alpha \in (0,1)$. The rate of filling vacancies is $q(\theta) = \mu(\theta)/\theta$. When deciding on a contract to post, entrepreneurs anticipate that their choice of contract affects the tightness of the market for the contract through its effect on worker flows.

[3]It is not important that their measures be the same, however.

Asset markets Entrepreneurs can trade in complete asset markets. Let $p(t)$ denote probability normalized Arrow-Debreu prices in these markets and W_0 initial financial wealth. Workers are excluded from these markets, and consume their income each period. The asset markets have zero net supply.

Shock Processes Business cycles are driven by aggregate shocks affecting labor productivity, which leads to a procyclical job finding rate $\mu(\theta)$, while the separation rate δ is assumed constant. This is motivated by the findings of Shimer (2005b) that cyclical variation in unemployment derives mainly from variation in the job-finding rate, while the separation rate has a significantly smaller impact.

Workers face uninsured idiosyncratic unemployment risk because separation shocks lead to an unemployment spell with reduced consumption. The length of the spell depends on the cyclical job-finding rate. Because entrepreneurs employ a measure of workers, they do not face corresponding risk. A law of large numbers leads to entrepreneurs facing a depreciation rate of δ in their labor force and a hiring rate of $q(\theta)$ in vacancy creation.

Labor productivity follows a continuous time stochastic process meant to approximate the labor productivity data reported by the BLS. I model it as a jump process with small jumps. Suppose $z(t)$ is otherwise constant, but at Poisson arrival rate $\eta > 0$ aggregate shocks arrive that change the value of z. Conditional on a shock, the new value of productivity z' satisfies $E[z' - 1|z] = \xi(z - 1)$ with $\xi \in (0, 1)$, and where z is the value before the jump. All model variables are functions of the history of the aggregate shock process and I denote by $\mathcal{F}_t$ the corresponding information set at time t.

Wage Contract $\sigma(t)$, signed at time t given $\mathcal{F}_t$, is a collection of random variables $w(t, t+s)$ specifying state contingent wage payments for all continuation histories $\mathcal{F}_{t+s}, s \geq 0$ of $\mathcal{F}_t$. These payments are conditional on the separation shock not having hit.[4] I use $\Sigma(t)$ to denote the set of all such contracts given history $\mathcal{F}_t$.

Worker and Entrepreneur Problems The only decision workers make is which contract to apply for when unemployed. The measure used for comparing alternative contracts is the utility value of wages $E_t \int_0^\infty e^{-(\rho+\delta)s} u(w(t, t + s))ds$.

Entrepreneurs face a more complicated problem. First, they are endowed with a belief function $\Theta : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_+$, which states the entrepreneur's beliefs about the market tightness

[4] I rule out severance pay from contracts. The model can be viewed as representing an extreme form of incomplete markets for workers, underlining the painfulness of unemployment. Further, empirical evidence suggests severance pay is not very prevalent: Chetty (2006) examines a pool of unemployed workers and reports that 19% received severance pay upon job loss. Bishow and Parsons (2004) examine a larger pool of employed workers and report that approximately 24% have formal severance packages.

to prevail in the market for any contract he may contemplate offering as

$$\Theta(E_t \int_0^\infty e^{-(\rho+\delta)s} u(w(t, t+s))ds, t).$$

In equilibrium these beliefs must be consistent with worker behavior. If an entrepreneur offers a contract with a high utility value to workers, he expects to attract many workers per vacancy, and hence a low θ. Accordingly, the Θ-function is assumed to be decreasing, convex and differentiable in the first argument.

Indexing entrepreneurs by $i \in [0, 1]$, the sequence problem of an entrepreneur at time zero reads: Given the process for prices, $p(t) > 0$, and a belief function $\Theta(., t)$ for all $\mathcal{F}_t, t \geq 0$, choose $c^i(t), v^i(t), \sigma^i(t)$ for all $\mathcal{F}_t, t \geq 0$, to

$$\max E_0 \int_0^\infty e^{-\rho t} u\big(c^i(t)\big) dt \tag{P}$$

$$\text{s.t. } E_0 \int_0^\infty p(t)\big[\int_{-\infty}^t n^i(\tau, t)[z(t) - w^i(\tau, t)]d\tau - \kappa v^i(t) - c^i(t)\big]dt + W_0^i = 0, \tag{1}$$

$$n^i(\tau, t) = e^{-\delta t} n^i(\tau, 0) \text{ for } \tau < 0, \tag{2}$$

$$n^i(\tau, t) = e^{-\delta(t-\tau)} q\big(\Theta(E_\tau \int_0^\infty e^{-(\rho+\delta)s} u(w(\tau, \tau + s))ds, \tau)\big) v^i(\tau) \text{ for } \tau \geq 0, \tag{3}$$

with $W_0^i, n^i(\tau, 0), \sigma^i(\tau)$ given for all $\tau < 0$.

The initial conditions concern the size and composition of the initial workforce, corresponding wage contracts, and the entrepreneur's initial financial wealth. Here $n^i(\tau, 0)$ denotes the measure of workers hired in period $\tau < 0$ who are still working in period zero and $\sigma^i(\tau)$ is the contract they were hired with.[5] The choice variables are the measures of vacancies to post $v^i(t)$, the wage contracts $\sigma^i(t)$ to offer, as well as the entrepreneur's consumption $c^i(t)$. Implicit in this is also a choice of trades in the asset market.

Equation (1) is a present value budget constraint. Note that prices are normalized by probabilities, so taking expectations gives the correct summation over states. Each period t the entrepreneur has production output $\int_{-\infty}^t n^i(\tau, t)z(t)d\tau$ due to the workforce hired in periods $\tau \in (\infty, t]$. Workers are paid according to their contracts so total wage costs equal $\int_{-\infty}^t n^i(\tau, t)w^i(\tau, t)d\tau$. The entrepreneur also pays the costs of vacancy posting. Equations (2) and (3) are laws of motion for the size of the labor force hired in a particular period. The first expresses how many of the initial workers hired in periods $\tau < 0$ are still working in period t. The second applies to workers hired after period zero.

Next I define a contract posting/competitive search equilibrium for an economy with a measure of entrepreneurs indexed by $i \in [0, 1]$, in the spirit of Moen (1997).[6]

[5] For notational convenience I impose the same contract for all workers hired by the entrepreneur at a given time. It will become clear later that this is not a restrictive assumption since there turns out to be a unique optimal choice.

Definition 1. An *equilibrium* consists of, for all $\mathcal{F}_t, t \geq 0$, prices $p(t) > 0$, unemployment values $V^u(t)$ and entrepreneur beliefs $\Theta : \mathbb{R}\times\mathbb{R} \to \mathbb{R}_+$, as well as for all entrepreneurs: consumption $c^i(t) \geq 0$, vacancies posted $v^i(t) \geq 0$, contracts offered $\sigma^i(t)$, vacancy-unemployment ratios $\theta^i(t)$, and resulting labor forces $n^i(\tau,t) \geq 0$ for $\tau \leq t$ with economy-wide unemployment $n_u(t) = 1 - \int_i \int_{\tau\leq t} n^i(\tau,t)d\tau di$ such that

1. Given $\{p(t), \Theta(.,t)\}$, for all $\mathcal{F}_t, t \geq 0$, the allocation $\{c^i(t), v^i(t), \sigma^i(t), n^i(\tau,t)\}$ solves the entrepreneur's problem (P) for all $\mathcal{F}_t, t \geq 0, i \in [0,1]$.
2. The entrepreneurs' beliefs are consistent with worker decision-making: Define $V^u(.)$ by

$$\begin{aligned}\rho V^u(t) &\\ = u(b) + \mu\big(\theta^i(t)\big)\big[E_t \int_0^\infty e^{-(\rho+\delta)s}[u(w^i(t,t+s)) + \delta V^u(t+s)]ds - V^u(t)\big] &\\ + \eta\big[E_{t,+}V^u(t) - V^u(t)\big] + \frac{d}{dt}V^u(t), \quad (4)\end{aligned}$$

   for any $\mathcal{F}_t, t \geq 0, i \in [0,1]$. The belief function is defined implicitly such that for any $\sigma(t) \in \Sigma(t)$ and $\mathcal{F}_t, t \geq 0$, the value $\theta(t) = \Theta(E_t \int_0^\infty e^{-(\rho+\delta)s}u(w(t,t+s))ds, t)$ satisfies

$$\begin{aligned}\rho V^u(t) = u(b) &\\ + \mu\big(\theta(t)\big)\big[E_t \int_0^\infty e^{-(\rho+\delta)s}[u(w(t,t+s)) + \delta V^u(t+s)]ds - V^u(t)\big] &\\ + \eta\big[E_{t,+}V^u(t) - V^u(t)\big] + \frac{d}{dt}V^u(t). \quad (5)\end{aligned}$$

3. Goods market clearing: $\int_i [\int_{-\infty}^t n^i(\tau,t)(z(t) - w^i(\tau,t))d\tau - \kappa v^i(t) - c^i(t)]di = 0$, for all $\mathcal{F}_t, t \geq 0$ where $w^i(\tau,t)$ is specified by $\sigma^i(\tau)$.
4. Labor market clearing: $\int_i \frac{v^i(t)}{\theta^i(t)}di = n_u(t)$, for all $\mathcal{F}_t, t \geq 0$.

To understand how the labor market works, note first that all unemployed workers are identical. Hence it makes sense to define a value of unemployment $V^u(t)$, reflecting that while unemployed, a worker consumes b each period, and that whichever equilibrium contract

[6] One could add the constraint $\int_i W_0^i di = 0$, but this is implied by integrating over budget constraints and using goods market clearing:

$$\int_i W_0^i di = E_0 \int_0^\infty p(t) \left[\int_i c^i(t) - \int_{-\infty}^t n^i(\tau,t)(z(t) - w^i(\tau,t))d\tau + \kappa v^i(t) di\right] dt = 0.$$

$\sigma^i(t)$ the worker applies for, the job-finding rate is given by $\mu\big(\theta^i(t)\big)$, and once employed, the worker consumes wages specified by $\sigma^i(t)$ until the separation shock arrives and the worker returns to unemployment. This is spelled out in equation (4). I use notation $E_{t,+}V^u(t)$ for the expected value of V^u at time t, conditional on a jump occurring at t and pre-jump information $\mathcal{F}_t$. The expectations are taken with respect to the new value of productivity upon a jump.

If more than one contract is offered at the same time, workers must be indifferent between them. If a worker's utility value from signing one contract is higher than signing another, there must be more workers applying for the first contract, driving down the market tightness θ in the market for that contract and making the job harder to get. In equilibrium workers are indifferent between applying for either contract, because the gain in utility from signing a high value contract is offset by the lower job-finding rate in that market.

An entrepreneur considering offering an out of equilibrium contract anticipates a market tightness making workers indifferent between equilibrium contracts and the new contract. In doing so, entrepreneurs are assumed to take as given the market value of search V^u.[7] In equilibrium there cannot exist an out of equilibrium contract which would give entrepreneurs higher profits while leaving workers indifferent.

2 Equilibrium Properties

This section first shows a useful aggregation result and then gives a system of dynamic equations which characterize equilibrium.

2.1 Representative Entrepreneur

Without loss of generality, the analysis can be simplified to a representative entrepreneur. The following results characterizing the equilibrium combine to an aggregation result in Proposition 1.

Lemma 1. *Equilibrium Properties* *If an equilibrium exists, it has the properties:*

1. *Unique labor market:* $\sigma^i(t) = \sigma(t)$, $\theta^i(t) = \theta(t)\ \forall i \in [0,1], \mathcal{F}_t, t \geq 0$.
2. *Consumption growth is equalized between entrepreneurs and employed workers:*

$$e^{-\rho s}\left(\frac{c^i(t+s)}{c^i(t)}\right)^{-\gamma} = e^{-\rho s}\left(\frac{w(\tau,t+s)}{w(\tau,t)}\right)^{-\gamma} = \frac{p(t+s)}{p(t)},$$

[7] This implies an assumption about not offering too many out of equilibrium contracts such as to affect the market value of search.

for all $i \in [0,1], \tau \leq t, s \geq 0$, and all information sets such that $\mathcal{F}_\tau \subset \mathcal{F}_t \subset \mathcal{F}_{t+s}$.

