Volatility, Growth, and Large Welfare Gains from Stabilization Policies*

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May 8, 2006

Abstract

This paper proposes a simple endogenous-fluctuations growth model to show: 1) long-run growth and short-run fluctuations can be intimately linked; in particular, the rate of long-run growth can be negatively affected by volatilities; 2) imperfect competition can cause endogenous fluctuations, and it reduces not only the level of output but also its mean growth rate by amplifying the volatility of the economy; and 3) the welfare gain of stabilization policy can be enormous (e.g., as high as 25% of annual consumption when calibrated to the U.S. data) because policies designed to reduce sunspots-driven fluctuations can generate permanently higher rates of growth.

Keywords: Endogenous Growth, Welfare Cost of Business Cycle, Stabilization Policy, Sunspots, Imperfect Competition, Indeterminacy.

JEL codes: E12, E32, O40.

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1 Introduction

Business cycles and growth are undoubtedly the two most important issues in macroeconomics. Yet they have been traditionally treated as separate areas of macroeconomics, as if fluctuations and growth are completely unrelated. This dichotomy is illustrated most clearly by the independent development of the neoclassical growth model (Solow 1956) and the Keynesian IS-LM model (Hicks, 1937). Although the modern real business cycle (RBC) theory, developed by Kydland and Prescott (1982) and Long and Plosser (1983), intends to end this dichotomy by using a common general-equilibrium framework and by hypothesizing a common driving force for both growth and fluctuations (i.e., technology), it nonetheless maintains a fundamental assumption that the mean growth rate of output is independent of the random shocks to the economy.[1] Based on this fundamental assumption, although temporary fluctuations may have permanent effects on the level of output, they do not affect the mean growth rate of output (i.e., a distinction between a level effect and a growth effect). Thus, long-run growth and short-run fluctuations are still viewed as unrelated and determined by fundamentally different forces. Therefore, by merely postulating a common driving force for growth and business cycles, the RBC theory has not ended the classical dichotomy. If anything, it may have helped reinforce the dichotomy in a particular way.[2] This is further highlighted by the popularity of the Hodrick-Prescott filter developed in the RBC literature, which is used widely by macroeconomists to decompose aggregate output into two seemingly independent components: a trend (growth) component and a cyclical component (see, Hodrick and Prescott, 1997). The underling assumption behind this decomposition is that growth and fluctuations can be studied in isolation.

One of the most far-reaching implications of this classical dichotomy between growth and fluctuations is that the welfare gains of eliminating fluctuations are trivial compared to that of stimulating long-run growth (Lucas, 1987). This famous calculation made by Lucas has survived numerous robustness analyses and is a major challenge to the old Keynesian belief that stabilization policies are desirable (see Lucas 2003, and the references therein). In fact, the policy implication of the Lucas calculation is even more robust than its welfare implication because even if we can find models in which the welfare cost of fluctuations is large, the gain from stabilization policy may

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[1] The RBC theory usually assumes that technology shocks follow a random walk with a constant drift, where the drift is independent of the innovations to technology.

[2] This is not to deny that the RBC theory has greatly advanced our understanding of business cycles. Nevertheless, it has not ended the dichotomy. For a clear presentation of the underling dichotomy in the RBC theory, also see King, Plosser, and Rebelo (1988). According to this leading article in the RBC literature, business cycles are viewed as temporary fluctuations around a long-run steady state. Although the requirement for balanced steady-state growth places restrictions on the structure of RBC models, the steady state itself cannot be affected by business cycles (by definition).

still be small (see, e.g., Kiley 2003, and Barlevy, 2004a). The fundamental reason for this is that general-equilibrium business-cycle models typically imply volatile consumption as optimal allocation because the fundamental sources of fluctuations (e.g., technology) are exogenous and such fluctuations are independent of long-run growth.

Yet there is a growing awareness that the dichotomy between volatility and growth is hard to square with the facts. For example, Ramey and Ramey (1995) present convincing evidence that there is a clear link between business cycle volatility and long-run growth. In particular, countries with high output volatility tend to have low output growth. The robustness of this negative relationship between volatility and growth has also been validated by more recent independent empirical studies.[3]

There is also a large theoretical literature lending support to the old Keynesian belief that business cycles can be endogenous, driven largely by animal spirits or self-fulfilling expectations (see, e.g., Azariadis 1981, Cass and Shell 1983, Woodford 1986, Boldrin and Motrucchio 1986, and Benhabib and Farmer 1994, among others).[4] Recent development of this literature suggests that stochastic dynamic general equilibrium models driven by self-fulfilling expectations may provide better explanations of business cycles (including procyclical productivity) than models driven by exogenous technology shocks.[5]

If growth is negatively related to volatility and fluctuations are largely endogenous, then the fundamental assumption behind the Lucas calculation is incorrect and should be re-evaluated in light of new models that can truly integrate growth and fluctuations. This paper proposes such a model and uses it to demonstrate three key points: 1) long-run growth and short-run fluctuations can be negatively linked; 2) imperfect competition can cause fluctuations in the mean growth rate of output via coordination failures, and consequently, inefficiencies due to imperfect competition exist not only in the level of output (Okun's gap) but also in its long-run growth rate (growth gap); and 3) the welfare cost of business cycles and the associated gain of stabilization policy can be enormous (as high as $25 \sim 50\%$ of consumption when the model is calibrated to the U.S. data).

The model is a simple extension of the AK growth model (e.g., Rebelo 1991). The additional new features include variable capacity utilization, imperfect competition, and imperfect information, all of which are key ingredients of traditional Keynesian theory. Due to imperfect information, imperfectly competitive firms face extrinsic uncertainty regarding other firms' price-setting behavior and the level of aggregate demand (even in the absence of fundamental shocks). Because of strategic complementarity among firms' actions, which arises from imperfect substitutability of firms' output

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[3] See, e.g., Aghion, Angeletos, Banerjee, and Manova (2005), Easterly, Islam, and Stiglitz (2000), Hnatkovska and Loayza (2004), Kroft and Lloyd-Ellis (2002), and Mobarak (2005), among others.

[4] There is also a literature studying the possibility of endogenous and deterministic growth cycles. See, e.g., Goodwin (1967) and Benhabib and Nishimura (1985), among others.

[5] See, e.g., Farmer and Guo (1994), Gali (1994), Wen (1998), and Jaimovich (2006), among others.

in the goods market, extrinsic uncertainty can be self-fulfilling and, consequently, the economy can suffer from coordination failures and endogenous fluctuations. In an environment where the firm's rate of capacity utilization is endogenous, fluctuations in the marginal cost translate directly into fluctuations in the rate of output growth. These stochastic growth paths, driven by firms' speculations about aggregate demand under imperfect information, yield a strictly lower mean growth rate than the fundamental-equilibrium growth path – a path in the absence of extrinsic uncertainty (i.e., under full information). Under parameter values calibrated to the U.S. data, the model predicts that a more volatile growth path has a lower mean growth rate. Namely, growth and volatility are negatively related, as in the data. Because of this, the welfare cost of business cycles can be hundreds of times larger than that calculated by Lucas under the assumption of the dichotomy. Since expectations-driven fluctuations are inefficient, the welfare gain from stabilization policy is equally significant.

Our analysis is closely related to the work of Barlevy (2004a). Using an AK endogenous growth model featuring adjustment costs in investment, Barlevy is able to show that volatility and growth can be negatively related. Consequently, the welfare gain of eliminating fluctuations can be large since it enhances long-run growth. However, the policy implication of Barlevy's model is fundamentally different from ours. In Barlevy's model, there is little scope for stabilization policies despite the potentially large welfare gains from eliminating fluctuations. This is so because fluctuations in Barlevy's model are optimal responses to exogenous shocks. Thus, there is no gain from stabilizing the economy. In our model, fluctuations are caused by coordination failures and self-fulfilling expectations, and are themselves inefficient regardless of technology shocks. For this reason, our model provides a better framework to meaningfully gauge the welfare cost of business cycles than models in which fluctuations are nothing but optimal.[6]

2 The Model

2.1 Firms

There is a final good production sector in the economy. The final good producers behave competitively and the households buy the final good for both consumption and investment. The final good is produced by using intermediate goods according to the Dixit-Stiglitz technology:

$$Y = \left(\int_0^1 y(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

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[6] For comprehensive literature reviews on the issue of welfare cost of business cycles and the benefits of stabilization, see Lucas (2003) and Barlevy (2004b). For previous works that evaluate welfare cost by linking endogenous growth to exogenous fluctuations, see Blackburn and Pelloni (2005), de Hek (1999), Epaulard and Pommeret (2003), Jones, Manuelli, Siu, and Stacchetti (2005), and Krebs (2003), among others.

where $\epsilon > 1$ measures the elasticity of substitution among intermediate goods $y(i)$. The price of the final good is normalized to one and the price of intermediate good i is denoted $p(i)$. Profit maximization in the final good sector yields the demand function for intermediate goods, $y(i) = p(i)^{-\epsilon} Y$. Substituting this into the production function yields the aggregate price index, $\int_0^1 p(i)^{1-\epsilon} di = 1$.

