Sovereign Debt, Defaults and Bailouts*

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Abstract

Governments default on their foreign debts with surprising regularity. Given the costs of default, to both creditors and to the defaulting country, is it possible to reform the international financial system to minimize the incidence and severity of debt crises while still promoting efficient capital flows? This paper uses evidence from a new historical dataset on the relationship between sovereign borrowing and defaults to discipline the development of a model in

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which defaults occur in equilibrium. In the context of this model, the optimal form of supra-national intervention ("bailouts") is derived.

INTRODUCTION

Governments default on their foreign debts with surprising regularity. In a typical year, approximately ten percent of sovereign governments fail to meet their financial obligations to foreign bondholders and commercial banks (Suter 1992). During systemic crises such as the Great Depression, as many as half the countries in the world have been in arrears on part or all of their international debts. Given the costs of default, not only to creditors but also to the defaulting country, it is natural to ask whether and how the international financial system might be reformed to minimize the incidence and severity of sovereign debt crises.

Before scholars and policymakers can talk sensibly about reforming the "international financial architecture," however, we need a clearer understanding of the incentives facing sovereign governments and private lenders, and the way they interact to produce defaults in practice. Moreover, we need to understand the ways in which policy makers – whether multilateral organizations of creditor country governments – can affect these incentives. Towards this end, this paper begins by studying a new database of sovereign loans, defaults and economic conditions in history with the view of isolating a series of stylized facts about the relationship between capital flows, economic activity and default. Guided by these results, we then develop a model of sovereign debt and default which we use to diagnose appropriate policy prescriptions, and show that a form of "bailout" attains the optimum.

The paper begins by examining the empirical phenomenon of default. Perhaps surprisingly, there is some controversy over how one should define a default in practice. To partially get around this controversy, we argue that it is necessary to take an empirical approach that emphasizes patterns in the levels and terms of capital flows. Drawing on data from a number of different time periods, we argue that "default episodes" are associated with dramatic declines in capital inflows, followed by an resumption of capital flows. Moreover, these data suggest that resumptions occur without prejudice: the terms of re-access do not appear to be sensitive to the default experience.

These facts suggest that defaults can be best thought of as episodes in which trade in capital between creditors and a sovereign government temporarily halts. Motivated by this observation, we present a model of default in which sovereign governments borrow in order to finance a productive investment opportunity. Contracts are limited by both the limited enforceability of contracts, and by asymmetries in information across creditors and sovereign borrowers. Obviously, it is implausible to think that creditors are completely uninformed about economic conditions in a country, and so we take care to allow for the possibility that these information differences are small. Creditors and sovereigns are assumed to interact in a way that allows them to achieve the optimal level of trade, ex ante, within the class of contracts studied. As a result of these information asymmetries, we show that it is often true that these agents will commit to contracts ex ante in which outcomes are very inefficient ex post. We interpret these ex post inefficiencies as defaults, and show that equilibria exist in which capital flows follow the patterns identified in the data.

We then use this model as a laboratory for assessing the optimality of intervention by a third party, which we think of as either a creditor country government or a supra national institution. We allow this policy maker to transfer resources to and from both creditors and sovereigns subject to a zero expected profit constraint under various assumptions on the information available to the policy maker. We interpret such transfer policies as "bailouts." We are particularly concerned with the extent to which a policy that eliminated default may also lower the efficiency of capital flows.

We first show that if the policy maker is completely uninformed about production opportunities, a bailout policy can be devised that not only eliminates default but also implements the efficient level of capital flows between countries. However, the "bailout" do not look like any bailout previously observed in practice. Not only are transfers to and from the policy maker positive even if the absence of default, they tend to take the form of transfers to the policy maker in states of the world when we would have otherwise observed a default. Moreover, although the policy maker makes zero profits in expected value, the transfers can be quite large and along some sample path can require that the policy maker make arbitrarily large losses. We then go on to explore numerically how this policy varies when the policy maker also has some private information about production opportunities.