3. *Posting vacancies gives zero profit:*

$$\kappa = q(\theta(t))E_t \int_0^\infty e^{-\rho s} \frac{p(t+s)}{p(t)} [z(t+s) - w(t,t+s)]ds, \ \forall \mathcal{F}_t, t \geq 0.$$

4. *Only aggregate measures of vacancies posted and labor force size are determined in equilibrium, not their distribution among entrepreneurs.*

The basic ideas underlying these results are simple (Appendix A contains the proofs): Notice that in the entrepreneur's problem (P), the decision of what contract to offer at any $\mathcal{F}_t, t \geq 0$, can be isolated from other decisions. The problem reads: Given $\mathcal{F}_t, t \geq 0$, the processes $p(t+s), V^u(t+s)$, for all $s \geq 0$, and continuations $\mathcal{F}_{t+s}$, choose $\theta^i(t), w^i(t,t+s)$, for all $s \geq 0$, and all continuations $\mathcal{F}_{t+s}$ to

$$\max -p(t)\kappa + q(\theta^i(t))E_t \int_0^\infty e^{-\delta s} p(t+s)[z(t+s) - w^i(t,t+s)]ds \tag{P1}$$

s.t.

$$\rho V^u(t) = u(b) + \mu\big(\theta^i(t)\big)\big[E_t \int_0^\infty e^{-(\rho+\delta)s}[u(w^i(t,t+s)) + \delta V^u(t+s)]ds - V^u(t)\big] + \eta[E_{t,+}V^u(t) - V^u(t)] + \frac{d}{dt}V^u(t).$$

Here I have substituted in the equilibrium condition (5) governing the beliefs of entrepreneurs. Entrepreneurs take as given the market value of search $V^u(t)$. Because this problem is independent of entrepreneur-specific factors, if there exists a unique solution, it must be independent of such factors as well. The problem further separates conveniently into two parts: 1) What is the optimal timing of wage payments, and 2) What is the optimal way to solve the tradeoff between the level of wages and market tightness?

Differentiating with respect to wages, the first question is answered by the optimality condition:

$$e^{-\rho s} \frac{u'\big(w^i(t,t+s)\big)}{u'\big(w^i(t,t)\big)} = \frac{p(t+s)}{p(t)} \tag{6}$$

Even though workers don't have direct access to asset markets, through the contracts they nevertheless have their marginal rates of substitution across time and states equated to the corresponding price ratios in the asset market. Because the entrepreneurs themselves can access these markets, also $e^{-\rho s} \frac{u'\left(c^i(t+s)\right)}{u'\left(c^i(t)\right)} = \frac{p(t+s)}{p(t)}$.

Equation (6) implies that once the initial wage level in a contract is pinned down, the timing of wage payments is determined by market prices. Imposing this structure on contracts, what remains is simply the choice of initial wage level. This problem turns out to have a unique solution under the Cobb-Douglas matching function.

The zero profit condition on vacancy creation must hold since each entrepreneur can create as many vacancies as he wishes. Finally, there is nothing pinning down how vacancy creation is divided across entrepreneurs.

Lemma 1 implies that the distribution of production and consumption among entrepreneurs is not relevant for the evolution of the economy, and the problem can be simplified by aggregating to a representative entrepreneur. Appendix A lays out a definition for a representative entrepreneur equilibrium, analogously with Definition 1 in the last section.

Proposition 1. ***Aggregation***

(a) *Given an equilibrium with a continuum of entrepreneurs (Definition 1), the aggregated allocation* $c(t) := \int_i c^i(t)di$, $v(t) := \int_i v^i(t)di$, $n(t, t+s) := \int_i n^i(t, t+s)di$ *along with* $p(t), V^u(t), \theta(t), \sigma(t)$ *constitutes a representative entrepreneur equilibrium (Definition 2 in Appendix A), if the initial conditions of the single entrepreneur are defined as* $n(\tau, 0) := \int_i n^i(\tau, 0)di$, $\sigma(\tau) := \{w(\tau, t) \forall t \geq \tau\}$, *with* $w(\tau, t) := \frac{1}{n(\tau,0)} \int_i n^i(\tau, 0) w^i(\tau, t)di$ *and* $W_0 := 0$.

(b) *Given a representative entrepreneur equilibrium (Definition 2), and individual initial conditions* $\{\sigma^i(\tau), n^i(\tau, 0), W_0^i, \forall \tau < 0, i \in [0, 1]\}$, *that*

1. *Integrate up to the initial conditions of the representative entrepreneur equilibrium, and*
2. *For all* $i \in [0, 1]$, *satisfy*
$E_0 \int_0^\infty p(t) \big[\int_{-\infty}^0 n^i(\tau, t)[z(t) - w^i(\tau, t)]d\tau - \kappa v^i(t) \big] dt + W_0^i > 0$,

then an equilibrium with a continuum of entrepreneurs (Definition 1) can be recovered, and it is unique in $c^i(t), \theta^i(t) = \theta(t), \sigma^i(t) = \sigma(t)$ *and aggregates* $\int_i v^i(t)di, \int_i n^i(\tau, t)di$ *for* $\mathcal{F}_t, t \geq 0$.

From here on I restrict attention to a representative entrepreneur.

2.2 Equilibrium Characterization

The following proposition gives a set of dynamic equations which are useful for solving for equilibrium.

Proposition 2. ***Characterization***

(a) *If a representative entrepreneur equilibrium exists, then it satisfies the following for all $t, s \geq 0$ and information sets:*

1. *Wage contracts have form:* $w(t, t+s) = a(t)c(t+s)$, *for all* $t, s \geq 0, \mathcal{F}_t \subset \mathcal{F}_{t+s}$.
2. $F(t) = \frac{1-\alpha}{\alpha} \frac{[V(t) - V^u(t)]}{u'(w(t,t))}$, *(wage-tightness tradeoff)*
3. $\kappa = q\big(\theta(t)\big)F(t)$, *(zero profit)*
4. $c(t) = \int_{-\infty}^{t} n(\tau, t)[z(t) - w(\tau, t)]d\tau - \kappa\theta(t)n_u(t)$, *(resource constraint)*
5. $\rho V^u(t) = u(b) + \mu\big(\theta(t)\big)\big[V(t) - V^u(t)\big] + \eta\big[E_{t,+}V^u(t) - V^u(t)\big] + \frac{d}{dt}V^u(t)$, *(value of unemployment)*
6. $\dot{n}_u(t) = -\mu\big(\theta(t)\big)n_u(t) + \delta(1 - n_u(t))$, *(law of motion for unemployment)*

where

$$F(t) := E_t \int_0^\infty e^{-(\rho+\delta)s} \frac{u'(c(t+s))}{u'(c(t))}[z(t+s) - w(t, t+s)]ds$$

and

$$V(t) := E_t \int_0^\infty e^{-(\rho+\delta)s}[u\big(w(t, t+s)\big) + \delta V^u(t+s)]ds.$$

(b) *If $V^u(t), \theta(t) > 0, \sigma(t), n(t, t+s), n_u(t), c(t) > 0$, for all $t, s \geq 0, \mathcal{F}_t \subset \mathcal{F}_{t+s}$ satisfy conditions 1.-6. in Proposition 2 (a), then they correspond to a unique representative entrepreneur equilibrium (up to price level scaling), with $\frac{p(t+s)}{p(t)} := e^{-\rho s}\frac{u'(c(t+s))}{u'(c(t))}$ for the corresponding continuation histories and $v(t) := \theta(t)n_u(t)$.*

The key part of this result is the characterization of wage contracts in part (a.1). It follows directly from the observation that the equilibrium contracts equalize the marginal rates of substitution across time and states between workers and the representative entrepreneur. With identical CRRA preferences this implies that the consumption growth of a worker and the representative entrepreneur are equalized in all contingencies, for as long as the worker remains employed. When a worker is hired, the entrepreneur commits to paying a fixed share of his own consumption to the worker for as long as the worker remains employed. I use a to denote this consumption share.

Condition 2 reflects the division of match surplus between worker and entrepreneur. Condition 3 is the zero profit condition on vacancy creation. Condition 4 is the resource constraint. Condition 5 is the dynamic equation for the value of unemployment.

To solve for equilibrium one can restrict attention to solving the equations in Proposition 2.

3 Wage Commitments and Employment as State Variables

Solving for equilibrium using Proposition 2 appears problematic due to a large state space. The state of the economy involves measures of workers hired at different points in time, along with their wage contracts. Each wage contract in itself contains a large amount of information on state contingent wages. Despite this, the equilibrium can be solved for in a very straightforward way, with only two state variables needed. This section focuses on an economy where z is constant in order to demonstrate how to solve for equilibrium, show that the equilibrium is unique and illustrate the novel dynamic features of the equilibrium.

3.1 Reduced State Space

Suppose the economy has been running for a long time, so that all existing contracts are of the form: $w(\tau, t) = a(\tau)c(t)$ for all $\tau \leq t$, where $c(t)$ is consumption of the representative entrepreneur and $a(\tau)$ is a consumption share parameter which is fixed for the duration of the contract. Notice that heterogeneity across contracts at any point in time is completely captured by differences in a's.

Examining the dynamic equations in Proposition 2, one can see that solving for equilibrium does not require information on the cross-sectional distribution of the a's, but knowing the average level is sufficient. Because of this, the state-space boils down to two state variables: total employment and an employment weighted average of a's allocated to workers. The first determines total output and the second total wage costs at each point (relative to entrepreneurial consumption).

To show how the conditions of Proposition 2 can be expressed with these state variables, this section specializes to log-utility. Appendix B considers other CRRA utility functions, which require a small change in the procedure below. The appendix also shows that one can easily allow for differences in risk aversion and discount rate between workers and entrepreneurs.

Denote total employment by $m(t) := \int_{-\infty}^{t} n(\tau, t)d\tau$. The law of motion for $m(t)$ is

$$\dot{m}(t) = -\delta m(t) + \mu\big(\theta(t)\big)\big(1 - m(t)\big). \tag{7}$$

Employment decreases due to separation shocks at rate $\delta m(t)$ and increases via hiring at rate $\mu\big(\theta(t)\big)\big(1 - m(t)\big)$.

In the beginning of an employment relationship each worker is assigned a consumption share a, which is fixed for the duration of the job. Denote an employment weighted average of these consumption shares in the existing workforce as $\Phi(t) := \frac{1}{m(t)} \int_{-\infty}^{t} n(\tau, t)a(\tau)d\tau$. The law of motion for $\Phi(t)$ is

$$\dot{\Phi}(t) = \frac{\mu\big(\theta(t)\big)\big(1-m(t)\big)}{m(t)}[a(t)-\Phi(t)]. \tag{8}$$

The average consumption share of workers increases when new hires get an above-average consumption share. The impact of this increase depends on the measure of new hires relative to total workforce.

To express the equilibrium conditions in Proposition 2 using these two state variables, I define the auxiliary jump-variables:

$$X(t) := \int_0^\infty e^{-(\rho+\delta)s}[\log c(t+s) + \delta V^u(t+s)]ds - V^u(t),$$
$$Y(t) := \int_0^\infty e^{-(\rho+\delta)s}\frac{z}{c(t+s)}ds.$$

Variables $X(t), Y(t)$ have laws of motion[8]

$$(\rho+\delta)X(t) = \log c(t) - \log b - \mu\big(\theta(t)\big)[\frac{\log a(t)}{\rho+\delta} + X(t)] + \dot{X}(t), \tag{9}$$

$$(\rho+\delta)Y(t) = \frac{z}{c(t)} + \dot{Y}(t). \tag{10}$$

Using these variables, the conditions 2.-4. in Proposition 2 collapse to the algebraic equations

$$[Y(t) - \frac{a(t)}{\rho+\delta}] = \frac{(1-\alpha)}{\alpha}a(t)[\frac{\log a(t)}{\rho+\delta} + X(t)], \tag{11}$$
$$\kappa = q\big(\theta(t)\big)c(t)[Y(t) - \frac{a(t)}{\rho+\delta}], \tag{12}$$
$$c(t) = \frac{m(t)z - \kappa\theta(t)(1-m(t))}{1+m(t)\Phi(t)}. \tag{13}$$

Solving for equilibrium becomes equivalent to solving a homogenous system of differential equations (7)-(10) in predetermined states m, Φ and jump-states X, Y, along with the static non-linear equations (11)-(13) for a, c, θ. Since the economy is stationary, I look for a solution converging to a steady-state.