The economy has a continuum of monopolistic intermediate good producers of measure one, each producing a single differentiated good $y(i)$. Intermediate goods are produced by using capital (k). The production function for intermediate goods is identical across firms and is given by:

$$y(i) = Au(i)k(i), \tag{2}$$

where A denotes the level of technology common to all firms and $u(i)$ denotes the rate of capacity utilization for firm i. Intermediate good producers are assumed to be price takers in the input market. Let r denote the market interest rate, and let $\delta(i)$ denote the rate of capital depreciation for firm i. Following Greenwood et al. (1988), the rate of capital depreciation is assumed to depend on its usage rate:

$$\delta(i) = \frac{\alpha}{1+\theta} u(i)^{1+\theta}, \quad \theta > 0. \tag{3}$$

Hence the user's cost of capital facing firm i is $r + \delta(i)$.[7]

The cost function of an intermediate firm can be found by minimizing $[r + \delta(i)]\, k(i)$ subject to $Au(i)k(i) \geqslant y(i)$. Denoting ϕ as the Lagrangian multiplier for the above constraint, cost minimization yields the relationship, $r + \delta(i) = \phi Au(i)$ and $\alpha u(i)^{\theta} = \phi A$.These first-order conditions imply $\delta(i) = \delta = \frac{1}{1+\theta} \alpha^{-\frac{1}{\theta}} (\phi A)^{\frac{\theta+1}{\theta}}$ and

$$r = \theta\delta = \frac{\theta}{1+\theta} (\alpha)^{-\frac{1}{\theta}} (\phi A)^{\frac{\theta+1}{\theta}}. \tag{4}$$

Since the technology has constant returns to scale and firms face the same interest rate, the marginal cost ϕ is the same across all firms. Consequently, the optimal rates of capital utilization and depreciation are also the same across firms. Thus, firms' output differ from each other if and only if their capital stocks differ.

A key variable determining the endogenous growth rate in an AK model is the interest rate r. Notice that the equilibrium interest rate in this model is always positive. This is in sharp contrast to the standard AK model where the interest rate $(r = A - \delta)$ can be negative if the returns to capital (A) is less than the rate of capital depreciation (δ). Consequently, an unpleasant feature of the standard AK model is that the long-run growth rate can be either positive or negative,

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[7] The importance of capacity utilization in understanding business cycles and growth has been emphasized by Greenwood et al. (1988), King and Rebelo (1999), Wen (1998), and Chatterjee (2003), among others.

depending on the relative magnitudes of A and δ. This unpleasant feature of the standard AK model is eliminated here due to endogenous capital utilization and depreciation, which renders the real interest rate always positive.[8]

Each intermediate firm faces a downward sloping demand curve, $y(i) = p(i)^{-\epsilon}Y$, and sets prices to maximize profits. Since firms have no influence on the aggregate quantity Y, there exists a strategic complementarity among firms' actions, in the language of Cooper and John (1988). Namely, every firm will opt to set lower prices to induce higher demand if they all anticipate that the other firms will set lower prices to boost the aggregate demand. This strategic complementarity, however, is a necessary but not sufficient condition for multiple Nash equilibria in this model. Another key condition for multiple equilibria is imperfect information regarding aggregate economic conditions.

A key feature of the model is that intermediate good firms each choose a price while taking as given the prices set by other firms, with quantities being then determined by demand at these prices in general equilibrium. This sequential feature of the model permits imperfect information. That is, in each period t, intermediate good firms must set prices without knowing the aggregate economic conditions (such as aggregate demand) that may prevail in period t. These aggregate economic conditions depend crucially on the actions of the other firms over which an individual firm has no influence. Thus, each individual firm, without knowing how the other firms will set their prices, must form expectations for the level of aggregate demand (Y) when setting its own prices.

Without loss of generality, assume that there are no fundamental shocks in the economy; then the only type of uncertainty, if any, is extrinsic uncertainty in the language of Cass and Shell (1983) (i.e., due to sunspots). An intermediate good firm's objective function is then to solve

$$\max_{p(i)} E\left[(p(i) - \phi)\, y(i)\right] \tag{5}$$

subject to the demand function $y(i) = p(i)^{-\epsilon}Y$. The optimal price is given by $p(i) = \frac{\epsilon}{\epsilon-1}\frac{E(\phi Y)}{EY}$. Assuming that firms are rational and have the same information sets, then they all set the same prices. Thus, $p(i) = p = 1$ and

$$E(\phi Y) = \frac{\epsilon - 1}{\epsilon} EY. \tag{6}$$

In the limiting case where $\epsilon \to \infty$, the model converges to a perfectly competitive economy. Our analysis of sunspots equilibria is independent of ϵ, hence it applies equally to perfectly (or near-

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[8] In fact, the rate of optimal capital utilization can be interpreted as maximizing the interest rate, $r = uA - \delta(u)$, which gives rise to $A = \delta'(u)$ or $r = \frac{1}{1+\theta}A^{\frac{1+\theta}{\theta}}$. Consequently, optimal capital utilization also maximizes the growth rate in an AK model.

perfectly) competitive economies where firms set prices equal to marginal cost with zero markup in the steady-state. Figure 1 illustrates the sequence of events in the model economy.

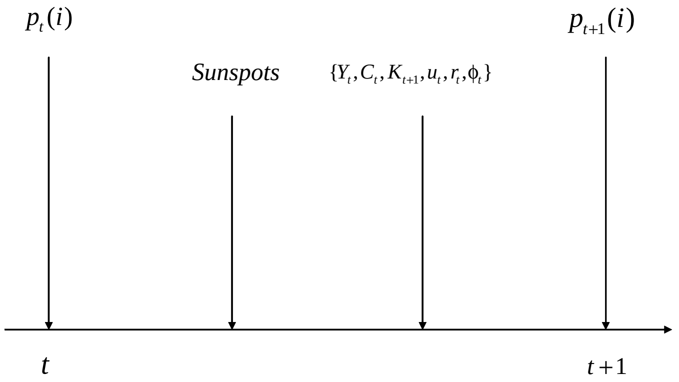

Figure 1. Sequence of Events and Timing of Sunspots.

Define $\tilde{\Omega}_t$ as the information set available to price-setting firms in period t, which includes the entire history of the economy up to period t except the realizations of sunspots (if any) in period t. Denote Ω_t as the information set that includes $\tilde{\Omega}_t$ and any realization of sunspots in period t. Thus we have $\Omega_t \supseteq \tilde{\Omega}_t \supseteq \Omega_{t-1}$. Since we do not consider fundamental shocks in this paper, we have $\tilde{\Omega}_t = \Omega_{t-1}$. Extension of the analysis to including fundamental shocks is straightforward.[9] Based on this definition of information sets, Equation (6) can also be written as $E_{t-1}(\phi_t Y_t) = \frac{\epsilon-1}{\epsilon} E_{t-1} Y_t$.

2.2 Households

There is a continuum of infinitely lived identical households of measure one. The representative agent chooses paths of consumption ($\{C_t\}_{t=0}^{\infty}$) and capital holdings ($\{K_t\}_{t=1}^{\infty}$) to solve

$$\max E_0 \sum_{t=0}^{\infty} \log(C_t) \tag{7}$$

subject to $K_0 > 0$ given and the budget constraint,

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[9] Notice that intrinsic uncertainty (due to fundamental shocks) can trigger extrinsic uncertainty in our model economy because without perfect foresight firms must form expectations and such expectations can be self-fulfilling even if the uncertainty is originally caused by fundamental shocks.

$$C_t + K_{t+1} = (1 + r_t)K_t + D_t, \tag{8}$$

where D_t denotes real profits distributed from intermediate good firms. The first-order condition is given by $\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}(1 + r_{t+1})$, plus the transversality condition, $\lim_{T\to\infty} \beta^T \frac{K_{T+1}}{C_T} = 0$.