This paper is related to the considerable theoretical literature on the incentives governing sovereign defaults. Much of the literature on sovereign debt has focused on the answering the question of why sovereigns ever repay their debts. Some theorists emphasize that governments repay to obtain future loans or protect their reputations more generally (e.g. Eaton and Gersovitz 1981), whereas others cite the threat of trade sanctions or diplomatic and military pressure (Bulow and Rogoff 1989a; Rose and Spiegel forthcoming; Finnemore 2003). Theorists have modeled the incentives of creditors, as well, focusing chiefly on the credibility of coordinated lending embargoes (Bulow and Rogoff 1989b; Kletzer and Wright 2000; and Wright 2003a) and the role of institutions in facilitating inter-creditor cooperation (Wright 2003b). In this paper, concern for ones reputation supports repayment, but defaults nevertheless occur in equilibrium as a result of the asymmetric information between creditors and sovereign governments.

A number of recent papers have studied the quantitative implications of variants of the Eaton and Gersovitz (2982) model of default. In recent studies, Arellano (2004) Aguiar and Gopinath (2004), Bai and Zhang (2005), Lizarazo (2005), Cuadra and

Sapriza (2005) and Yuen (2005) follow that paper and model default as providing insurance against bad economic outcomes in a world in which contracts are exogenously limited in their complexity. By contrast we model default as arising in a world in which enforcement and information frictions combine to endogenously limit contracts. This is important for policy: when contracts are limited exogenously, as in the studies listed, optimal policies tend to have the feature that they act to complete the market. However, this runs into the danger that these policies may prove infeasible or undesirable for the very reasons that markets were incomplete in the first place. Instead, in our theory, contractual limitations are modeled explicitly, and both the government and private sector agents are subject to the same limitations. Additionally, some of these studies have difficulty replicating the incidence of default when agents are assumed to have conventional levels of patience. In our model, we show that defaults occur even in the limit when agents are very patient. One other paper that is able to replicate a number of facts on default is the signalling model of Cole, Dow and English (1993). However, in contrast to this model where the pattern of lending inherits the stochastic properties of the exogenous process governing the governments type, in our model equilibrium lending follows a non-stationary process of defaults despite a stationary lending environment.

Finally, this paper is related to the substantial literature on the implementation of efficient allocations in the presence of both enforcement and information frictions. Like the model of Green and Porter (1991), we examine situations in which all agents have private information, and it is optimal to design interactions ex ante such that outcomes can be very inefficient ex post (in their model, a price war). However, unlike that paper which relies on a very specific information structure that limits the ability of players to determine the actions of their opponents, our environment allows observation of actions, but it is the robustness of the mechanism with respect to information differences which produces ex post inefficiencies, or default, in equilibrium.

In particular, our robustness requirement draws upon the work of Miller (2004a,b), also explored in Athey and Miller (2004), who introduces the notion of ex post perfect public equilibrium, and develops the mechanism design approach to contracting in this environment, along the lines initially laid out in Abreu, Pearce and Stachetti (1996).

The rest of this paper is structured as follows, Section 2 introduces our database on sovereign default, borrowing and economic conditions, and uses it to construct the stylized facts on default. Section 3 outlines our model of default and shows that it can produce defaults in equilibrium and that it can replicate these facts. Section 4 derives the form of optimal supranational intervention, while Section 5 concludes.

SOME FACTS ABOUT SOVEREIGN DEFAULT

Defaults

In order to understand the phenomenon of defaults, one must first ask "What is a default?" In the narrowest sense, a default is a well defined contractual and legal provision in a debt contract. For example, modern bonds typically include a range of provisions regarding grace periods and cross default provisions after which the fiscal agent for the bond, or alternatively the creditors holding the bond, can declare that bond to be "in default". The bond typically also includes provisions governing the consequences of such a declaration, including the legal jurisdiction governing the bond, as well as statements regarding the classes of assets that may attached in fulfillment of the bond.

For our purposes, however, this may be both too narrow a definition of a default, and too broad a definition. It may be too narrow because it requires creditors to declare a default, which they may fail to do for a number of reasons even though the bonds or debt contracts provisions have not been met. Perhaps the best known examples of this concern the rescheduling of sovereign bank debts during the 1980s where it has been argued that banks were reluctant to declare the loans in default because of concerns about prudential requirements for write downs, despite the fact that these loans were not being serviced. Similarly, a bond or debt contract may be restructured in advance of an actual missed payment on interest or principal, in such a way as to reduce the value of the security to its holders.