[8]We have $X(t) = \Delta t[u(c(t))+\delta V^u(t)]+e^{-(\rho+\delta)\Delta t}[X(t+\Delta t)+V^u(t+\Delta t)]-V^u(t) \approx \Delta t[u(c(t))+\delta V^u(t)]+X(t)-(\rho+\delta)\Delta t[X(t)+V^u(t)]+\Delta t[\dot{X}(t)+\dot{V}^u(t)]$, which implies $(\rho+\delta)X(t) = u(c(t))-\rho V^u(t)+\dot{V}^u(t)+\dot{X}(t)$. Combining this with the equation for $V^u(t)$ implies the equation in the text.

Lemma 2. *1. The system* (7)-(13) *has a unique steady-state.*

2. In the neighborhood of the steady-state, equations (11)-(13) *implicitly define continuously differentiable functions* $\tilde{a}(X,Y)$, $\tilde{c}(m,\Phi,X,Y)$, $\tilde{\theta}(m,\Phi,X,Y)$.

3. The system (7)-(13) *has eigenvalues* $\lambda_1 \ldots \lambda_4$ *s.t.* $\lambda_1 < -\delta < \lambda_2 < 0 < \lambda_3 = \rho < \rho + \delta < \lambda_4$ *if* $\frac{z}{\kappa(1-\alpha)} > \frac{\mu(\theta)+\rho+\delta}{\mu(\theta)+\rho+2\delta}\frac{\rho+2\delta}{\mu(\theta)}$.

The sufficient condition for the eigenvalues[9] holds for example if $\alpha \geq \frac{1}{2}$ or $\frac{b}{z} \geq \frac{1}{2}$. This implies a uniqueness result:

Proposition 3. ***Unique Equilibrium Path*** *Given initial values* (m, Φ) *close enough to the steady-state and parameters satisfying the condition in Lemma 2, generically there is a unique equilibrium path converging to the steady-state.*

The next section discusses the economics of these state variables, in particular the impact of wage commitments, as represented by Φ, on employment.

3.2 The Slow Dynamics of Wage Commitments and Their Impact on Employment

The equilibrium has an interesting parallel with traditional macroeconomic analysis of labor markets, where wage rigidities play an important role in explaining changes in employment. Here optimal contracts generate a form of wage rigidity in the sense that upon hiring, employers commit to paying each worker a fixed share of their own consumption at all times. The total extent of these promises affects affects the hiring of new workers – both the wages they get and how much hiring takes place. Wages of existing workers have an impact on unemployment, a feature which is absent from the standard Mortensen-Pissarides model. Quantitatively, the dynamics of wage commitments are slow, well beyond the business cycle time frame. Hence, wage commitments generally do not adjust to fully accommodate business cycle frequency changes in aggregate conditions. To illustrate these dynamics, as well as the impact of wage commitments on employment, I calibrate the model.

Calibration The calibration is consistent with that of Shimer (2005a), though the thrust of the results is not particularly sensitive to reasonable changes in parameters. Time units are chosen to be months. The monthly discount rate is set to $\rho = 0.05/12 = 0.004$. Productivity z is normalized to one. To be consistent with the empirical separation and job-finding rates I set $\delta = 0.035$ and calibrate to make sure $\mu(\theta) = 0.46$. Lack of good data on vacancies

[9] The constraints have to do with separating the two negative roots. A nicer condition can probably still be found.

leaves the vacancy cost associated with a given level of θ indeterminate. This can be solved as follows: One chooses the units of vacancies such that $\theta = 1$. To be consistent with the job-finding rate, one then sets $k = 0.46$. The vacancy cost is used as a free parameter to make sure the zero profit condition: $\kappa = q(1)\frac{z-w}{\rho+\delta}$ holds for any level of w. The matching function elasticity is set to $\alpha = 0.72$ following Shimer (2005a). I consider alternative configurations of b, γ. How b should be calibrated is not immediate, but the empirical evidence on how much consumption falls upon unemployment is as follows: Aguiar and Hurst (2005) report that food consumption drops by 5% for the unemployed, Gruber (1994) and Stephens (2001) report food expenditures dropping by $7 - 10\%$, Browning and Crossley (2001) report a broader set of consumption expenditures dropping by 14%.

The dynamics of the model are characterized by a dichotomy in speeds of convergence. Employment dynamics are strongly affected by the faster eigenvalue of the system, which has a half life of approximately 1.5 months in the calibrated model (independent of b, γ). This is analogous to the employment dynamics of the Mortensen-Pissarides model, where employment in the sole state variable. The second eigenvalue can be associated with the dynamics of wage commitments Φ. When varying b between $0.4 - 0.9$ and γ between $0 - 5$, the half-life of this slower eigenvalue varies between $50 - 7000$ months, so the dynamics are clearly beyond business cycle frequency.[10] One would expect these dynamics to be relatively slow because: i) Wage commitments only change through turnover in the workforce and jobs last on average 2-3 years. ii) Because of feasibility entrepreneurs may be constrained in their ability to make large changes to the wages of new workers relative to existing ones.

How does Φ affect employment dynamics? The variable characterizes the extent of the entrepreneur's wage costs and so affects entrepreneurial consumption. Suppose these wage commitments are high relative to steady-state today, but employment is at steady-state. The entrepreneurial sector will reduce their level over time as it hires new workers. The speed of adjustment is constrained by the rate of turnover in the workforce as well as the infeasibility of making large changes at once. This adjustment allows entrepreneurial consumption to rise over time, which is reflected in an interest rate that is initially high and decreases over time. A high interest rate implies investment into vacancy creation is less profitable. This leads to a fall in the vacancy-unemployment ratio, which in turn causes a drop in hiring and leads to a rise in unemployment.

Figure 1 illustrates this for the calibrated model, contrasting linear and concave preferences. Suppose that initially Φ is 10% above steady-state.[11] When agents have linear preferences, the adjustments in consumption caused by changes in Φ have no impact on the interest rate nor the profitability of investment. Hence employment is unaffected. With concave preferences, the figure shows the high interest rate associated with rising consumption, as

[10] For the faster root higher b generally speeds up convergence while higher γ slows it down. For the slower root higher b generally slows convergence while higher γ speeds it up.

[11] Note first that with $\gamma = 4$, a two percent decrease in productivity leads to roughly 0.3 percent decrease in the steady-state employment level and a 10 percent increase in the steady-state level of Φ.

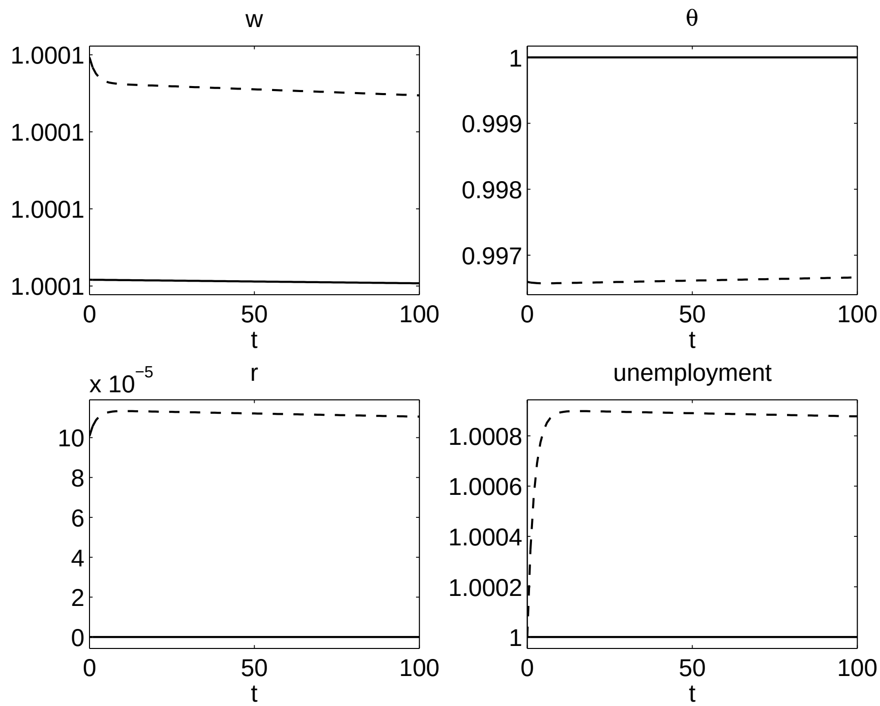

Figure 1: Effect of wage commitments on employment

Notes: The figure shows the adjustment path to steady-state when initially Φ is ten percent above steady-state and employment is at steady-state value, for linear utility (solid line) and risk aversion $\gamma = 4$ (dashed line). All variables are reported as percentage deviations from steady-state values, apart from the interest rate which is in percentage points. $b = 0.8$. The wage refers to the aggregate wage i.e. the cross sectional average.

well as the resulting drop in the vacancy-unemployment ratio and rise in unemployment. Over time all variables return to the steady-state level.

Two quantitative observations deserve mention: i) The dynamics in unemployment induced by the consumption share variable are very slow and suggestive of a hysteresis effect. ii) In terms of magnitudes, the effects of Φ are small. Because of this, in the calibration discussed, these dynamics are overshadowed.

3.3 Connection to the Mortensen-Pissarides Model

There is a close relationship between this model and the standard Mortensen-Pissarides model. This is because the contract posting equilibrium has a close relationship with the equilibrium of of an economy with random search and bilateral wage bargaining, as usually seen in the context of the Mortensen-Pissarides model with linear preferences. In particular, if i) the Hosios (1990)-condition holds, and ii) the bargaining takes place over the same set of contracts as in the posting equilibrium, then the two are equivalent. Because of this, in the risk neutral limit with $\gamma = 0$, the model inherits the employment dynamics of the Mortensen-Pissarides model. One can easily relate the changes in employment dynamics stemming from incomplete markets to that benchmark by varying the preference parameter γ. This makes the quantitative results shown here directly comparable to existing literature discussing the dynamics of the Mortensen-Pissarides model, as it is very common in that literature to impose the Hosios (1990)-condition. I show this connection between the models in Appendix E.

The next section returns to the case with aggregate productivity shocks and examines the quantitative impact of incomplete markets on the business cycle dynamics of the model.

4 Volatility of the Labor Market

This section examines quantitatively the impact of incomplete markets on the amplification properties of the model, and whether it can explain the unemployment-volatility puzzle. I show that incomplete markets work in the right direction by increasing the volatility of the vacancy-unemployment ratio, but that the ability of this mechanism to explain the large discrepancy between model and data is limited. I then explore the sources of the amplification to see what are the roles of i) concavity in preferences *per se* versus incomplete markets, and ii) differences in preferences between workers and entrepreneurs. The presence of aggregate shocks doesn't change the way the state space of the problem reduces and Appendix C discusses solving the model in this case.

4.1 Amplification from Incomplete Markets

The following proposition suggests that incomplete markets will amplify the response of the vacancy-unemployment ratio to productivity shocks and dampen that of wages. The proof is in Appendix D.

Proposition 4. *Consider the steady-state of an economy without aggregate shocks. As the steady-state productivity increases, wages and the vacancy-unemployment ratio increase. As agents' willingness to substitute across time and states decreases, the increase in wages becomes muted.*

In an economy without aggregate shocks, workers periodically transition between low consumption during unemployment and higher consumption during employment. The steady-state wage level and market tightness depend on the preferences of the unemployed. When workers are less willing to substitute, they become more willing to accept a lower wage in return for finding a job faster. An increase in steady-state productivity increases both the wage and market tightness. However, when workers are less willing to substitute consumption, their gain in utility from getting hired with a high wage diminishes. This shifts the benefits of increased productivity away from wages and toward market tightness. These last two changes in the responses of wages and market tightness are exactly what one would hope to find in the dynamic model, to bring the model closer to data.

Simulation results from the model with aggregate shocks not only show that the steady-state responses predict correctly the direction of change in the volatilities of wages and market tightness, but they are also quantitatively informative of the magnitudes of these changes. This is perhaps due to the productivity shocks being relatively persistent compared to the speed of the dynamics of these variables in the model.[12]

Turning to the simulation results of the model with aggregate shocks, Figure 2 shows the impact of incomplete markets on the business cycle volatility that the model produces, changing the willingness of agents to substitute consumption. The point of reference is the darkest line, which corresponds to the case of linear preferences and in which case access to asset markets is irrelevant.[13] I focus on wages and the vacancy-unemployment ratio and examine two alternative measures of volatility: the standard deviation relative to that of labor productivity and the elasticity with respect to labor productivity. The second measure takes into account the fact that while in the model wages and the vacancy-unemployment ratio are perfectly correlated with productivity, in the data the correlations are lower (0.6 for wages and 0.4 for the vu-ratio).