2.3 Symmetric Rational Expectations Equilibrium

Since the economy's technology is symmetric with respect to all the intermediate inputs, the attention in this paper is restricted to symmetric equilibria where $y(i) = Y$ and $k(i) = K$ for all $i \in [0, 1]$. Notice that in the absence of extrinsic uncertainty, Equation (6) implies that the marginal cost is constant, $\phi = \frac{\epsilon-1}{\epsilon}$. Given the value of ϕ, the value of interest rate is then fully determined, as is the balanced growth rate. However, as will be shown shortly, constant marginal cost is not the only possible equilibrium in this model. There are also multiple Nash-sunspots equilibria that feature stochastic marginal cost and stochastic interest rate.

The equilibrium conditions in this economy can be summarized by the following equations:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left(1 + \frac{\theta}{1+\theta} \alpha^{-\frac{1}{\theta}} \left(\phi_{t+1} A\right)^{\frac{\theta+1}{\theta}}\right), \tag{9}$$

$$C_t + K_{t+1} = Y_t + (1 - \delta_t)k_t = \left[1 + \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t^{\frac{1}{\theta}} \left(1 - \frac{\phi_t}{1+\theta}\right)\right] K_t, \tag{10}$$

$$E_{t-1}\phi_t^{\frac{1+\theta}{\theta}} = \frac{\epsilon - 1}{\epsilon} E_{t-1}\phi_t^{\frac{1}{\theta}}; \tag{11}$$

where the last equation is derived from Equation (6).[10] These three equations, in conjunction with a transversality condition, fully determine the equilibrium paths of the marginal cost, consumption, and the capital stock. In particular, given any path of the marginal cost (ϕ) as specified by Equation (11), Equations (9) and (10) fully determine the paths of consumption and the capital stock.

Notice that Equation (11) implies $E\phi^{\frac{1}{\theta}}(E\phi - \frac{\epsilon-1}{\epsilon}) = -cov(\phi^{\frac{1}{\theta}}, \phi) \leq 0$, hence any stochastic process $\{\phi_t\}_{t=0}^{\infty}$ satisfying $E\phi \leq \frac{\epsilon-1}{\epsilon}$ and $cov(\phi^{\frac{1}{\theta}}, \phi) = E\phi^{\frac{1}{\theta}}(\frac{\epsilon-1}{\epsilon} - E\phi)$ constitutes a rational expectations equilibrium path for the marginal cost.[11] The fundamental equilibrium (in the absence of extrinsic uncertainty or sunspots) corresponds to the case where $cov(\phi^{\frac{1}{\theta}}, \phi) = 0$ and $\phi = \frac{\epsilon-1}{\epsilon}$. The fundamental equilibrium is clearly unique.[12] But there also exists multiple sunspots equilibria.

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[10] Note that K_t is a state variable known to firms in the beginning of period t.

[11] To avoid complex values, the condition $E\phi \geq 0$ must be imposed.

[12] Notice that the uniqueness is regardless of fundamental shocks. For example, suppose the technology A is a stochastic process, then in the fundamental equilibrium, we still have $\phi = \frac{\sigma-1}{\sigma}$.

To construct such sunspots equilibria, consider the process $\phi_t = \frac{\epsilon-1}{\epsilon}\varepsilon_t$, where ε denotes sunspots shocks. Equation (11) implies

$$E_{t-1}\varepsilon_t^{\frac{1+\theta}{\theta}} = E_{t-1}\varepsilon_t^{\frac{1}{\theta}}. \tag{12}$$

Clearly, any random variable satisfying the distribution,

$$E_{t-1}\varepsilon_t \in [0,1], \quad cov(\varepsilon_t^{\frac{1}{\theta}}, \varepsilon_t) = E_{t-1}\varepsilon_t^{\frac{1}{\theta}}\left(1 - E_{t-1}\varepsilon_t\right), \tag{13}$$

constitutes an equilibrium. This paper restricts attention to *i.i.d.* sunspots shocks with mean $E\varepsilon = \bar{\varepsilon} \in [0,1]$.

Definition 1 *A balanced growth path in the model is defined as an equilibrium path along which consumption, the capital stock, and output all grow at the same expected rate.*

Proposition 2 *For any and each i.i.d. sunspots shock process, there always exists a balanced growth path along which the stochastic growth rates of consumption and capital are both given by* $\ln\left[s(1+\varphi_t)\right]$*, where* $\varphi_t \equiv \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t^{\frac{1}{\theta}} \left(1 - \frac{\phi_t}{1+\theta}\right)$ *and* $s \equiv \beta E_t \frac{1+r_{t+1}}{1+\varphi_{t+1}}$*; and the growth rate of output is given by* $\ln\left[\left(\frac{\phi_t}{\phi_{t-1}}\right)^{1/\theta} s(1+\varphi_t)\right]$.

Proof. Since ϕ_t is *i.i.d.*, any function of ϕ_t is also *i.i.d.* An educated guess of the equilibrium paths of consumption and the capital stock is given by

$$C_t = (1-s)(1+\varphi_t)K_t, \tag{14}$$

$$K_{t+1} = s(1+\varphi_t)K_t, \tag{15}$$

where $s = \beta E_t \frac{1+r_{t+1}}{1+\varphi_{t+1}}$ denotes the optimal rate of savings, which is a constant under the *i.i.d.* assumption and is derived from the intertemporal Euler equation

$$\frac{1}{(1-s)(1+\varphi_t)K_t} = \beta E_t \frac{1+r_{t+1}}{(1+\varphi_{t+1})(1-s)s(1+\varphi_t)K_t}. \tag{16}$$

Using Equations (14) and (15), it can be shown that $\frac{C_{t+1}}{C_t} = s(1+\varphi_{t+1})$. Hence the balanced growth rates of consumption and capital are both given by $g = \ln\left[s(1+\varphi_t)\right]$. The growth rate of output is given by $g_y = \ln \frac{u_t K_t}{u_{t-1}K_{t-1}} = \frac{1}{\theta}\left(\ln\phi_t - \ln\phi_{t-1}\right) + \ln\left[s(1+\varphi_t)\right]$, which has the same (unconditional) expected value as g. ■

Proposition 3 *In the absence of extrinsic uncertainty, the model has a unique balanced growth path with its growth rate determined by*

$$g = \ln s(1+\varphi) = \ln \left[\beta \left(1 + \frac{\theta}{1+\theta} \alpha^{-\frac{1}{\theta}} \left(\frac{\epsilon - 1}{\epsilon} A \right)^{\frac{\theta+1}{\theta}} \right) \right]. \tag{17}$$

Proof. In the absence of extrinsic uncertainty, Equation (6) implies that the marginal cost is constant, $\phi = \frac{\epsilon-1}{\epsilon}$. Hence r and φ are all constant. Consequently, the fundamental (no-sunspots) growth rate in the economy is uniquely determined by $\ln \beta(1 + r(\frac{\epsilon-1}{\epsilon}))$. ■

Proposition 1 and Proposition 2 imply that stochastic growth paths driven by sunspots (i.e., sunspots equilibria) are not mere randomizations over fundamental growth paths (i.e., fundamental equilibria). This is in sharp contrast to DSGE models that rely on the indeterminacy of the steady state or multiple fundamental equilibria to generate sunspots equilibria via randomization (see, e.g., Benhabib and Farmer 1994). As Cass and Shell (1983) theorized, however, sunspots equilibria can exist in economies where the fundamental equilibrium is unique. The Cass-Shell theory was based on an overlapping generations model with incomplete markets. Here we show that this theory remains valid in an infinite-horizon DSGE model with incomplete information.

Proposition 4 *If* $\epsilon > \frac{1+\theta}{\theta} + 2A^{\frac{1+\theta}{\theta}}$*, the mean growth rate of a stochastic growth path is strictly less than the deterministic growth rate without uncertainty (*$\phi_t = \frac{\epsilon-1}{\epsilon}$*), i.e.,* $E\left[s(1+\varphi(\phi_t))\right] < \beta(1 + r(\frac{\epsilon-1}{\epsilon}))$.