One way to adjust for this concern is to look for instances in which the value of securities and debt contracts have been reduced from the perspective of creditors. Such an approach also has problems in that it requires a lot of information about the details of each security and debt contract, and inevitably requires that the researcher use a lot of their own judgement. However, this approach may be too broad a definition of default to the extent that many debt contracts are part of long term relationships which may include implicit provisions and understandings under which debts will be renegotiated voluntarily to reduce explicit payment terms.

Despite these problems, we adopt this second approach, bearing in mind the ways in which these problems may affect the analysis. Perhaps the best known effort to catalogue default episodes was the enormous undertaking by Suter (1992) which has since been modified and updated on a regular basis by the private sector ratings agency Standard & Poors. This series has also been the basis for a number of empirical studies of default including Reinhart, Rogoff and Savastano (2003).

In using this definition, we exclude provincial defaults, despite the fact that Suter includes the USA as a defaulting nation as a result of the state defaults of 1842-4. This makes it consistent with the exclusion of other provincial defaults in Argentina and Australia in 1930's. A more difficult issue is whether or not we should include defaults on internal debts when they were held by foreigners. During the 1860s, many international creditors appear to have accepted Austria right to impose a tax (partially default) on interest payments on its internal debts. For the time being, we ignore this distinction, which also means that we include Russia recent default on its internal debts. Finally, we also exclude all defaults on official debts, which amongst other things means we exclude the inter-allied war debts, as well as most debts covered by the highly indebted poor countries initiative.

In addition, we also attempt where possible to verify the dates provided by Suter (1992) for the onset and end of defaults using primary sources. The resulting series closely mirrors the Suter/Standard and Poors series, but contains some differences in coverage and timing.

Based on this series, from the end of the Napoleonic Wars and the rise of international lending in 1824 to the present day (2003), 94 countries defaulted a total of 235 times. As has been documented by Reinhart et al (2003), although some countries have few default experiences, others have defaulted a large number of times. Amongst the most common "serial defaulters" are the modern countries (and their historical antecedents) of Uruguay, Mexico, and Costa Rica which each defaulted 8 times according to our measure.

Some of these defaults were very substantial. One of the largest was the Russian repudiation of 1917, which is estimated to have involved sums totalling 1.7 billion pounds sterling. Adjusting for inflation, this is close to the Argentine default of 2001 which is estimated to involve securities with a face value of approximately $90 billion. However, almost half of the Russian repudiation involved its share of the interallied war debts, which were official lending, making Argentina the unquestioned largest default on privately sourced debts.

Credit Market Access

Of primary interest is the question of what the consequences were to a default. The first question we examine concerns the effect on market access. Towards an answer we collected data from a wide number of sources on debt issues throughout history. For the 20th Century, much of the work as regarding bond debt has been completed by Michael Adler and his students at Columbia University who collected data for US securities markets using primarily Moody's Bond Record and the annual reports of the Foreign Bondholders Protective Council (FBPC). This was supplemented by data from the UK from the Stock Exchange Yearbook.

We have extended this collection in several ways. Regarding bond debts, we have extended our coverage back into the 19th Century, and into the German and French markets. Our main sources here included the Course of the Exchange, and Fenn's Compendium of the English and Foreign Funds for the UK, and for France and Germany we used the Manuel des Fonds Publics, Saling's Börsen-Papiere, Annuaire-chaix, Les principales sociétés par actions, Annuaire Desfossés. We also verified and cross checked the Adler database for the US using Fitch & Kimber's Record of Government Debts, and White, Weld and Company's Foreign Dollar Bonds and International Bonds.

All of these data sources concern international bonds, which were the primary source of finance throughout the 19th Century, as well as for much of the 20th Century. The exception is last few decades of the 20th Century where there has also been a substantial amount of bank lending to sovereigns. Our primary data source here is the official publication Borrowing in International Capital Markets. This data source is however limited in its time coverage and we are currently exploring some other sources of bank lending information. Partly as a result of this weakness we focus on the earlier period of bond lending in assessing the loss of market access. There is also another reason for this focus: the current institutional environment governing sovereign defaults appears to be moving towards a system in which most lending is undertaken through bonds, and there is limited official intervention. To the extent that this is true, the earlier period is likely to be more informative about future prospects that is data from the era of bank lending, and where creditor government and supranational intervention has been more substantial.