The figure shows how the volatility in the vacancy-unemployment ratio increases as the drop in consumption upon unemployment decreases (b/w is the consumption of the unemployed relative to the steady-state wage). When workers are more indifferent about being unemployed, the surplus from matching is smaller and productivity variation causes bigger percentage variation in the surplus. Hence the vacancy-unemployment ratio varies more in response. This is familiar from the literature, e.g. Hagedorn and Manovskii (2006).

Decreasing the willingness of agents to substitute consumption across time and states decreases the volatility of wages, and increases the volatility of the vu-ratio. This moves the model in the right direction relative to data on both counts. Despite a significant increase in the volatility of the vu-ratio, the difference between model and data is too large to be explained by the incomplete markets mechanism, however. In particular, incomplete markets make less of a difference when workers are relatively indifferent between working and not, in which case the difference between model and data is smaller.[14]

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[12]Shimer (2005a) and Nagypál and Mortensen (2005) utilize this approach to evaluate mechanisms of amplification.

[13]To place the results in the context of the literature, note that the linear utility case corresponds exactly to the standard Mortensen-Pissarides model (see Appendix E).

[14]This result is not sensitive to the high transition rates characterizing the US labor market. A calibration exercise in the spirit of Blanchard and Portugal (2001) setting the job-finding rate and separation rate to a

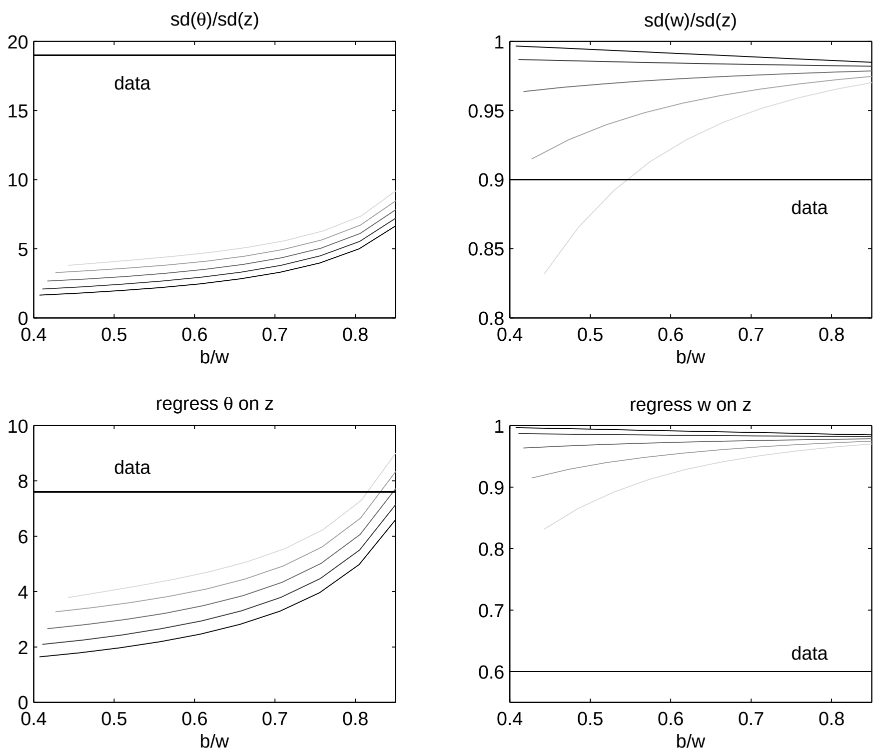

Figure 2: Volatility of the labor market under incomplete asset markets

Notes: The figure plots volatility measures based on simulations of the incomplete markets model, comparing them to data. See Appendix C for details on the simulations and data. Risk aversion levels $\gamma = 0, 1, 2, 3, 4$, grey color fades out with higher γ. The wage refers to the aggregate wage i.e. the cross sectional average. As b, γ vary, the steady-state is re-calibrated to match observed unemployment.

The next two sections explore the origins of this amplification, separating the roles of concave preferences versus asset market structure, as well as differences between the preferences of workers and entrepreneurs.

4.2 Market Structure versus Willingness to Substitute

The previous section examined how decreasing the willingness of agents to substitute over time and states affects the equilibrium of the incomplete markets economy. Because the existing literature generally uses linear preferences, in which case access to asset markets is irrelevant, this is a useful exercise to see how results change when agents with concave preferences face incomplete markets. It does not answer the question: For given concave preferences, what is the impact of being able to access to asset markets for the dynamics

third of the above values would lead to the same conclusion.

of the model? To examine this I construct a corresponding complete markets economy where not only are workers able to access asset markets, but they can also insure away the idiosyncratic unemployment risk.[15]

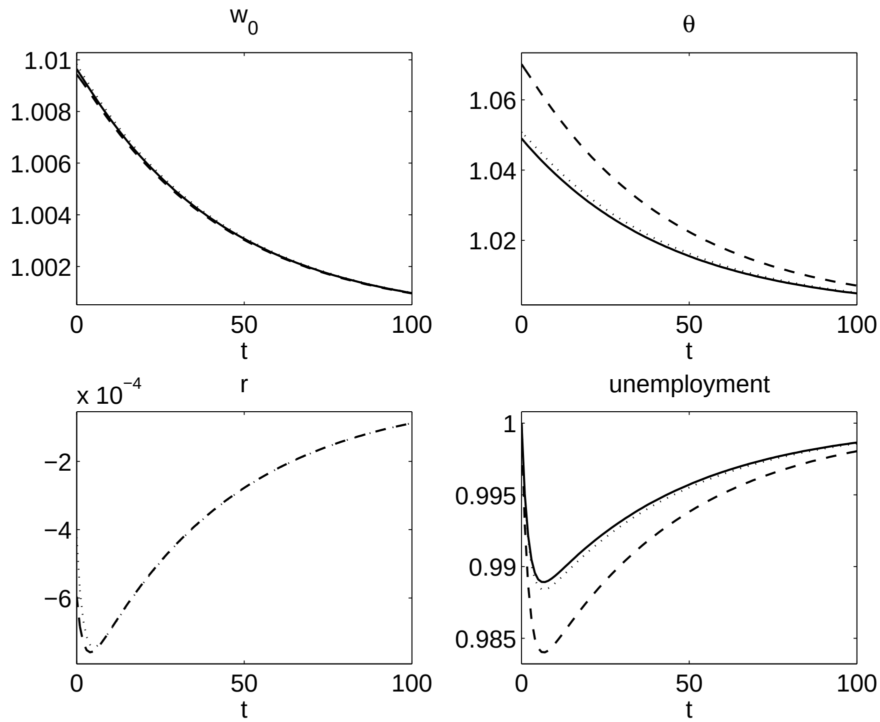

Figure 3: Impulse responses in three environments

Notes: The figure plots impulse responses of endogenous model variables to a one percent positive productivity shock. See Appendix C for details. All variables are reported as percentage deviations from steady-state values, apart from the interest rate which is in percentage points. Three cases: i) the incomplete markets model with linear preferences (solid line), ii) the incomplete markets model with risk aversion $\gamma = 4$ (dashed line), and iii) the complete markets model with risk aversion $\gamma = 4$ (dotted line). $b = 0.8$. Under complete markets, the plotted wages are based on continual re-bargaining. Under incomplete markets, the wage refers to the starting wage in a new job. Time is in months.

Figure 3 compares impulse responses to a labor productivity shock in three cases: i) when agents have linear preferences (and market structure doesn't matter), ii) when agents have CRRA preferences and workers face complete markets, and iii) when agents have CRRA preferences and workers face incomplete markets. In the figure productivity z starts one percent above steady-state and the other state variables m, Φ at steady-state. Since z is mean reverting the conditional expectation $E[z(t)|z(0)]$ returns to steady-state over time. The

[15]Appendix F shows how to solve this case, which is not novel to the literature. Examples include (Merz 1995) and (Andolfatto 1996).

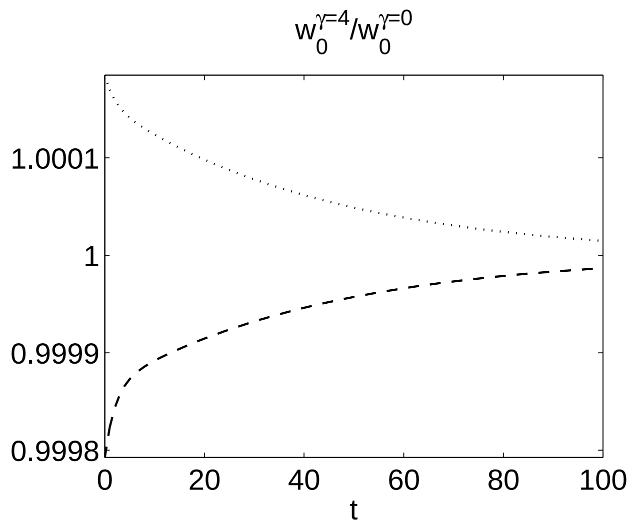

Figure 4: Effect of concavity in preferences on wage responses

Notes: The figure re-plots the wages of Figure 3. The dashed line is the starting wage level in the incomplete markets model under risk aversion $\gamma = 4$ relative to the same under linear utility. The dotted line is the starting wage level in the complete markets model under risk aversion $\gamma = 4$ relative to the same under linear utility. In the complete markets case the wages shown are based on continual re-bargaining. $b = 0.8$. Time is in months.

figure plots the time paths of conditional expectations of starting wages, market tightness, interest rate and unemployment in response to this.

To understand the figure, note that the model works as follows: Vacancy creation takes place under a zero profit condition. This means that in response to increased productivity, entrepreneurs will either increase the starting wage level, or increase the measure of vacancies per unemployed (which leads to increased hiring costs). Both actions make unemployed workers better off. The figure shows a combination of these two responses: both wages and the vacancy-unemployment ratio rise on impact. The high vacancy-unemployment ratio accelerates hiring, leading to a fall in unemployment. When agents have non-linear preferences, the interest rate responds as well, here falling on impact.

When workers have access to complete markets, concavity in preferences changes the dynamics quantitatively relatively little. The interest rate responds to the positive productivity shock by falling, enhancing further the profitability of investing and leading to amplified responses of both starting wages and market tightness. Figure 4 re-plots the wages in Figure 3 to show the direction of change in wages. The magnitude of the change in the wage figure appears very small compared to the differences in market tightness. This is explained by the fact that profits per worker are also very small, so small differences in wages translate into larger percentage differences in profits.

When workers are unable to smooth consumption through asset markets, concavity in preferences amplifies the response of the vacancy-unemployment ratio more strongly, while the response of the starting wage level is now dampened (see Figure 4). This outcome can be viewed as a combination of two effects: First, when agents dislike substituting consumption over time and states, the interest rate is affected by shocks and this affects the profitability of investment. This amplifies the responses of wages and θ to shocks, but the magnitude of this effect is relatively small and more generally the direction is ambiguous.[16] Second, entrepreneurs respond to the workers' preference for consumption smoothing by reducing the response of the starting wage to shocks. As starting wages respond less to productivity changes, profits from new employment relationships respond more, and so does vacancy creation. This is the effect referred to in Proposition 4 on how the steady-state wage level responds to changes in the steady-state productivity when workers dislike substituting consumption across time and states. The incomplete markets outcome in Figure 3 combines these two effects. For θ they work in the same direction, amplifying the response of θ to shocks. For wages they work in opposite directions, but on net wage responses are dampened. The consumption smoothing effect appears to dominate quantitatively.

Figure 5 illustrates these effects for simulated moments across a broader range of parameter values. Under complete markets the impact of decreasing willingness to substitute is relatively small.

4.3 Preference Heterogeneity

One reason why wealth is concentrated in the hands of the entrepreneurial sector could be that entrepreneurs are more patient than workers. Extending the model to allow for differences in discount rates is straightforward.[17] Will such differences have a significant impact on the amplification properties discussed? I find that doubling the discount rate of workers does not substantially affect the conclusions.

A tradition in economics dating back to Knight (1921) argues that through selection workers are more risk averse than entrepreneurs (see esp. Kihlstrom and Laffont (1979)). The model is straightforward to extend to allow also for such differences.[18] To examine the roles of worker versus entrepreneur risk aversion, I repeat the simulations varying the risk aversion levels of the two parties separately. The results show that entrepreneurial risk aversion matters little for the volatility of the vu-ratio. The amplification resulting from decreasing the willingness of agents to substitute shown in Figure 2 is effectively due to changes in workers' preferences. This is consistent with the idea that responses of the steady-state are informative of the dynamic responses as well. Only the workers' willingness to substitute affects this steady-state relationship.