Proof. See the Appendix. ■

As an example, consider the limiting case where $\epsilon = \infty$. In this case, the deterministic (gross) growth rate is given by $g^* = \beta(1+r) = \beta\left(1 + \frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\right)$, and the price equation (11) becomes

$$E\phi_t^{\frac{1}{\theta}+1} = E\phi_t^{\frac{1}{\theta}}. \tag{18}$$

Since we restrict our attention to the interval, $0 \leq \phi \leq 1$, the only distribution that can satisfy the above relationship for the marginal cost is the binary distribution, $\phi_t = \{0, 1\}$ with probability $\{1-p, p\}$. Under this distribution, we have $r_t = \varphi_t$, hence $s = \beta E\frac{1+r_{t+1}}{1+\varphi_{t+1}} = \beta$ and $E\varphi_t = p\frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}$. The mean (gross) growth rate is hence given by

$$\bar{g} = \beta\left(1 + p\frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\right), \tag{19}$$

which is strictly less than the deterministic (gross) growth rate g^* for any $p \in (0, 1)$. In this limiting case, the condition, $\epsilon > \frac{1+\theta}{\theta} + 2A^{\frac{1}{\theta}+1}$, is trivially satisfied.

3 Calibration

Let the time period to be a year and the time discounting rate $\beta = 0.98$.[13] For most economies such as the U.S. economy, the markup is around $10\% \sim 20\%$. This implies that $\phi = 0.9 \sim 0.8$ or $\epsilon = 10 \sim 6$. Let the real annual interest rate be 6% and the annual rate of depreciation be 10% in the deterministic economy without sunspots.[14] Hence Equation (4) implies $\theta = r/\delta = 0.6$. Since $\alpha u^{\theta} = \phi A$ and $\delta = \frac{1}{1+\theta}\alpha^{-\frac{1}{\theta}}\left(\phi A\right)^{\frac{\theta+1}{\theta}}$, these two equations can help pin down the values of $\{\alpha, A\}$ once the value of the utilization rate (u) is given. Let $u = \phi = 0.9$ in the deterministic economy (which implies $\epsilon = 10$); then the above two relationships imply $\alpha = 0.189\,38$ and $A = 0.19753$. Given these values, the condition required in Proposition 3, $\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}$ (≈ 3.1), is more than satisfied. It can be shown easily that this condition is still satisfied under other plausible parameter configurations, such as when the annual real interest rate in the deterministic equilibrium is as low as 1.5%.

Based on the calibrated parameter values, the deterministic growth rate is given by $\ln s(1+\bar{\varphi}) = \ln \beta(1+r) \simeq 0.0381$; in other words, the fundamental growth rate is about 4% a year. To compute the mean growth rate of a stochastic growth path, we generate a time series for $\phi_t = \frac{\epsilon-1}{\epsilon}\varepsilon_t$, where the sunspots shock (ε) has the log-normal distribution $\ln \varepsilon \sim N(\mu, \sigma^2)$ with

$$e^{\frac{\sigma^2}{\theta}} E\varepsilon_t = 1. \tag{20}$$

Notice that this distribution satisfies Equation (12) and the condition, $0 < E\varepsilon_t < 1$.

Based on these calibrated parameter values, Table 1 shows the statistical relationship between volatility and mean growth rate for the range of σ that yields empirically plausible mean growth rates. The statistics reported in the table are estimates based on simulated time series with sample size of 10^6. The table shows that, as the standard deviation of the sunspots shock (σ) increases, the standard deviation of the stochastic growth rate (σ_g) also increases, while the mean growth rate of the economy ($\bar{g}$) tends to decrease. Table 2 shows that the same result is also confirmed for a uniform distribution of sunspots shocks.[15] This prediction of a negative relationship between volatility and growth is consistent with the empirical regularity documented by Ramey and Ramey (1995) in cross-country data.

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[13] This value is based on the empirical analysis of Reis (2005).

[14] The average interest rate in the model can be significantly lower under the influence of sunspots than it is in the deterministic equilibrium.

[15] Even with the large sample size, the standard deviation of the growth rate (σ_g) is quite large for the log-normal distribution, suggesting that the estimated mean growth rate can have large standard errors. Despite this, the tendency for the mean growth rate to decline as the growth volatility increases is clear from Table 1. When a uniform distribution is assumed instead for sunspots shocks, the standard error of the growth rate (σ_g) is much smaller and the mean growth rate is more tightly estimated, which makes the negative relationship between volatility and growth even clearer (see Table 2). Note that under the uniform distribution the growth rate of the model is always positive when the parameters of the distribution (mean and variance) of sunspots shocks satisfy Equation (12).

Table 1. Predicted Volatility and Growth (Log-Normal Distribution)

σ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\bar{g}(\%)$	3.81	3.68	3.32	2.83	1.83	1.54	0.62	-0.11	-1.01	-1.22	-2.79
σ_g	0	0.003	0.01	0.02	0.03	0.05	0.07	0.09	0.11	0.15	0.32

Table 2. Predicted Volatility and Growth (Uniform Distribution)

σ	0	0.029	0.058	0.087	0.12	0.14	0.17	0.23	0.29	0.35	0.40
$\bar{g}(\%)$	3.81	3.79	3.77	3.70	3.63	3.54	3.41	3.10	2.69	2.17	1.70
σ_g	0	0.001	0.002	0.003	0.004	0.005	0.007	0.011	0.016	0.022	0.026

Figure 2 shows simulations of the stochastic growth paths of consumption and the implied log consumption levels for each of the distributions considered above. In particular, the simulation under the log-normal distribution is presented in the first row windows (A and B), and the simulation based on uniform distribution is presented in the second row windows (C and D). The growth rate series are graphed in the left column windows (A and C) and the log output level series are graphed in the right column windows (B and D). In windows showing the growth series (Window A and C), the horizontal line is the deterministic growth rate in the absence of sunspots shocks, the solid lines represent a growth rate series under the influence of a particular sunspots process, and the dashed lines represent the annual consumption growth of the U.S. economy for the period 1947-2005. It is seen that the model is able to generate similar volatility in growth rate to the U.S. data. Since the mean growth rate in the model under a particular sunspots process is lower than that of the U.S. data, the implied consumption level (Window B or D) is stochastically dominated by the U.S. consumption level. Notice that a mean growth rate similar to the actual U.S. data can also be generated from the model by using sunspots shocks with a smaller variance than the one represented by the solid lines. As suggested by Windows B and D, along a lower consumption growth path due to a higher volatility, the loss in consumption is irreversible (unrecoverable) even if the mean growth rate later recovers to the previous level due to a decrease in volatility. The fact that such a large and ever increasing gap in consumption levels, in sharp contrast to the Okun's gap and the random walk phenomenon, can be caused by business cycles (volatility) alone is striking. The lesson is that, when growth is endogenous, fluctuations can affect not only the consumption level permanently, but also its long-run growth rate permanently.

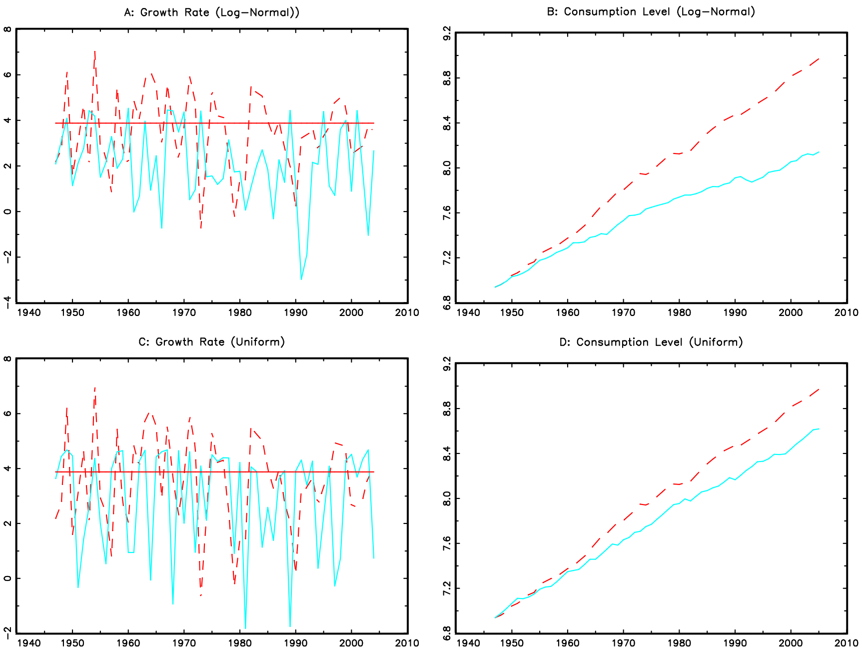

Figure 2. Volatility and Growth (—— Model; - - - Data).