Tables One and Two present data on the length of exclusion of countries that were in default during the period 1824 to 1969. This period covers 126 defaults according to our measure, and we have confidence in our lending data for 111 of these default-year pairs. Examining this data, we look for the years at which bonds were issued and compare them to the dates of defaults. Bonds that were obviously not new issues were excluded: for example, bonds that were listed as "funding" or "interest arrears" bonds were excluded for the purpose of this calculation. In some other cases, a reading of primary source also reveals that some bonds not carrying such a designation also existed that were also not new issues. These have been excluded on a case by case basis. However, we suspect that some other funding bonds remains in our database. To the extent that this is true, the numbers presented below understate the length of time for which countries were excluded form international financial markets following a default.

Table One Default and Credit Market Access

	1824 -1829	1830 -1839	1840 -1849	1850 -1859	1860 -1869	1870 -1879	1880 -1889	1890 -1899	1824 -1899
# defaults	15	3	2	3	5	13	2	17	60
# w. loan data	14	3	2	3	3	6	2	14	47
mean excl.	49	23	17	12	19	22	11	8	24
median excl.	38	31	17	12	23	17	11	7	19

	1900 -1909	1910 -1919	1920 -1929	1930 -1939	1940 -1949	1960 -1969	1900 -1969	1824 -1969
# defaults	6	14	4	30	6	3	66	126
# w. loan data	6	14	3	30	6	3	64	111
mean excl.	4	21	9	21	24	4	19	21
median excl.	2	7	2	11	18	4	12	14

Two patterns emerge from an inspection of these data. The first is that there is a tremendous amount of heterogeneity in the amount of times countries were excluded from financial markets, both across countries and over time. To partially account for the heterogeneity across countries, and in particular to remove the influence of a small number of outliers, we focus on the median length of exclusion by time period. These numbers show that about half of all defaults led to exclusion from capital markets for a period of more than 12 years.

This number may overstate the length of exclusion to the extent that it is affected by two phenomenon that also show up over time. The first is the fact that exclusion lengths were very high in the first half of the 19th Century, presumably as a result of the rapid increase in the number of sovereign countries, many of whom became first time borrowers. To the extent that different concerns govern new borrowers, as opposed to defaults by countries with long histories, these numbers may overstate the exclusion length period.

The second concern that arises from the tables concerns the rise in exclusion length in the middle of the 20th century. This is conceivably the result of the relative closure of international capital markets during the post war period, and not the result of the defaults alone. To the extent that this is true, again we might expect that the median number for the whole period overstates the exclusion length picture.

Nevertheless, if one looks at the last few decades of the 19th Century and the first few decades of the 20th Century, during which creditor organizations were well organized and active, many countries had developed some histories following independence, and during which capital markets were relatively open, one sees that it was not uncommon for countries to be excluded from capital markets for periods in excess of seven years. There were some default cases in which exclusion periods were smaller, and during the first decade of the 20th Century the median exclusion period was only 2 years, but despite these declines (and the presuming optimism that these speedy default resolutions presumably engendered) exclusion lengths rose again later in the Century.

These figures on substantial exclusion periods may seem obvious to the casual observer of sovereign defaults, but stand in contrast to the view held by many economists that exclusions periods were relatively modest. Perhaps this view is driven by the development of some models of default which assume a permanent exclusion from financial markets. The results also stand in contrast to some recent empirical work that finds very small exclusion periods. For example, Gelos et al (2004) found that a default engendered an average of 5 years of exclusion in 1980s, and only 4 months of exclusion in 1990s. One reason for these smaller numbers may be the fact that many banks delayed designating their sovereign loans in default during the 1980s and instead re-loaned to countries experiencing payments difficulties in order to cover their repayment obligations. To the extent that this is true, these numbers understate the true exclusion periods because these new loans should be more correctly thought of as funding loans that do not include new lending. During the 1990s, these numbers may also be understated by the fact that some default episodes had not yet ended. Our own preliminary results for the 1990s are somewhat higher at a median of two years. This number is still quite low, which is a reflection of the fact that a number of bonds were restructured before going into default, and thus enter the database as a default with a very short (or even zero) period of exclusion.