[16]With more persistence in productivity, the interest rate rises on impact.
[17]See Appendix B.
[18]See Appendix B.

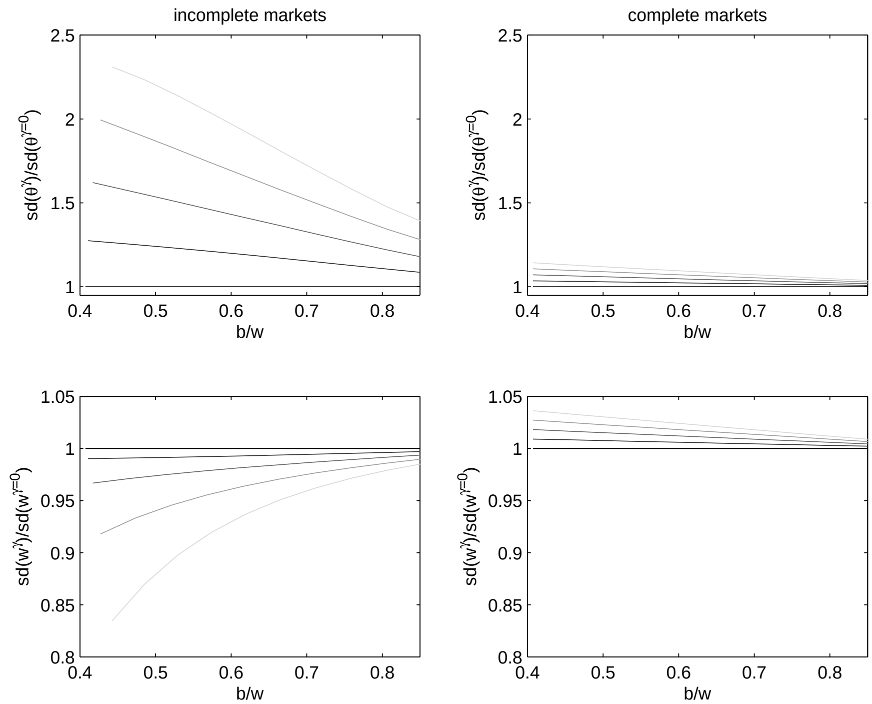

Figure 5: Effect of decreasing willingness to substitute on volatility under complete vs. incomplete markets

Notes: The figure plots volatility measures based on simulations of the incomplete and complete markets models. See Appendix C for details on the simulations. Risk aversion levels $\gamma = 0, 1, 2, 3, 4$, grey color fades out with higher γ. In the complete markets case the wages are based on continual re-bargaining. The wage refers to the aggregate wage i.e. the cross sectional average. As b, γ vary, the steady-state is re-calibrated to match observed unemployment.

Entrepreneurial risk aversion is an important factor for the cyclicality of aggregate wages however, because the degree of risk sharing in contracts depends on the relative risk aversion levels of the two parties. If entrepreneurs are risk neutral and workers risk averse, contract wages are constant over time and the model produces extremely rigid aggregate wages (by aggregate wage I refer to the cross sectional average wage).[19] By varying entrepreneurial risk aversion from zero to the worker risk aversion level, the model produces wage cyclicality ranging from negligible to high relative to data.

It may seem puzzling that the cyclicality of of the aggregate wage appears to have no impact on the volatility of the vu-ratio, whereas the previous sections argued that changes in wage responses are closely linked to changes in the responses of the vu-ratio. The cyclicality of the aggregate wage is affected by two separate factors: i) the cyclicality of wages within a contract, and ii) the cyclicality of the starting wage level. The cyclicality of vacancy creation

[19] This setting is explored in Rudanko (2007).

hinges on the latter, because the wage level of new workers is important for determining the present value of profits that a filled vacancy can create, and hence how much vacancy creation takes place. Entrepreneurial risk aversion appears to have quantitatively only a small effect on this compared to worker risk aversion. On the other hand, entrepreneurial risk aversion is important for the cyclicality of wages within a contract, and hence also the aggregate wage.[20]

The ability of workers and entrepreneurs to commit to wage contracts appears to be important for the equilibrium. Depending on the preferences of the two parties, this is not necessarily the case, however. When both parties have the same preferences, risk sharing implies that wages within each contract are relatively procyclical. This means that wages rise in booms, when the workers' outside opportunities of looking for a new job improve. Wages decrease in downturns, when entrepreneurs face declining profitability. In fact, when checking numerically whether the parties would prefer to leave their contract as aggregate conditions vary in the simulations, I find that the contracts generally are self-enforcing when preferences are identical.[21] If entrepreneurial risk aversion were low relative to worker risk aversion, more wage smoothing would take place, and the assumption of commitment would be more restrictive.[22]

5 Related Literature on Incomplete Markets

Empirical evidence suggests that in modelling the behavior of unemployed individuals, an assumption of complete consumption insurance against unemployment risk is not appropriate. Even if the average unemployment spell is relatively short, workers appear not to hold much wealth to protect themselves against this risk. Mankiw and Zeldes (1991) documented that only a fraction of all households hold financial wealth that would allow for short term consumption smoothing. A literature exploring whether individuals face liquidity constraints, e.g. Hall and Mishkin (1982), Zeldes (1989), Campbell and Mankiw (1990) and others, suggests that a significant fraction of the population is subject to such constraints. In particular, Chetty (2006) focuses on the pool of unemployed, showing that in the US approximately half of unemployment benefit claimants held no liquid wealth at the time of job loss and arguing that their behavior is suggestive of liquidity constraints. Jappelli (1990) documents that the unemployed are less likely to have access to credit than the average consumer.

Several recent papers examine the effects of incomplete markets faced by workers on business

[20]The simulation figures of the previous two sections display changes in the aggregate wage, but since the preferences of the agents remained equal as the level of risk aversion was changed, the changes in the cyclicality of the aggregate wage can be viewed as reflecting changes in the cyclicality of starting wages rather than within contract wages.

[21]In the sense of Thomas and Worrall (1988).

[22]The dynamics of self-enforcing contracts in this case are explored in Rudanko (2007).

cycles in the labor market: Bils, Chang, and Kim (2007), Costain and Reiter (2005), Krusell, Mukoyama, and Sahin (2007) and Shao and Silos (2007).[23] The papers complement the analysis here in a useful way. The clearest departure between this paper and the others is that they allow workers to save while I do not.[24] This assumption delivers significant parsimony, allowing me to solve the model without the black box of heavy computational machinery.[25] The assumption is meant to capture the fact that unemployed workers hold little wealth, and so job loss is relatively painful, while the other papers face the challenge of endogenously producing wealth distributions reflecting this. All these papers find market incompleteness to have a quantitatively small amplifying effect on unemployment and vacancy creation. Given the extreme form of incomplete markets in this paper, one could view my results as an upper bound on how far incomplete markets can go in explaining the volatility puzzle.

6 Conclusions

This paper presented a parsimonious equilibrium model of frictional labor markets where risk averse workers and entrepreneurs differed in their ability to access capital markets. Entrepreneurs used long term contracts to attract workers, providing them consumption smoothing through the contract, but were unable to insure workers against the idiosyncratic risk of job loss. Despite a seemingly intractable state space, the equilibrium could be characterized with just two state variables: employment and a measure of wage commitments.

The paper explored whether incomplete markets can resolve the unemployment-volatility puzzle facing search and matching models of unemployment. While the mechanism was shown to work in the right direction, it was found to have limited ability to account for the large discrepancy between model and data, despite the extreme form of market incompleteness faced by workers in the model. This suggests that incomplete markets cannot resolve the unemployment volatility puzzle.

An interesting application of the model would be to examine the positive and welfare effects of cyclical labor market policies, such as counter-cyclical benefits. The welfare effects of such policies are not clear, because although they provide workers insurance, they may also lead to increased volatility in the labor market.

[23]The effects of market incompleteness on steady states in an equilibrium model of job search have been studied by Acemoglu and Shimer (1999), Acemoglu and Shimer (2000) and Alvarez and Veracierto (2001).

[24]One justification for this could be that workers are more impatient than entrepreneurs, something I allow for in the paper.

[25]Solving for an equilibrium with saving is complicated by the fact that heterogeneity in wealth holdings leads to heterogeneity in labor market outcomes. In particular individual wages are generally affected by individual wealth.

Appendix A Proofs

Proof of Lemma 1: *Equilibrium Properties*
Lemma 1 is implied by the following Lemmas 3-6.

First, assume an equilibrium exists. As discussed in the text, the choice of contract at any point t solves the problem (P1) in the text. This problem separates into a problem of finding the structure of optimal wage contracts (P1.a) and a problem of finding the optimal level of wages given this structure (P1.b).

The wage contract problem reads: Given $\mathcal{F}_t, t \geq 0$, the processes $p(t+s), V^u(t+s)$, for all $s \geq 0$, and continuations $\mathcal{F}_{t+s}$, and a corresponding value $\bar{V} > V^u(t)$, choose $w^i(t, t+s)$, for all $s \geq 0$, and continuations $\mathcal{F}_{t+s}$, to

$$\begin{aligned} &\max E_t \int_0^\infty e^{-\delta s} p(t+s)[z(t+s) - w^i(t,t+s)]ds \\ &\text{s.t. } \bar{V} \leq E_t \int_0^\infty e^{-(\rho+\delta)s}[u\big(w^i(t,t+s)\big) + \delta V^u(t+s)]ds. \end{aligned} \tag{P1.a}$$

Lemma 3. *With equilibrium prices, the wage contract problem* (P1.a) *has a unique solution with the property*

$$e^{-\rho s}\frac{u'\big(w^i(t,t+s)\big)}{u'\big(w^i(t,t)\big)} = \frac{p(t+s)}{p(t)} \tag{14}$$

for all $t, s \geq 0$, *and* $\mathcal{F}_t \subset \mathcal{F}_{t+s}$.

Pf. This problem can be written as

$$\begin{aligned} &\min E_t \int_0^\infty e^{-\delta s} p(t+s) w^i(t,t+s) ds \\ &\text{s.t. } \underline{V} \leq E_t \int_0^\infty e^{-(\rho+\delta)s} u\big(w^i(t,t+s)\big) ds \end{aligned} \qquad \text{or} \qquad \begin{aligned} &\max E_t \int_0^\infty e^{-(\rho+\delta)s} u\big(w^i(t,t+s)\big) ds \\ &\text{s.t. } E_t \int_0^\infty e^{-\delta s} p(t+s) w^i(t,t+s) ds \leq \underline{B} \end{aligned}$$

This is a standard problem where the FOC characterize a unique optimum. *Q.E.D.*

Optimal wage contracts equate marginal rates of substitution between different dates and states to the corresponding price ratio.[26] Hence, once one knows the initial wage in a contract, future wages are pinned down as $w^i(t,t+s) = w^i(t,t)(e^{\rho s}\frac{p(t+s)}{p(t)})^{-\frac{1}{\gamma}}$ according to prices. Using the shorthand $w^i(t,t+s) = w^i(t,t) f(t,t+s)$, **the problem of the optimal wage level** reads: Given $\mathcal{F}_t, t \geq 0$ and the processes $p(t+s), V^u(t+s)$, for all $s \geq 0$, and continuations $\mathcal{F}_{t+s}$, choose $\theta^i(t), w^i(t,t)$ to

[26] Note that prices are weighted by probabilities in this notation.

solve

$$\max -p(t)\kappa + q(\theta^i(t))E_t \int_0^\infty e^{-\delta s} p(t+s)[z(t+s) - w^i(t,t)f(t,t+s)]ds \tag{P1.b}$$

s.t.

$$\rho V^u(t) = u(b) + \mu\big(\theta^i(t)\big)\big[E_t \int_0^\infty e^{-(\rho+\delta)s}[u\big(w^i(t,t)f(t,t+s)\big) + \delta V^u(t+s)]ds - V^u(t)\big] + \eta\big[E_{t,+}V^u(t) - V^u(t)\big] + \frac{d}{dt}V^u(t).$$

Lemma 4. *For equilibrium $p(.), V^u(.)$, the wage level problem* (P1.b) *has a unique solution, characterized by*

$$F_p(t) = \frac{1-\alpha}{\alpha}\frac{[V(t) - V^u(t)]}{u'(w(t,t))},$$

where

$$F_p(t) := E_t \int_0^\infty e^{-\delta s}\frac{p(t+s)}{p(t)}[z(t+s) - w(t,t+s)]ds,$$
$$V(t) := E_t \int_0^\infty e^{-(\rho+\delta)s}[u\big(w(t,t+s)\big) + \delta V^u(t+s)]ds.$$