4 Welfare Cost of Fluctuations

4.1 The Lucas Calculation

The Lucas calculation of the cost of business cycles is based on a simple yet fundamental assumption: volatility and growth are unrelated. Given this dichotomy and the fact that the aggregate consumption series is smooth, Lucas (1987 and 2003) concludes that the welfare cost of fluctuations is trivial in terms of consumption goods. Suppose that a representative consumer is endowed with the stochastic consumption stream,

$$c_t = Ae^{ut}e^{-(1/2)\sigma^2}\varepsilon_t, \tag{21}$$

where u is a deterministic growth rate and $\ln(\varepsilon_t)$ is a normally distributed random variable with zero mean and variance σ^2. Hence $Ee^{-(1/2)\sigma^2}\varepsilon_t = 1$. The preference over consumption is assumed to be $E\sum_{t=0}^{\infty}\left(\beta^t \frac{c_t^{1-\gamma}}{1-\gamma}\right)$. The welfare gain can be computed as the percentage increase in consumption one would get by eliminating all the volatility, namely:

$$E\sum_{t=0}^{\infty}\left[\beta^t \frac{((1+\lambda)c_t)^{1-\gamma}}{1-\gamma}\right] = \sum_{t=0}^{\infty}\left[\beta^t \frac{\left(Ae^{ut}\right)^{1-\gamma}}{1-\gamma}\right], \tag{22}$$

where λ measures the welfare gain. Since growth and fluctuations are unrelated, λ can be computed easily by comparing the utilities in a single period:

$$E\left((1+\lambda)c_t\right)^{1+\gamma} = \left(Ae^{ut}\right)^{1+\gamma}, \tag{23}$$

which implies $\lambda \approx \frac{1}{2}\gamma\sigma^2$. The annual U.S. real consumption growth in the period of 1947-2005 is about 3.5% with a standard deviation of 0.0165. Assuming log utility ($\gamma = 1$), the welfare cost is estimated to be $\lambda \approx \frac{1}{2}(0.0165)^2 \approx 0.014\%$. This is less than 1.5¢ for every \$100 of annual consumption.[16]

4.2 Calculation based on Hall's (1978) Random Walk

A crucial feature of the Lucas calculation is that random shocks to consumption have no permanent effect on the consumption level. According to the permanent income theory, however, consumption follows a random walk, hence transitory shocks can have permanent effects (Hall, 1978). Adopting the random walk framework, the consumption path can be described by

$$c_t = c_{t-1}(e^{u-\frac{\sigma^2}{2}}\varepsilon_t), \tag{24}$$

where u is a drift term in the random walk specification of log consumption, which determines the average growth rate of consumption. This characterization of consumption is also an implication of the RBC theory where technology shocks follow random walks. Suppose that the initial consumption level is given by $c_0 = A$. Equation (24) implies that in the absence of uncertainty (i.e., $\varepsilon_t = e^{\sigma^2/2}$ for all t), consumption grows at the rate $u : c_t = Ae^{ut}$. It also implies that under random shocks consumption evolves according to

$$c_t = Ae^{\left(u-\sigma^2/2\right)t}\varepsilon_1\varepsilon_2...\varepsilon_t. \tag{25}$$

The welfare cost of fluctuations can then be computed as the solution (λ) to Equation (22) based on the random-walk consumption in (25). Again assuming log utility ($\gamma = 1$) and $\ln\varepsilon_t \sim N(0, \sigma^2)$, Equation (22) implies

$$\frac{\ln(1+\lambda)}{1-\beta} - \frac{\sigma^2}{2}\left(\beta + 2\beta^2 + 3\beta^3 + ...\right) = 0 \tag{26}$$

Solving for λ we get

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[16] Of course, a higher γ can increase the estimation. Micro evidence suggests that $\gamma \in [1, 4]$. But even with $\gamma = 100$, the annual cost of business cycle is still less than 1.5 percent of consumption.

$$\lambda \approx \frac{\sigma^2}{2}\frac{\beta}{(1-\beta)}. \tag{27}$$

Notice that the welfare measure under the random walk assumption is a multiplier, $\frac{\beta}{1-\beta}$, times the welfare measure of Lucas. This is the result obtained by Obstfeld (1994).[17] This multiplier exists because a one dollar increase in consumption today is translated into a $\sum_{t=1}^{\infty}\beta^t = \frac{\beta}{1-\beta}$ dollar increase in life-time consumption. This suggests that when shocks to consumption have permanent effects, the welfare cost of business cycles can be potentially much larger. Letting $\beta = 0.98$ and $\sigma = 0.0165$, we get $\lambda \approx 0.67\%$. This is more than 47 times larger than the welfare gain under the Lucas specification of the consumption path. However, it is still small in absolute magnitude: less than one dollar for every \$100 of annual consumption. Notice that this calculation is still based on the assumption that volatility and growth are unrelated. Namely, even if shocks have permanent effects on the level of consumption, they have no effects on the average growth rate of consumption. Consequently, the welfare cost of fluctuations is small.

4.3 Calculation based on Ramey and Ramey (1995)

According to the empirical studies of Ramey and Ramey, volatility and growth are negatively related. Hence eliminating volatility should increase the growth rate, which implies a large welfare cost of business cycles, consistent with Lucas's (1987) analysis on the welfare effect of long-run growth. But Lucas did not relate business cycle to growth, hence he failed to appreciate the welfare cost of fluctuations. To illustrate this, consider a counterfactual experiment where completely removing uncertainty can increase the growth rate by π percent from u to $u(1+\pi)$. Then Equation (22) becomes

$$E\sum_{t=0}^{\infty}\left[\beta^t\frac{((1+\lambda)c_t)^{1-\gamma}}{1-\gamma}\right] = \sum_{t=0}^{\infty}\left[\beta^t\frac{\left(Ae^{u(1+\pi)t}\right)^{1-\gamma}}{1-\gamma}\right]. \tag{28}$$

Under the random-walk consumption path (25), Equation (28) then becomes

$$\frac{\ln(1+\lambda)}{1-\beta} - \frac{\sigma^2}{2}\left(\beta+2\beta^2+3\beta^3+...\right) = \pi u\left(\beta+2\beta^2+3\beta^3+...\right), \tag{29}$$

which implies $\lambda \approx \left(\frac{\sigma^2}{2}+\pi u\right)\frac{\beta}{1-\beta}$. According to Ramey and Ramey (1995, p1141), one standard deviation of the volatility in growth rate of output translates into about one-third of a percentage point of the mean growth rate. Applying this estimate to consumption, it means that by decreasing

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[17] Also see Reis (2005) for a more general ARMA specification of the consumption process.

the consumption volatility from $\sigma = 0.0165$ to zero, the gain in growth rate is about $\frac{0.0165}{3} = 0.55\%$, which is about 16% of the current mean consumption growth rate for the U.S. economy ($u = 3.5\%$). This implies that $\pi = 16\%$ and $\pi u = 0.55\%$. Assuming $\beta = 0.98$, we have $\lambda \approx 28\%$. This is an enormous welfare gain: more than a quarter of total annual consumption.[18]

4.4 Calculation Based On Our Model

Consumption in our model follows the path $c_t = c_{t-1}[s(1+\varphi_t)]$, where $c_0 = (1-s)(1+\varphi_0)k_0$. Notice that since the sunspots shocks are *i.i.d.*, we have $E_0 g(\varphi_1) = E_0 g(\varphi_2) = \cdots = E_0 g(\varphi_t)$ for all $t > 0$. Hence the expected life-time utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t(1+\lambda)) = \frac{\ln(1+\lambda)}{1-\beta} + \frac{\ln c_0}{1-\beta} + \frac{\beta\left(\ln s + E\ln(1+\varphi)\right)}{(1-\beta)^2}. \tag{30}$$