Terms of Access

In addition to quantity measures of market access, we are also concerned with the terms access was obtained under. This issue has been investigated a number of times before, and has turned out to be very controversial. On the one hand, Eichengreen and Portes (1988), and Lindert and Morton (1989), who examine the interwar period, and the period from 1870 to 1914 respectively, found that there was no interest premium for reborrowers who had defaulted in these periods. In contrast, Ozler (1991) has argued, based on an examination of the experience of countries that defaulted in the 1930's, that there was as much as a 20-30% premium, paid by countries with a default record. Similarly, Tomz (2004) has examined borrowing in the 18th and 19th Centuries and has argued that substantial premia were paid by defaulting countries.

The relevant question, in this context, concerns the expected rate of return creditors required in order to make loans to re-borrowers. As a result, it is necessary that we form an estimate of creditors expectations of repayment. This can be done in a large number of ways, and is presumably the source of the differences in findings form the studies listed above. Our approach follows that of Lindert and Morton (1989) in using actual returns to identify creditors expectations of returns. To the extent that creditors were unusually optimistic, or pessimistic, about repayment likelihoods, this has the potential to either underestimate, or overestimate, creditors true expectations of returns.

Bearing this caveat in mind, Tables Three and Four present our estimates of the rates paid on borrowings by defaulters based on our database of sovereign bonds and loans, relative to a benchmark interest rate. In constructing these estimates, there were two primary difficulties we encountered. The first was the absence of substantial amounts of secondary market price data. To the extent that bonds when issued by previous defaulters trade at a larger discount on their face value than for nondefaulters, this will mean that these numbers understate the rates paid by defaulters on reaccessing international financial markets. For those countries for which we have secondary market price data, this does not appear to be the case, although must inevitably be a large selection bias here in that the countries for which secondary market prices exist tend to be larger borrowers who might arguably be of higher quality.

The second problem is that ex post rates of return are quite sensitive to the terms of restructuring of debts in the event of future defaults. In turn these terms are quite complicated, often involving exchanges of securities who's market values we do not observe, and even in some cases involving repayment in kind. A complete resolution of this problem requires large amounts of time spent studying the details of each restructuring agreement. As a first cut at the problem, we took a sample of twenty defaults from the period 1870 to 1914 which were relatively well documented in the Corporation of Foreign Bondholders Annual reports. Average restructuring terms were calculated for this sample, and then applied to all bonds to compute ex post rates of return. Obviously, to the extent that restructuring terms differ substantially across time periods, this measure will introduce some errors, and our results should be best be viewed as preliminary.

Table Two: Rates Charged on Reborrowing

	1824 -1829	1830 -1839	1840 -1849	1850 -1859	1860 -1869	1870 -1879	1880 -1889	1890 -1899	1824 -1899
# defaults	15	3	2	3	5	13	2	17	60
# w. loan data	14	3	2	3	3	6	2	14	47
mean int premium.	2.5	2.6	0.8	0.9	0.1	0.1	1.0	3.0	1.5
median int premium.	0.8	1.2	0.8	1.2	-0.1	0.1	0.1	0.7	0.9

	1900 -1909	1910 -1919	1920 -1929	1930 -1939	1940 -1949	1960 -1969	1900 -1969	1824 -1969
# defaults	6	14	4	30	6	3	66	126
# w. loan data	6	14	3	30	6	3	64	111
mean int premium.	0.2	0.5	0.3	2.4	1.1	1.5	1.3	1.4
median int premium.	-0.3	-0.0	0.2	1.1	0.3	1.0	1.0	1.0

Bearing in mind these caveats, our results are presented in Table Two which presents both the mean and median premium in percentage points over the rate on UK securities, for the 19th Century, and over the rate for the US for the 20th century, for all countries defaulting for which we had data, by the decade in which the default occurred. Strikingly, all of the numbers appear quite low: mean interest premia are rarely more than one percent, and are often negative, despite the fact that the comparison is against the largest creditor countries of their time, which are presumably the safest investments available. There has been a small reduction in premia over time, particularly from the start of the 19th Century, and the countries that defaulted in the 1930s do appear to have paid higher premia, in line with, but much less significant than in the findings of Ozler (1991). Nonetheless, the overall picture is on in which defaulters paid very little premium over the rate paid by the governments of the largest creditor countries.