Pf. In equilibrium $V^i - V^u \geq 0$, where V^i is the value to a worker from signing a new contract i. Denote starting wages by $\hat{w}$ and define the net utility to a worker from a new contract as

$$g(\hat{w},t) := E_t \int_0^\infty e^{-(\rho+\delta)s}[u\big(\hat{w}f(t,t+s)\big) + \delta V^u(t+s)]ds - V^u(t),$$

for $\hat{w}$ large enough s.t. $g(\hat{w},t) \geq 0$. The function g is strictly increasing in the wage. For equilibrium contracts $V^i(t) - V^u(t) = g(w^i(t,t),t) \geq 0$. Define entrepreneurial profit $h(\hat{w},t) := \int_0^\infty e^{-\delta s}p(t+s)[z(t+s) - \hat{w}f(t,t+s)]ds$. For equilibrium contracts $h(w^i(t,t),t) > 0$. If an equilibrium exists there must be a range of starting wages $(\underline{w}, \overline{w})$ where one can solve for the corresponding $\hat{\theta}$ from the unemployment value equation and have positive profits. Substituting this into the maximand and differentiating w.r.t $\hat{w}$ we get:

$$\bar{V}(t)^{-\frac{\alpha}{1-\alpha}} g(\hat{w},t)^{\frac{\alpha}{1-\alpha}}\big[\frac{\alpha}{1-\alpha}\frac{g_1(\hat{w},t)}{g(\hat{w},t)}h(\hat{w},t) - E_t \int_0^\infty e^{-\delta s}p(t+s)f(t,t+s)ds\big]$$

where $\bar{V}(t) := \rho V^u(t) - u(b) - \frac{d}{dt}V^u(t)$. Within $(\underline{w}, \overline{w})$, the term in brackets is strictly decreasing. Close to $\underline{w}$ it is positive and close to $\overline{w}$ it is negative. There is hence only one optimal starting wage, and it must set the derivative to zero, delivering the equation in the lemma. *Q.E.D.*

Hence in equilibrium all entrepreneurs offer the same contract: $\theta^i(t) = \theta(t), \sigma^i(t) = \sigma(t)\ \forall \mathcal{F}_t, t \geq 0, i$. Because this holds independent of initial conditions, even if the economy started out with non-optimal contracts, eventually it would converge toward having only contracts of the optimal form.

The lemma characterizes how entrepreneurs solve the tradeoff between attracting workers through higher wages versus faster employment. The outcome depends on the elasticity of the matching function, which reflects how changes in θ affect job-finding and worker-finding rates μ, q. It also depends on how the wage level affects the worker's and entrepreneur's surpluses from matching. The ratio of the elasticities of μ and q with respect to θ is $\frac{1-\alpha}{\alpha}$. The ratio of elasticities of worker and firm surplus with respect to w is

$$\frac{\frac{d}{dw}\log(V - V^u)}{-\frac{d}{dw}\log(F_p)} = \frac{u'(w)F_p}{V - V^u}.$$

The optimal choice equalizes these ratios.

Lemma 5. *In equilibrium:* $\kappa = q\big(\theta(t)\big)F_p(t)$ *for all* $\mathcal{F}_t, t \geq 0$.

Pf. If a positive measure of vacancies is to be posted, profits from posting a vacancy must be non-negative: $-\kappa + q\big(\theta(t)\big)F_p(t) \geq 0$. Vacancies enter into the budget constraint linearly. If profits from vacancy posting were strictly positive, an infinite amount of vacancies would be posted, θ would be infinitely large and hence $q(\theta)$ would be zero. If the present value of output is bounded, as it should be in equilibrium, the present value from a contract, $F_p(t)$, is bounded from above. Then $-\kappa + q\big(\theta(t)\big)F_p(t) < 0$ and we have a contradiction. Hence $-\kappa + q\big(\theta(t)\big)F_p^i(t) = 0$. *Q.E.D.*

In equilibrium entrepreneurs must be indifferent between investing into vacancies or the asset market and so the present value of posting vacancies must be zero. The linear technology implies it is not pinned down which entrepreneur actually hires workers.

The second part of the entrepreneur's problem (P) is **the consumption problem**:
Given prices $p(t) > 0$, for all $\mathcal{F}_t, t \geq 0$,

$$\max_{c^i(t)} E_0 \int_0^\infty e^{-\rho t} u(c^i(t))dt \tag{P2}$$

$$\text{s.t. } E_0 \int_0^\infty p(t)c^i(t)dt = W^i_{TOT}, \text{ with}$$

$$W^i_{TOT} := E_0 \int_0^\infty p(t) \int_{-\infty}^0 n^i(\tau, t)[z(t) - w^i(\tau, t)]d\tau dt + W^i_0 \text{ given.}$$

Lemma 6. *As long as initial wealth* W^i_{TOT} *is not negative, given equilibrium prices, the consumption problem* (P2) *has a unique solution, characterized by*

$$e^{-\rho s}\frac{u'\big(c^i(t+s)\big)}{u'\big(c^i(t)\big)} = \frac{p(t+s)}{p(t)} \text{ for all } t, s \geq 0, \mathcal{F}_t \subset \mathcal{F}_{t+s}.$$

Pf. This is a standard problem, where a unique bounded optimum is characterized by the FOC. *Q.E.D.*

The entrepreneurs trade in complete asset markets, choosing to equate their marginal rates of substitution to corresponding price ratios.

Definition 2. A *representative entrepreneur equilibrium* consists of, for all $\mathcal{F}_t, t \geq 0$, prices $p(t) > 0$, unemployment values $V^u(t)$, entrepreneur beliefs $\Theta : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_+$, consumption $c(t) \geq 0$, measure of vacancies posted $v(t) \geq 0$, contract offered $\sigma(t)$, vacancy-unemployment ratio $\theta(t)$, and resulting labor force $n(\tau, t) \geq 0$ for $\tau \leq t$ with economy-wide unemployment $n_u(t) = 1 - \int_{\tau \leq t} n(\tau, t) d\tau$ such that

1. Given $\{p(t), \Theta(., t)\}$, for all $\mathcal{F}_t, t \geq 0$ the allocation $\{c(t), v(t), \sigma(t), n(\tau, t)\}$ solves the entrepreneur's problem (P) for all $\mathcal{F}_t, t \geq 0$.
2. The entrepreneur's beliefs about worker behavior are consistent with worker optimization: Define $V^u(.)$ by

$$\begin{aligned}&\rho V^u(t)\\ &= u(b) + \mu\big(\theta(t)\big)\big[E_t \int_0^\infty e^{-(\rho+\delta)s}[u(w(t, t+s)) + \delta V^u(t+s)]ds - V^u(t)\big]\\ &\qquad + \eta\big[E_{t,+}V^u(t) - V^u(t)\big] + \frac{d}{dt}V^u(t),\end{aligned}$$

   for any $\mathcal{F}_t, t \geq 0$. The belief function is defined implicitly such that for any $\sigma(t) \in \Sigma(t)$ and $\mathcal{F}_t, t \geq 0$, the value $\theta(t) = \Theta(E_t \int_0^\infty e^{-(\rho+\delta)s} u(w(t, t+s))ds, t)$ satisfies

$$\begin{aligned}&\rho V^u(t) = u(b)\\ &+ \mu\big(\theta(t)\big)\big[E_t \int_0^\infty e^{-(\rho+\delta)s}[u(w(t, t+s)) + \delta V^u(t+s)]ds - V^u(t)\big]\\ &\qquad + \eta\big[E_{t,+}V^u(t) - V^u(t)\big] + \frac{d}{dt}V^u(t).\end{aligned}$$

3. Goods market clearing: $\int_{-\infty}^t n(\tau, t)(z(t) - w(\tau, t))d\tau - c(t) - \kappa v(t) = 0$, for all $\mathcal{F}_t, t \geq 0$ where $w(\tau, t)$ is specified by $\sigma(\tau)$.
4. Labor market clearing: $\frac{v(t)}{\theta(t)} = n_u(t)$, for all $\mathcal{F}_t, t \geq 0$.

Proof of Proposition 1 ***Aggregation***
(a) Aggregating up the multi-agent problem allocations for c^i, v^i, n^i along with the economy-wide θ, σ one obtains a feasible possible solution to the representative entrepreneur's problem with the initial conditions described in Proposition 1. This possible solution satisfies the optimality conditions and other equilibrium conditions of the representative entrepreneur equilibrium, so it must constitute such an equilibrium.

(b) Each entrepreneur in the multi-agent problem consumes a constant fraction of the aggregate over time: $c^i(t) = d^i c(t)$. The budget constraint pins down the individual d^i according to wealth and income: $d^i E_0 \int_0^\infty p(t)c(t)dt = W_0^i + E_0 \int_0^\infty \int_{-\infty}^0 p(t)n^i(\tau, t)[z(t) - w^i(\tau, t)]d\tau dt$. Given a representative entrepreneur equilibrium and individual initial conditions, one can then back out an equilibrium for the multi-agent case. *Q.E.D.*

Proof of Proposition 2 ***Characterization***
(a) Assume a representative entrepreneur equilibrium exists. Condition 1 holds because Lemma 1, part two implies that the growth rate of wages across time and states within any contract equals that

of entrepreneurial consumption. Condition 2 holds because of Lemma 4 and Lemma 6. Condition 3 holds because of Lemma 5 and Lemma 6. Condition 4 is the goods market clearing condition combined with labor market clearing. Condition 5 is the dynamic equation for unemployment value. Condition 6 is the dynamic equation for unemployment.

(b) Given variables $V^u(t), c(t), \theta(t), \sigma(t), n_u(t)$ satisfying the conditions of Proposition 2 for all $t \geq 0$, one can use the consumption path $c(t)$ to construct prices for which $c(t), \theta(t), \sigma(t)$ satisfy the optimality conditions of the agent ($\frac{p(t+s)}{p(t)} := e^{-\rho s}\frac{u'(c(t+s))}{u'(c(t))}$) and with $v(t) = \theta(t)n_u(t)$, these are budget feasible for the agent as well as clear markets. *Q.E.D.*

Proof of Lemma 2 1. Reduce the steady state equations into one equation in the wage:

$$\frac{z-w}{\rho+\delta}[\rho+\delta+k(\frac{z-w}{\rho+\delta}\frac{k}{\kappa})^{\frac{1-\alpha}{\alpha}}] = \frac{1-\alpha}{\alpha}w\log(w/b).$$

This equation has a unique solution in (b, z). The other variables can be expressed as functions of the wage:

$$\theta = q^{-1}(\frac{\kappa(\rho+\delta)}{z-w}), \quad m = \frac{\mu(\theta)}{\mu(\theta)+\delta}, \quad c = \frac{\rho\kappa}{q(\theta)+\delta/\theta}, \quad a = \Phi = \frac{w}{c}.$$

2. Equation (11) determines a: $\frac{Y}{a} = \frac{1-\alpha}{\alpha}(\frac{\log a}{\rho+\delta} + X) + \frac{1}{\rho+\delta}$. Given that Y is positive, there is a unique strictly positive solution for any X. In steady-state $\bar{Y} - \frac{\bar{a}}{\rho+\delta} = \frac{1}{\bar{c}}\frac{z-\bar{w}}{\rho+\delta} > 0$ and hence $Y - \frac{a}{\rho+\delta}$ is positive when Y, X are close to steady-state.

The equation is continuously differentiable in a, X, Y and the derivative w.r.t a is non-zero, so that according to the implicit function theorem one can solve for a differentiable $\tilde{a}(X, Y)$ with continuous partial derivatives.

From equation (12), we have $c = \frac{\kappa}{q(\theta)(Y - \frac{a(\tilde{X},Y)}{\rho+\delta})}$ and plugging this into equation (13): $c(1+m\Phi) = mz - \kappa\theta(1-m)$, one can show that a unique solution $\theta > 0$ exists. Because the expression is continuously differentiable, and the partial w.r.t. θ is non-zero, there exists a continuously differentiable $\theta(m, \Phi, X, Y)$. This then extends to c by the expression above.