In the absence of uncertainty, the model implies $\phi = \frac{\sigma-1}{\sigma}$, and the fundamental growth rate of consumption is given by $\ln \beta(1+r) = 3.81\%$. The life-time value of the deterministic consumption path is given by

$$\sum_{t=0}^{\infty} \beta^t \ln\left(c_0 (1.0388)^t\right) = \frac{\ln c_0}{1-\beta} + \frac{\beta \ln(1.0388)}{(1-\beta)^2}. \tag{31}$$

Comparing the two expressions in (30) and (31) gives the welfare gain:

$$\lambda \approx \frac{\beta}{1-\beta}\left(0.0381 - E\ln s(1+\varphi)\right), \tag{32}$$

Notice that the welfare gain is the multiplier $(\frac{\beta}{1-\beta})$ times the difference between the maximum sustainable growth rate under full information and the mean of the stochastic growth rate under sunspots shocks. As Proposition 3 shows, the mean growth rate of a stochastic growth path is strictly less than the fundamental growth rate. Hence λ is always positive. Furthermore, as Table 1 and Table 2 both show, when the volatility of sunspots shocks increases in the model, the mean of the stochastic growth rate, $E\ln s(1+\varphi)$, decreases, which increases the value of λ. For example, under the assumption of a log-normal distribution (Table 1), a standard deviation of 0.3 for sunspots shocks (ε_t) implies a stochastic consumption growth path with a standard deviation $\sigma_g = 0.02$, which is similar to the U.S. consumption data. Under this volatility, the mean consumption growth is 2.83%. Substituting this number into Equation (32) implies $\lambda = 24\%$. Under the assumption of a uniform distribution (Table 2), a standard deviation of 0.29 for sunspots shocks implies a

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[18] Interestingly, this estimate is very close to the estimate obtained by Alvarez and Jermann (2004) using a non-parametric asset-pricing approach.

stochastic growth path with a standard deviation $\sigma_g = 0.016$, which almost exactly matches the U.S. consumption data. Under this volatility, the mean consumption growth is 2.69%. Substituting this number into Equation (32) implies $\lambda = 27\%$.[19] Thus, based on our endogenous growth model, the welfare cost of business cycles with volatility similar to the U.S. data is about a quarter of annual consumption. The estimates are quite consistent with the estimate based on Ramey and Ramey's empirical studies and the estimates obtained by Alvarez and Jermann (2004) based on a nonparametric asset-pricing approach. Although our quantitative estimates of the welfare cost depend on the calibrated parameter values of the model (such as β), their qualitative scales are robust to small changes in the parameter values because fluctuations in the model can significantly decrease the average growth rate for a wide range of plausible parameter values, and the welfare cost of a small decrease in growth is very large (as realized by Lucas, 1987).

5 Welfare Gain from Stabilization Policy

A large welfare cost of fluctuations by no means implies an equally large welfare gain from stabilization policy. The reason is that volatile consumption can itself be optimal. For example, Barlevy (2004a) provides a model in which the welfare cost of fluctuations can be at least as large as $7 - 8$ percent of annual consumption. But, since volatile consumption is an optimal response to technology changes in his model, there is no gain from reducing or eliminating consumption fluctuations. Thus, despite the large welfare cost of business cycles, the policy implication of Barlevy's model is the same as the Lucas calculation: stabilization policy is counter-productive and hence undesirable. However, fluctuations in the real world can be highly inefficient as they are in our model. In this case, the welfare gain from stabilizing consumption is as large as the welfare cost of fluctuations.

5.1 Pareto Optimal Allocation

Consider the Pareto optimal allocation first. Without loss of generality, assume $\alpha = 1$. The Pareto allocation is determined by solving the following social planner problem,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t) \tag{33}$$

subject to

$$C_t + K_{t+1} = Au_t K_t + (1 - \delta_t) K_t, \tag{34}$$

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[19] Suppose we use the actual U.S. consumption growth rate ($u = 3.5\%$) instead. Equation (32) implies that the welfare cost of volatility is about 7% of annual consumption.

and

$$\delta_t = \frac{1}{1+\theta} u_t^{1+\theta}. \tag{35}$$

Define $\varphi_t \equiv Au_t - \delta_t$. It can be shown that under optimal capacity utilization we have $\delta_t = \frac{A^{(1+\theta)/\theta}}{1+\theta}$ and

$$\varphi_t = \frac{\theta}{1+\theta} A^{(1+\theta)/\theta}. \tag{36}$$

Thus, in the absence of technology change, δ_t and φ_t are constant. The optimal allocation is thus given by

$$C_t = (1-\beta)\,(1+\varphi)K_t, \tag{37}$$

$$K_{t+1} = \beta(1+\varphi)K_t, \tag{38}$$

where the balanced growth rate $\beta(1+\varphi)$ is given by $\beta(1+\frac{\theta}{1+\theta}A^{(1+\theta)/\theta})$. The result also holds for the case where A is stochastic.

5.2 Optimal Policy without Sunspots

Under imperfect competition and in the absence of extrinsic uncertainty (i.e., no sunspots), the Pareto optimal allocation can be achieved by subsidizing monopolistic firms for production, which is a standard result in the literature. To see this, consider that the government subsidizes the intermediate producers by the amount τ for each unit of good it sells. The profit maximization problem for each intermediate good producer becomes

$$\max\,(p+\tau-\phi))\,p^{-\epsilon}y. \tag{39}$$

The optimal price is given by $p = \frac{\epsilon}{\epsilon-1}(\phi-\tau)$, which is lower than the monopolistic price $\frac{\epsilon}{\epsilon-1}\phi$. In equilibrium, $p=1$, hence the optimal rate of subsidy must satisfy $\tau = \phi - \frac{\epsilon-1}{\epsilon}$. Since Pareto allocation requires $\phi = 1$, the optimal subsidy is given by $\tau = \frac{1}{\epsilon}$. Notice that a positive price requires $\tau < 1$, which is satisfied since $\epsilon > 1$. The equilibrium allocation of consumption and capital is given by

$$C_t = (1-\beta)\left(1+\frac{\theta}{1+\theta}A^{\frac{1+\theta}{\theta}}\right)K_t, \tag{40}$$

$$K_{t+1} = \beta\left(1+\frac{\theta}{1+\theta}A^{\frac{1+\theta}{\theta}}\right)K_t, \tag{41}$$

which is Pareto optimal.

Notice that the optimal policy allows monopolist firms to make positive profits that are the same as the amount they would make without subsidies. To finance the amount of subsidies, τY, the government can use a non-distortionary lump-sum tax (T) on household income. A balanced budget implies $T_t = \tau Y_t$.

5.3 Optimal Policy under Sunspots

When there exists imperfect information, there is an additional source of inefficiency in the economy - the decrease of the average growth rate due to sunspots-driven fluctuations. Hence, we can design two separate policies to deal with the two source of inefficiency: one is the subsidizing policy (τ) discussed above, and another is a stabilization policy (ω) to deal with volatility specifically. Since fluctuations arise from firms' expectation about other firms' production levels, the stabilization policy can focus on stabilizing firms' output level via subsidizing capacity utilization. Consider a policy that subsidizes a firm's marginal cost of production by the amount, ω_t, for each additional unit of output produced or for each unit-increase in capacity utilization. The cost minimization problem of the intermediate good producer becomes to minimize $(r + \delta(i) - \omega A u(i))\, k(i)$ subject to $Au(i)k(i) \geqslant y(i)$. The first order conditions are

$$r + \delta(i) - \omega A u(i) = \phi A u(i) \tag{42}$$

$$u(i)^{\theta} - \omega A = \phi A. \tag{43}$$

These equations imply $u_t = (\phi_t + \omega_t)^{\frac{1}{\theta}} A^{\frac{1}{\theta}}$ and $r_t = \theta \delta_t = \frac{\theta}{1+\theta} \left[(\phi_t + \omega_t) A\right]^{\frac{\theta+1}{\theta}}$. The output level is given by

$$y(i) = (\phi_t + \omega_t)^{\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} k(i). \tag{44}$$