3. The result on the eigenvalues is shown by considering the characteristic polynomial $\pi(x) = \det(A - xI)$, where A is the system matrix. We know that $\pi(x) = (\lambda_1 - x)(\lambda_2 - x)(\lambda_3 - x)(\lambda_4 - x)$. By tedious algebra, one can show that $\pi(0) > 0, \pi(\rho) = 0, \pi(\rho+\delta) < 0, \pi(-\delta) < 0$. See Mathematica notebook posted at `http://people.bu.edu/rudanko/papers/eigenvalues.nb` *Q.E.D.*

Proof of Proposition 3 ***Unique Equilibrium Path*** The statement is that there is a unique point in the stable manifold that will, when mapped with an orthogonal projection to the (m, Φ)-plane, map onto the point (m, Φ). The system is smooth and hyperbolic, so the theory of differential equations gives useful results. Consider first the linearized system. The stable subspace is spanned by the two stable eigenvectors, denote them by v^1, v^2. Points in that space can be represented as linear combinations $d^1v^1 + d^2v^2$ for some $d^1, d^2 \in \mathbb{R}$. Points corresponding to (m, Φ) must satisfy $d^1v^1_{1:2} + d^2v^2_{1:2} = (m, \Phi)'$, which means that a unique representation exists iff the vectors $v^1_{1:2}, v^2_{1:2}$ are linearly independent. To consider the original system then, note that the stable manifold is both smooth and tangent to the stable subspace at the steady-state. *Q.E.D.*

Appendix B Other CRRA Preferences

This section adapts the reduced state space representation for constant relative risk aversion preferences $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ where $0 < \gamma \neq 1$. I also allow preference heterogeneity –workers and entrepreneurs can have differences in discount rates and willingness to substitute consumption across time and states.

Suppose workers have discount rate $\hat{\rho}$ and risk aversion parameter $\hat{\gamma}$ while those of entrepreneurs are ρ and γ. Due to the optimal contracts (see equation (6)), in equilibrium the marginal rates of substitution between dates and states across all agents are equated: For any $\mathcal{F}_t, t \geq 0, s \geq 0$ and $\mathcal{F}_{t+s}$ a continuation of $\mathcal{F}_t$,

$$e^{-\rho s}\left(\frac{c(t+s)}{c(t)}\right)^{-\gamma} = e^{-\hat{\rho}s}\left(\frac{w(t,t+s)}{w(t,t)}\right)^{-\hat{\gamma}} \Rightarrow \frac{w(t,t+s)}{w(t,t)} = e^{-\frac{\hat{\rho}-\rho}{\hat{\gamma}}s}\left(\frac{c(t+s)}{c(t)}\right)^{\frac{\gamma}{\hat{\gamma}}}.$$

This implies contract wages satisfy $w(t,t+s) = e^{-\frac{\hat{\rho}-\rho}{\hat{\gamma}}s}a(t)c(t+s)^{\frac{\gamma}{\hat{\gamma}}}$, where $a(t) := w(t,t)c(t)^{-\frac{\gamma}{\hat{\gamma}}}$. The contract wage trends down over time if $\hat{\rho} > \rho$, with the worker's willingness to substitute consumption across time affecting how strong this trending is. The cross sectional heterogeneity among workers can now be captured by keeping track of the cross sectional average of the $e^{-\frac{\hat{\rho}-\rho}{\hat{\gamma}}s}a(t)$-terms. Define accordingly $\Phi(t) := \frac{1}{m(t)}\int_{-\infty}^{t} n(\tau,t)e^{-\frac{(\hat{\rho}-\rho)}{\hat{\gamma}}(t-\tau)}a(\tau)d\tau$, where m, n are measures of employment as before. The law of motion for Φ is $\dot{\Phi} = \frac{\mu(\theta(t))(1-m(t))}{m(t)}[a(t)-\Phi(t)] - \frac{(\hat{\rho}-\rho)}{\hat{\gamma}}\Phi$, where the last term captures any trending in contract wages over time.

To express the equilibrium conditions in Proposition 2 using state variables m, Φ, define new variables as follows:[27]

$$\begin{aligned}
X^1(t) &:= \int_0^\infty e^{-(\rho+\delta+\frac{(\hat{\rho}-\rho)}{\hat{\gamma}})s}\frac{c(t+s)^{\frac{\gamma}{\hat{\gamma}}(1-\hat{\gamma})}}{1-\hat{\gamma}}ds, \\
X^2(t) &:= \delta\int_0^\infty e^{-(\hat{\rho}+\delta)s}V^u(t+s)ds - V^u(t), \\
Y(t) &:= \int_0^\infty e^{-(\rho+\delta)s}c(t+s)^{-\gamma}zds.
\end{aligned}$$

The laws of motion for these variables read[28]

$$(\rho+\delta+\frac{(\hat{\rho}-\rho)}{\hat{\gamma}})X^1(t) = \frac{c(t)^{\frac{\gamma}{\hat{\gamma}}(1-\hat{\gamma})}}{1-\hat{\gamma}} + \dot{X}^1(t), \tag{15}$$

$$(\hat{\rho}+\delta)X^2(t) = -\frac{b^{1-\hat{\gamma}}}{1-\hat{\gamma}} - \mu\big(\theta(t)\big)[a(t)^{1-\hat{\gamma}}X^1(t) + X^2(t)] + \dot{X}^2(t), \tag{16}$$

$$(\rho+\delta)Y(t) = c(t)^{-\gamma}z + \dot{Y}(t). \tag{17}$$

[27]For notational reasons I leave out aggregate shocks.

The algebraic equations determining a, c, θ, (corresponding to (11)-(13)) are now

$$\kappa = q\big(\theta(t)\big)c(t)^{\gamma}[Y(t) - a(t)X^1(t)(1-\hat{\gamma})], \tag{18}$$

$$c(t)^{\gamma}[Y(t) - a(t)X^1(t)(1-\hat{\gamma})] = \frac{(1-\alpha)}{\alpha}(a(t)c(t))^{\hat{\gamma}}[a(t)^{1-\hat{\gamma}}X^1(t) + X^2(t)], \tag{19}$$

$$c(t) = m(t)z - m(t)\Phi(t)c(t)^{\frac{\gamma}{\hat{\gamma}}} - \kappa\theta(t)(1-m(t)). \tag{20}$$

We have a system of five differential equations in predetermined states m, Φ and jump-states X^1, X^2, Y given by (7),(8), (15)-(17) with the algebraic equations (18)-(20) determining a, c, θ. One can proceed exactly as in the log-utility case, with one additional jump variable.[29]

Appendix C Solving for Equilibrium with Aggregate Shocks

This section considers the problem of solving for equilibrium in the presence of aggregate shocks. I focus on the log-utility case for simplicity, but the extension to more general preferences, as in Appendix B, is straightforward.

When an aggregate shock hits, the value of z jumps. I assume the distribution of new values of productivity z', conditional on prevailing value of z, satisfies $z' - \bar{z}|z \sim N(\xi(z-\bar{z}), \sigma_\epsilon^2)$, where the draws are independent, $\xi \in (0,1)$ and the variance σ_ϵ^2 is small.

Solving Having shocks to z does not change how wages and entrepreneurial consumption co-move in equilibrium. For the log utility case, we continue to have $w(\tau, t) = a(\tau)c(t)$ and the average consumption share Φ remains the key state variable for collapsing the state space of the problem. As in the case without aggregate shocks, I consider equilibria where the state of the economy is fully characterized by the current values of $\psi := (m, \Phi, z)$. With no aggregate shocks we know that if (m, Φ) is close to steady-state, generically there exist unique values of X, Y that guarantee convergence to the steady-state. The equilibrium path can thus be represented by $(m, \Phi, \hat{X}(m, \Phi), \hat{Y}(m, \Phi))$, where $\hat{X}(m, \Phi), \hat{Y}(m, \Phi)$ take on these unique values. Analogously, postulate that to each ψ, there corresponds a unique pair $\hat{X}(\psi), \hat{Y}(\psi)$, corresponding to equilibrium in the presence of shocks. The equilibria represented by $(\psi, \hat{X}(\psi), \hat{Y}(\psi))$ must satisfy a dynamic system analogous to (7)-(10). With aggregate shocks arriving at Poisson rate η this system can be written in terms of $\hat{X}, \hat{Y}$ as:

$$\begin{aligned} 0 =& F(\psi, \hat{X}(\psi), \hat{Y}(\psi)) + \eta E[\hat{X}(\psi') - \hat{X}(\psi)|\psi] + \hat{X}_m(\psi)f(\psi, \hat{X}(\psi), \hat{Y}(\psi)) + \hat{X}_\Phi(\psi)g(\psi, \hat{X}(\psi), \hat{Y}(\psi)), \\ 0 =& G(\psi, \hat{X}(\psi), \hat{Y}(\psi)) + \eta E[\hat{Y}(\psi') - \hat{Y}(\psi)|\psi] + \hat{Y}_m(\psi)f(\psi, \hat{X}(\psi), \hat{Y}(\psi)) + \hat{Y}_\Phi(\psi)g(\psi, \hat{X}(\psi), \hat{Y}(\psi)) \end{aligned} \tag{21}$$

[28] We have $X^2(t) = \Delta t \delta V^u(t) + e^{-(\hat{\rho}+\delta)\Delta t}[X^2(t+\Delta t) + V^u(t+\Delta t)] - V^u(t) \approx \Delta t\delta V^u(t) + X^2(t) - (\hat{\rho} + \delta)\Delta t[X^2(t) + V^u(t)] + \Delta t[\dot{X}^2(t) + \dot{V}^u(t)]$, which implies $(\hat{\rho}+\delta)X^2(t) = -\hat{\rho}V^u(t) + \dot{V}^u(t) + \dot{X}^2(t)$. Combining this with the equation for $V^u(t)$, $\hat{\rho}V^u(t) = \frac{b^{1-\hat{\gamma}}}{1-\hat{\gamma}} + \mu(\theta(t))[a(t)^{1-\hat{\gamma}}X^1(t) + X^2(t)] + \dot{V}^u(t)$, implies the equation in the text.

[29] If workers have log preferences, then a slightly different treatment is necessary.

where $\psi' := (m, \Phi, z')$ and

$$\begin{aligned}
f(\psi, X, Y) :=& -\delta m + \mu\big(\tilde{\theta}(\psi, X, Y)\big)(1-m),\\
g(\psi, X, Y) :=& \frac{\mu\big(\tilde{\theta}(\psi, X, Y)\big)(1-m)}{m}[\tilde{a}(\psi, X, Y) - \Phi],\\
F(\psi, X, Y) :=& -(\delta+\rho)X + \log\tilde{c}(\psi, X, Y) - \log b - \mu\big(\tilde{\theta}(\psi, X, Y)\big)[\frac{\log\tilde{a}(\psi, X, Y)}{\rho+\delta} + X],\\
G(\psi, X, Y) :=& -(\delta+\rho)Y + \frac{z}{\tilde{c}(\psi, X, Y)}.
\end{aligned}$$

The functions $\tilde{a}, \tilde{c}, \tilde{\theta}$ are based on Lemma 2. The problem is now one of solving a system of partial differential equations, where one needs to further specify boundary conditions relating to stability.

Because the variance of labor productivity shocks over the business cycle is relatively small, I solve the problem by linearizing around the deterministic steady-state corresponding to average productivity: $\bar{m}, \bar{\phi}, \bar{z}$. We have

$$\begin{aligned}
\hat{X}(m, \Phi, z) &\approx \bar{X} + \bar{X}_m(m-\bar{m}) + \bar{X}_\Phi(\Phi-\bar{\Phi}) + \bar{X}_z(z-\bar{z}),\\
\hat{Y}(m, \Phi, z) &\approx \bar{Y} + \bar{Y}_m(m-\bar{m}) + \bar{Y}_\Phi(\Phi-\bar{\Phi}) + \bar{Y}_z(z-\bar{z}),
\end{aligned}$$

where $\bar{X} := \hat{X}(\bar{m}, \bar{\Phi}, \bar{z}), \bar{Y} := \hat{Y}(\bar{m}, \bar{\Phi}, \bar{z})$ and $\bar{X}_m, \bar{X}_\Phi, \bar{X}_z, \bar{Y}_m, \bar{Y}_\Phi, \bar{Y}_z$ are the corresponding partial derivatives evaluated at the steady-state. The unknowns are these partial derivatives, along with the steady-state. The derivatives can be found by differentiating the system (21) with respect to m, Φ, z and using the approximation for $\hat{X}, \hat{Y}$. Given the assumptions about the distribution of new productivity levels, we have $E[\hat{X}(m, \Phi, z') - \hat{X}(m, \Phi, z)|m, \Phi, z] \approx \bar{X}_z E[(z'-z)|m, \Phi, z] = \bar{X}_z(\xi-1)(z-\bar{z})$ and similarly for $\hat{Y}$.