The price-setting problem for an intermediate good firm is the same as before, which is to maximize the expected profits $E\left[(p(i) + \tau - \phi)\, p(i)^{-\epsilon} Y\right]$. The optimal monopoly price is given by

$$p(i) = \frac{\epsilon}{\epsilon - 1} \frac{E(\phi_t - \tau) Y_t}{E Y_t}. \tag{45}$$

In a symmetric equilibrium, $p(i) = 1$, $y(i) = Y$, and $k(i) = K$. Substituting out output in the above price equation using Equation (44) gives

$$1 = \frac{\epsilon}{\epsilon - 1} \frac{E(\phi_t - \tau)\, (\phi_t + \omega_t)^{\frac{1}{\theta}}}{E\, (\phi_t + \omega_t)^{\frac{1}{\theta}}}. \tag{46}$$

As before, we can set $\tau = \frac{1}{\epsilon}$. Hence the pricing rule requires

$$E\left(\phi_t+\omega_t\right)^{\frac{1}{\theta}}=\frac{1}{\epsilon-1}E\left(\phi_t+\omega_t\right)^{\frac{1}{\theta}}\left(\epsilon\phi_t-1\right). \tag{47}$$

Clearly, the optimal stabilization policy is given by

$$\omega_t=1-\phi_t. \tag{48}$$

Under this policy, the paths of consumption and capital are given by

$$C_t=(1-\beta)\left(1+\frac{\theta}{1+\theta}A^{\frac{1+\theta}{\theta}}\right)K_t, \tag{49}$$

$$K_{t+1}=\beta\left(1+\frac{\theta}{1+\theta}A^{\frac{1+\theta}{\theta}}\right)K_t, \tag{50}$$

which were shown to be Pareto optimal previously. Equation (46) implies that the marginal cost is given by $E\phi_t=1$. Although the marginal cost can be stochastic in equilibrium, its volatility has no consequence on the real variables in the economy under the stabilization policy ω_t. In order to have a balanced budget for the government, the government can simultaneously impose a lump-sum income tax on households such that:

$$\begin{aligned} T_t &= (\tau+\omega_t)Y_t \\ &= \left(\frac{1}{\epsilon}+(1-\phi_t)\right)Y_t. \end{aligned} \tag{51}$$

We can also combine the two policies together into one single policy that eliminates both types of inefficiencies simultaneously. Clearly, it is still required that $\omega_t=1-\phi_t$ so as to ensure constant growth rate. Setting $\tau=\omega$ and replacing τ by ω in Equation (46) gives $E\phi_t=\frac{2\epsilon-1}{2\epsilon}$. In this case, the expected profit is the same as before but the markup is smaller than the monopolistic markup but greater than zero.

5.4 Optimal Policy under Imperfect Information for the Government

5.4.1 Case 1: Without Technology Shocks

The stabilization policy, $\omega_t=1-\phi_t$, requires that the government have full information about the marginal cost, or that ϕ_t is observable to the Government. In reality, the marginal cost is difficult to observe directly. Here we discuss optimal stabilization policies which do not depend on the observability of the marginal cost. The only assumption required in this policy is that the government can observe the aggregate utilization rate of capital.

Without loss of generality, continue to assume $\alpha = 1$ and, in addition, assume $\epsilon = \infty$ so that the only source of inefficiency is from sunspots-driven fluctuations.[20] Denote $u = \int_0^1 u(i)di$ as the aggregate (average) capacity utilization rate, and denote $\omega_t = \omega(u_t)$ as the optimal subsidy to each firm's marginal cost of production via capacity utilization. A firm's total cost of production is given by $(r + \delta(i) - \omega(u)Au(i))k(i)$. In a symmetric equilibrium, the first-order condition from cost minimization is given by

$$u_t^\theta - \omega(u_t)A = \phi_t A, \tag{52}$$

where ϕ is the marginal cost. Notice that the Pareto optimal allocation is given by a constant capacity utilization rate, $u^* = A^{\frac{1}{\theta}}$. The key of the policy design is to find an incentive compatible subsidy policy $\omega(u_t)$ such that it is in the best interest of all firms to choose $u_t = u^*$ in equilibrium.

Proposition 5 *The subsidy policy,*

$$\omega(u) = \begin{cases} 0 & \text{if } u_t > u^* \\ \frac{u^\theta}{A} - \frac{A^{\frac{1}{\theta}}}{u} & \text{if } u_t \leq u^* \end{cases}. \tag{53}$$

achieves the Pareto allocation.

Proof. Equation (52) implies

$$\phi = \frac{u^\theta - \omega(u)A}{A}. \tag{54}$$

The monopolist price is determined by the equation $E\phi_t Y_t = EY_t$. Substituting out ϕ_t and Y in the price equation gives

$$E\left[u_t^{1+\theta} - \omega(u_t)Au_t - Au_t\right] = 0, \tag{55}$$

which is the firm's profit maximization condition or incentive compatibility condition. Define the function $P(u) \equiv u^{\theta+1} - \omega(u)Au - Au$. Substituting the subsidy policy into $P(u)$ gives $P(u) > 0$ for $u \neq u^*$ and $P(u) = 0$ for $u = u^*$. Since $EP(u) \neq 0$ is not optimal (or incentive compatible), firms will never choose $u_t \neq u^*$ under the above subsidy policy. Note that under the optimal capacity utilization u^*, the marginal cost is given by $\phi_t = 1$. Hence the allocation under $\omega(u)$ is Pareto optimal. ■

Clearly the functional form of the optimal policy is not unique. In fact, any policy function $\omega(u)$ such that it makes $P(u) = 0$ if $u_t = u^*$ and $P(u) \neq 0$ if $u \neq u^*$ is optimal. Whatever the optimal policy is, it must provide incentives to induce firms to choose u^* and penalize them when

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[20] Namely, we consider stabilization policies that are separate from τ.

$u \neq u^*$. Otherwise the policy is ineffective. For example, let $\omega(u) = \frac{u^\theta - A}{A}$. This policy is derived by setting $\phi = 1$ in equation (52). This policy is not effective in eliminating sunspots equilibria because under this policy, $P(u) = 0$ regardless of u. Hence it cannot eliminate sunspots-driven fluctuations.

5.4.2 Case 2: With Technology Shocks

The previous analyses have assumed away any fundamental shocks in order to simplify the exposition. Although allowing for fundamental shocks in the model will not change the results, it does complicate the issue of policy design when information is imperfect for the government. For example, since sunspots shocks to the marginal cost behave very much like technology shocks (i.e., sunspots shocks affect the marginal product of capital by affecting capacity utilization), it may not be possible for the government to distinguish where the shocks are coming from if neither sunspots shocks nor technology shocks are directly observable to the government. In this case, policies that completely stabilize the growth rate are no longer optimal if technology shocks dominate.

We show here that it is still possible to find stabilization policies that completely eliminate the undesirable effects of sunspots, provided that the government has access to certain types of information. Assume that the public information available to the government include firms' output (y) and capital stock (k), and the market interest rate r. To prevent the government from deducing the level of technology (A_t) from the production function, we assume that the government cannot observe the rate of capacity utilization (u). Define a firm's output-capital ratio as $z(i) = y(i)/k(i) = Au(i)$ and the aggregate (average) output-capital ratio as $z = \int z(i)di$. Because $z = Y/K$ is observable to the government, it can be used as the basis for designing stabilization policy. However, note that since $z = Au$, the government cannot differentiate whether movements in z are caused by technology or by capacity utilization driven by sunspots.

It can be shown that under technology shocks the Pareto optimal allocation is given by $u_t = A_t^{1/\theta}$, $Y_t = A_t^{(1+\theta)/\theta} K_t$, and $r_t = \frac{\theta}{1+\theta} A_t^{(1+\theta)/\theta}$. This implies that the Pareto optimal output-capital ratio is given by $z_t^* = \frac{1+\theta}{\theta} r_t$ or $r_t K_t / Y_t = \frac{\theta}{1+\theta}$. If there is influence from sunspots, however, it can be shown that $z_t = \frac{1+\theta}{\theta} \frac{r_t}{\phi_t}$ or $r_t K_t / Y_t = \frac{\theta}{1+\theta} \phi_t$. The key of the policy design is to find an incentive compatible subsidy policy such that it is in the best interest of all firms to choose $z_t = z_t^*$ in equilibrium.

Denote $\omega_t = \omega(r, z)$ as the optimal subsidy to each firm's marginal cost of production, which individual firms take as given. A firm's total cost of production is then given by $(r + \delta(i) - \omega(r, z) A u(i)) k(i)$. In a symmetric equilibrium, the first-order conditions can be expressed as

$$r + \delta = (\phi + \omega(r,z))z \tag{56}$$

$$u^{1+\theta} = (\phi + \omega(r,z))z \tag{57}$$

These imply $r = \theta\delta$ and

$$\phi = \frac{(1+\theta)}{\theta}\frac{r}{z} - \omega(r,z). \tag{58}$$

The optimal monopoly price is still determined by the pricing rule, $E\phi Y = EY$. Since $Y = zK$ and K is known to firms in the beginning of each period, substituting out ϕ and Y in the pricing rule gives

$$E\left[\frac{1+\theta}{\theta}r - \omega(r,z)z - z\right] = 0. \tag{59}$$

Define the function $P(r,z) \equiv \frac{(1+\theta)}{\theta}r - \omega(z)z - z$. Since $EP(r,z) \neq 0$ is not optimal (or incentive compatible) to firms, the key of the policy design is to set the subsidy rate $\omega(r,z)$ such that firms will never choose an output-capital ratio $z_t \neq z_t^*$ under the subsidy policy. Hence, any policy function $\omega(r,z)$ such that it makes $P(r,z) = 0$ if $z = z^*$ and $P(r,z) \neq 0$ if $z \neq z^*$ can achieve the Pareto allocation. For example, it is easy to check that the following policy can achieve the Pareto allocation:

$$\omega(r,z) = \begin{cases} 0 & \text{if } z_t \geq \frac{1+\theta}{\theta}r \\ \frac{2(1+\theta)}{\theta}\frac{r}{z} - 2 & \text{if } z < \frac{1+\theta}{\theta}z \end{cases}. \tag{60}$$

Under this policy, we have $EP(r,z) = 0$ if and only if $z = z^* = \frac{1+\theta}{\theta}r$. When $z = z^*$, we can show that $\phi_t = 1$, $u_t = A_t^{1/\theta}$, and $r_t = \frac{\theta}{1+\theta}A_t^{(1+\theta)/\theta}$. Hence the allocation under the policy $\omega(r,z)$ is Pareto optimal.

6 Conclusion

When business cycles and growth are intimately related and fluctuations are endogenous (e.g., due to animal spirits), both the cost of business cycles and the gain from stabilization can be enormous. This is true regardless of fundamental shocks because the impact of such shocks can be amplified by self-fulfilling expectations. For example, the reason oil shocks in the 1970s have had such a large impact on the U.S. economy could well be that they were amplified by the private sector's expectations and the public sector's lack of experience in stabilizing such shocks (see, e.g., Hamilton

1988a and 1988b, and Aguiar-Conraria and Wen 2006). This paper demonstrates the possibility of such links between growth and business cycles. It shows that ordinary market imperfections (such as imperfect competition and imperfect information) can cause coordination failures, hence making expectations self-fulfilling. In an endogenous growth environment, fluctuations in the marginal cost can directly translate into fluctuations in growth. Given that volatility due to endogenous fluctuations can reduce the mean growth rate, the welfare gain from stabilization can be potentially large. Calibrations to the U.S. data show that the welfare gain from further stabilizing the U.S. economy can be as large as about 25% of annual consumption, which is in sharp contrast to the Lucas (1987) calculation and the policy implications of that calculation. The optimal stabilization policy we found is consistent with those in practice: stabilize output growth (or capacity utilization) around a potential target.

Appendix: Proof of Proposition 3.

Proof. The key of the proof is to show that $E\phi_t^{\frac{1}{\theta}} \leq \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}$ and that the average growth rate is a strictly increasing function of $E\phi_t^{\frac{1}{\theta}}$ under certain conditions. In such a case, the maximum growth rate is achieved when $\phi_t = \frac{\epsilon-1}{\varepsilon}$.

The growth rate of the model is given by

$$g_t = s(1+\varphi_t), \tag{61}$$

where $\varphi_t = \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t^{\frac{1}{\theta}} (1 - \frac{\phi_t}{1+\theta})$, $s = \beta E \frac{1+r_t}{1+\varphi_t}$, and $r_t = \frac{\theta}{1+\theta} \alpha^{-\frac{1}{\theta}} (\phi_t A)^{\frac{1+\theta}{\theta}}$. The monopolistic price follows the rule, $E\phi_t^{\frac{1+\theta}{\theta}} = \frac{\epsilon-1}{\epsilon} E\phi_t^{\frac{1}{\theta}}$. Since $Ex^{1+\theta} \geq (Ex)^{1+\theta}$, we have $\frac{\epsilon-1}{\epsilon} E\phi_t^{\frac{1}{\theta}} = E\phi_t^{\frac{1+\theta}{\theta}} \geq \left(E\phi_t^{\frac{1}{\theta}}\right)^{1+\theta}$. It follows that

$$E\phi_t^{\frac{1}{\theta}} \leq \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}. \tag{62}$$

Given the definition of φ_t, we have

$$\begin{aligned} E\varphi_t &= \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \left(E\phi_t^{\frac{1}{\theta}} - \frac{1}{1+\theta} E\phi_t^{\frac{1+\theta}{\theta}} \right) \\ &= \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} E\phi_t^{\frac{1}{\theta}} \left(1 - \frac{\epsilon-1}{\epsilon} \frac{1}{1+\theta} \right) \\ &\leq \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}} \left(1 - \frac{\epsilon-1}{\epsilon} \frac{1}{1+\theta} \right). \end{aligned} \tag{63}$$

Notice that s can be approximated as

$$\begin{aligned} s &\simeq \beta E(1+r_t-\varphi_t) \qquad (64)\\ &= \beta E\left(1+\frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}\left(\phi_t A\right)^{\frac{1+\theta}{\theta}}-\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\phi_t{}^{\frac{1}{\theta}}\left(1-\frac{\phi_t}{1+\theta}\right)\right)\\ &= \beta\left(1+\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\left(E\phi_t^{\frac{\theta+1}{\theta}}-E\phi_t^{\frac{1}{\theta}}\right)\right)\\ &= \beta\left(1-\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\frac{1}{\epsilon}E\phi_t^{\frac{1}{\theta}}\right). \end{aligned}$$

Denoting $\bar{x}\equiv E\phi_t^{\frac{1}{\theta}}$, the mean growth rate is then given by

$$\begin{aligned} \bar{g} &= s\left(1+E\varphi_t\right) \qquad (65)\\ &= \beta\left[1-\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\frac{1}{\epsilon}\bar{x}\right]\left[1+\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)\bar{x}\right]. \end{aligned}$$

If the condition,

$$\bar{x}<\frac{\epsilon-\left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}}, \qquad (66)$$

is satisfied, then $\bar{g}$ is a strictly increasing function of $\bar{x}$. Since $\bar{x}\leq\left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}$ according to (62), hence the maximum growth rate will be achieved by the certainty equilibrium where $\phi_t=\frac{\epsilon-1}{\epsilon}$, provided that Condition (66) holds.

But a sufficient condition for (66) to hold is the condition,

$$\left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}<\frac{\epsilon-\left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}}. \qquad (67)$$

Notice that $\left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}\leq\frac{1+\theta}{\theta}$ since $\epsilon\leq\infty$, hence we have the following inequality for the right-hand side of Equation (67):

$$\frac{\epsilon-\left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-1/\theta}A^{(1+\theta)/\theta}}\geq\frac{\epsilon-\frac{1+\theta}{\theta}}{2\alpha^{-1/\theta}A^{(1+\theta)/\theta}}. \qquad (68)$$

If the right-hand side of the above equation is greater than one, namely, if

$$\epsilon>\frac{1+\theta}{\theta}+2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}, \qquad (69)$$

we then have

$$\frac{\epsilon-\left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}}\geq\frac{\epsilon-\left(1+\theta\right)/\theta}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}}>1\geq\left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}. \tag{70}$$

Hence, a sufficient condition for the inequality (66) to hold is (69). ■

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