Simulating Between aggregate shocks, the linearized system for m, Φ reads:

$$\begin{aligned}
\dot{m} = \bar{f}_m(m-\bar{m}) + \bar{f}_\Phi(\Phi-\bar{\Phi}) + \bar{f}_z(z-\bar{z}) &+ \bar{f}_X[\bar{X}_m(m-\bar{m}) + \bar{X}_\Phi(\Phi-\bar{\Phi}) + \bar{X}_z(z-\bar{z})]\\
&+ \bar{f}_Y[\bar{Y}_m(m-\bar{m}) + \bar{Y}_\Phi(\Phi-\bar{\Phi}) + \bar{Y}_z(z-\bar{z})]\\
\dot{\Phi} = \bar{g}_m(m-\bar{m}) + \bar{g}_\Phi(\Phi-\bar{\Phi}) + \bar{g}_z(z-\bar{z}) &+ \bar{g}_X[\bar{X}_m(m-\bar{m}) + \bar{X}_\Phi(\Phi-\bar{\Phi}) + \bar{X}_z(z-\bar{z})]\\
&+ \bar{g}_Y[\bar{Y}_m(m-\bar{m}) + \bar{Y}_\Phi(\Phi-\bar{\Phi}) + \bar{Y}_z(z-\bar{z})]
\end{aligned}$$

This has a closed form solution. Simulation can be done by drawing arrival times from an exponential distribution and drawing new productivity realizations from the appropriate normal distribution.

Impulse Responses Suppose $z(0) \neq \bar{z}$. From $E[z(t+\Delta t) - \bar{z}|z(t)] = \eta\Delta t\xi(z(t)-\bar{z}) + (1-\eta\Delta t)(z(t)-\bar{z})$ we have $\hat{z}(t) := E[z(t)|z(0)] - \bar{z}$, $\dot{\hat{z}} = -\eta(1-\xi)\hat{z}$ and hence $\hat{z}(t) = z(0)e^{-\eta(1-\xi)t}$. From the linearized system above we get a system for the responses of endogenous variables $\hat{m}(t) := E[m(t)|z(0)] - \bar{m}, \hat{\Phi}(t) := E[\Phi(t)|z(0)] - \bar{\Phi}$:

$$\dot{\begin{pmatrix} m \\ \Phi \end{pmatrix}} = \bar{A}\begin{pmatrix} m-\bar{m} \\ \Phi-\bar{\Phi} \end{pmatrix} + \bar{b}(z-\bar{z}) \Rightarrow \dot{\begin{pmatrix} \hat{m} \\ \hat{\Phi} \end{pmatrix}} = \bar{A}\begin{pmatrix} \hat{m} \\ \hat{\Phi} \end{pmatrix} + \bar{b}\hat{z} \Rightarrow \dot{\begin{pmatrix} \hat{m} \\ \hat{\Phi} \\ \hat{z} \end{pmatrix}} = \begin{pmatrix} \bar{A} & \bar{b} \\ 0 & -\eta(1-\xi) \end{pmatrix}\begin{pmatrix} \hat{m} \\ \hat{\Phi} \\ \hat{z} \end{pmatrix}.$$

Calibrating the productivity process The target for the calibration of the parameters of the productivity process $(\xi, \sigma_\epsilon, \eta)$ is the non-farm business sector labor productivity series reported by the Bureau of Labor Statistics. Taking logs and $HP(10^5)$-filtering[30] the quarterly labor productivity series has autocorrelation 0.89 and standard deviation 0.02. For a given set of productivity parameters, I calculate the corresponding numbers as averages from 100 realizations of simulated monthly data for 55 years, after aggregating to quarters, taking logs and filtering. To pin down productivity parameters I first pick $\eta = 0.1$, which leaves ξ, σ_ϵ to be pinned down by the observed standard deviation and autocorrelation. If one observed z with unit intervals, the observed process would have autocorrelation $\rho_z := 1 - \eta + \eta\xi$ and variance $\sigma_z^2 := \frac{\eta^2\sigma_\epsilon^2}{1-\rho_z^2}$. To obtain the observed moments, I use input values $\xi = 0.768$, $\sigma_\epsilon = 0.0191$. For impulse responses hence $\rho_z = 0.977$.

Computing simulated moments To compute moments from the model, I feed into the model simulated monthly productivity data. For each 55 years worth of data I aggregate to quarters, take logs, $HP(10^5)$-filter and calculate moments, then averaging over 100 samples.

Quality of linearized solution In the absence of aggregate shocks, solving the original nonlinear problem numerically is relatively straightforward, and comparing the solution of this problem with the corresponding linearization based solution allows me to shed some light on whether linearization is valid. Computing a solution to the nonlinear deterministic system involves a shooting problem with two predetermined variables and either two or three jump variables depending on the utility function used for workers.[31] I perturb the initial conditions $(m(0), \Phi(0))$ from steady-state to the steady-state of an economy with 2% higher or lower productivity and examine the adjustment paths of the two solutions. I find that the differences are practically indistinguishable to the eye for perturbations of this size, offering support for linearization.

Appendix D Steady-State of Equilibrium

Examining the steady-state of the economy is informative about the level effects of incomplete markets, but it turns out to also be useful for considering responses to shocks.

The steady-state is otherwise the same as in the Mortensen-Pissarides model, but now workers are risk averse individuals who consume their income each period, so the drop in consumption upon unemployment is felt differently. Vacancy-creation takes places under a zero profit condition, where entrepreneurs face a tradeoff between the cost of hiring workers and wage costs. Offering high wages will on the one hand increase the number of job applicants and so reduce the cost of hiring, but on the other reduce profits due to higher operating costs. In equilibrium, worker preferences determine the unique optimal balance between these costs. When workers face incomplete asset markets, they value income smoothing. Steady-state wages are reduced and vacancy creation hence increases as

[30]for consistency with the treatment of unemployment and vacancies adopted in the literature since Shimer (2005a)

[31]I used Matlab's collocation based bvp4c-function.

entrepreneurs take advantage of the lower wage costs. This drives up hiring costs until profits from vacancy creation are zero. With more vacancy creation, and workers find jobs faster. To sum up: With incomplete markets, unemployment spells are shorter and the change in consumption between employment and unemployment is smaller.

The steady-state is characterized by:

$$\frac{z-w}{\rho+\delta}[\rho+\delta+k(\frac{z-w}{\rho+\delta}\frac{k}{\kappa})^{\frac{1-\alpha}{\alpha}}]=\frac{1-\alpha}{\alpha}w^{\gamma}\frac{[w^{1-\gamma}-b^{1-\gamma}]}{1-\gamma},\ \text{if}\ \gamma\neq 1,$$

$$\text{or}\ \frac{z-w}{\rho+\delta}[\rho+\delta+k(\frac{z-w}{\rho+\delta}\frac{k}{\kappa})^{\frac{1-\alpha}{\alpha}}]=\frac{1-\alpha}{\alpha}w\log(w/b),\ \text{if}\ \gamma=1,$$

$$\text{and}\ \theta=q^{-1}(\frac{\kappa(\rho+\delta)}{z-w}),\quad m=\frac{\mu(\theta)}{\mu(\theta)+\delta},\quad c=\frac{\rho\kappa}{q(\theta)+\delta/\theta},\quad a=\Phi=\frac{w}{c}.$$

It is straightforward to verify that the wage equation has a unique solution.

Level effect of concavity in preferences Decreasing the willingness of workers to substitute across time and states implies that they are less willing to bear changes in consumption between spells of employment and unemployment. They accept lower wages in return for shorter unemployment spells. This means that the vacancy-unemployment ratio will tend to increase as unemployment decreases and vacancy-creation increases. (See below.)

Calibrating to match unemployment To calibrate the model one generally wants to match the level of unemployment in the data, so comparisons of the model with different preferences should respect this. The calibration approach involves changing the vacancy cost κ to target the level of unemployment. When workers' willingness to substitute decreases, the vacancy cost must be increased to keep unemployment from falling. Fixing the level of unemployment by imposing $\theta=1$ implies that $z-w=\frac{\kappa(\rho+\delta)}{q(1)}$ and the wage equation becomes an equation in κ instead

$$\frac{\kappa(\rho+\delta+k)}{k}=\frac{1-\alpha}{\alpha}[z-\frac{\kappa}{k}(\rho+\delta)]^{\gamma}\frac{[(z-\frac{\kappa}{k}(\rho+\delta))^{1-\gamma}-b^{1-\gamma}]}{1-\gamma},\ \text{if}\ \gamma\neq 1. \tag{22}$$

Response to changing productivity The higher is productivity, the higher are the wages and hiring costs that entrepreneurs can profitably pay. It is straightforward to verify above that both w and θ are increasing in z. Unemployment decreases in z while vacancy creation increases.

One indicator of the responses of model variables to persistent productivity shocks is the response of the steady-state to change in the level of productivity. It turns out that when workers are less willing to substitute consumption across time and states, the increase in steady-state wages from an increase in steady-state productivity is smaller, as workers gain less from a high wage level during employment. This shifts the productivity increase toward a larger increase in market tightness. (See below.) This is consistent with the numerical results on how the model responds to productivity shocks.

Lemma 7. *If* $f(x,\gamma) := x^{\gamma\frac{[x^{1-\gamma}-1]}{1-\gamma}}$ *and* $g(x,\gamma) := \frac{1-\gamma}{x^{1-\gamma}-1}$, *where* $x > 1$, *then* $f_x(\gamma, x) > 0$, $f_\gamma(\gamma, x) > 0$, $g_x(x,\gamma) < 0$, $g_\gamma(x,\gamma) > 0$.

Pf. We have $f_x(\gamma, x) = \frac{1-\gamma x^{\gamma-1}}{1-\gamma} > 0$ for all $\gamma > 0$. Also, $f_\gamma(\gamma, x) = \frac{x^{\gamma-1}(x^{1-\gamma}-1-(1-\gamma)\log x)}{(1-\gamma)^2} > 0$ for all $\gamma \neq 1$, because $x^{1-\gamma} = e^{(1-\gamma)\log x} > 1 + (1-\gamma)\log x$ for all $\gamma \neq 1$. We have $g_x(x,\gamma) = -(1-\gamma)^2(x^{1-\gamma}-1)^{-2}x^{-\gamma} < 0$, and $\frac{\partial}{\partial\gamma}\log g(x,\gamma) = \frac{1-x^{1-\gamma}+(1-\gamma)x^{1-\gamma}\log x}{(1-\gamma)(x^{1-\gamma}-1)}$. The denominator in the latter is clearly positive. The numerator $h(x,\gamma) := 1 - x^{1-\gamma} + (1-\gamma)x^{1-\gamma}\log x$ has $h_x = (1-\gamma)x^{-1}(1-x^{1-\gamma}) > 0$. Hence h increases in x and because $h(1,\gamma) = 0$, we have that $h(x,\gamma) > 0$ for $x > 1$. *Q.E.D.*

Step 1: We show that w is decreasing in γ in the calibration strategy which holds $\theta = 1$ by increasing κ. In this calibration, equation (22) determines the steady-state κ and wages can be found from $\kappa = k\frac{z-w}{\rho+\delta}$. Right hand side of equation (22) can be written as $\frac{1-\alpha}{\alpha}bf(\frac{z-\frac{\kappa}{k}(\rho+\delta)}{b}, \gamma)$. Using the lemma above we see that κ increases in γ and hence the wage decreases in γ.

Step 2: The partial w_z can be found from the steady-state wage equation above as $w_z = \frac{B}{B+\frac{z-w}{w}(1+g(\frac{w}{b},\gamma))}$ where $B := 1 + \frac{1-\alpha}{\alpha}\frac{\mu(\theta)}{\rho+\delta+\mu(\theta)}$. Imposing again the calibration strategy holding $\theta = 1$, we know that as γ increases, w decreases. Hence $\frac{d}{d\gamma}g(\frac{w}{b},\gamma) = g_x(\frac{w}{b},\gamma)\frac{w_\gamma}{b} + g_\gamma(\frac{w}{b},\gamma) > 0$, and we know that w_z decreases as γ increases.

External Appendixes as Supplementary Material:

Appendix E Connection to the Mortensen-Pissarides Model

Appendix F Complete Markets Economy

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