Output and Defaults

In order to discipline our understanding of the causes and consequences of sovereign defaults, we also examine the relationship between defaults and economic activity in history. For a number of reasons we focus our analysis on the period 1870 to 1914.

This period has a number of advantages for our purposes. First, a number of features of the 1870-1914 period resemble the evolving structure of sovereign debt markets today: like the current period, sovereign lending in this period was dominated by bonds. Second and similarly, like the earlier period, the current institutional environment is moving away from intervention by creditor country governments and supra-national institutions towards a more market based process for dealing with default. This contrasts with other resent default episodes such as the 1980s default crisis that was dominated by bank lending and involved substantial official intervention.

Third, and unlike some other earlier periods, the 1870-1914 period was characterized by relatively well developed financial markets with open flows of information, and creditor bodies that were well organized.

Towards this end we assembled data on a number of features of the borrowing experience. Data collection details are in the data appendix. First, because creditors never default, we first examined data on borrowing history to assemble a list of sovereign borrowers during this period. Second, this data was combined with indicators of whether or not the country was in default. This was combined with data on broad measures of economic activity for as many of these countries as possible.

Borrowers.—

As creditors do not default, attention was restricted to countries that had outstanding debts at some point during the period 1870-1914. Where a country first accessed debt markets during this period, years in which it was not a borrower were excluded. After this process, 79 countries were identified as being gross borrowers during this period. Of these, 56 were sovereign states for the entire period, while the remaining 23 were colonies or dominions. Two of the borrowers – the United Kingdom and the Netherlands were also net creditor countries, despite being gross borrowers.

Table 3: Number of Borrowers

period	#
1870-1879	54
1880-1889	61
1890-1899	71
1900-1909	77
1910-1914	79
1870-1914	79

Table gives the breakdown by decade within this period, which shows that there was a steady increase in the number of borrowing countries throughout the period. This is in part due to the increasing internationalization of capital markets through this period, as well as due to the opening up of borders by countries in Asia and central Europe.

Defaults.—

Table Four: Borrowers and Defaults

period	# borrower-years	# in default	proportion
1870-1879	504	105	21%
1880-1889	571	90	16%
1890-1899	680	65	10%
1900-1909	736	58	8%
1910-1914	392	22	6%
1870-1914	2883	340	12%

Table two combines data on the years that countries borrowed with data on defaults. A borrowing year is defined as a country-year pair for which a country had borrowed.

Summing over all 79 borrowers, there are just under 3000 borrower years in our sample. This is combined with data on whether or not a country was in default for that year. As shown in the figure, borrowers were in default for about 12% of the years in this sample. This number is slightly higher than the 10% figure estimated by Suter (1984) for the period 1820-1990.

The figure also shows that not all decades are equal: the decade of the 1870s recorded an average of 21% of borrowers being in default, which dropped to as little as six per-cent on the eve of the first world war. Interestingly, the decade of the 1890s was about average with 10 per-cent of countries being in default despite the declines in output experiences by many countries. These results are all robust to excluding the UK and the Netherlands form the sample.

Table Five: Sovereigns and Default

period	# borrower-years	# in default	proportion
1870-1879	193	105	54%
1880-1889	133	90	68%
1890-1899	249	65	26%
1900-1909	107	58	54%
1910-1914	111	21	19%
1870-1914	793	338	43%

Arguably, the incentives of colonies differ from those from sovereign nations: colonial powers may have other means of enforcing debts available to them, and colonial governments may aim to maximize the welfare of their colonial masters. As a result, one might expect that colonies behave differently. Figure three repeats this analysis for the subset of fully sovereign nations. The results are indeed quite different for these 56 sovereigns. For this total period, just under half of all borrowers were in default on average. The proportion rises to 68% for the decade of the 1880s, while falling to a low of 14% in the 14 years up to world war one.

Defaults and Output.—

To assess the relationship between defaults and economic activity, we focus our attention on broad measures of economic performance. This has the obvious advantage of being the best indicator of a nations capacity to service its debts. However, this approach has two potential disadvantages. The first is that, in examining a historical episode before the widespread availability of national income accounting statistics, we are looking at a variable which market participants themselves did not know about.

Table Six: Correlation with Output

	corr	>10%	>25%	<50%	<25%	<10%	<5%
All Years							
HP 6.25	-0.02	0.11	0.09	0.08	0.12	0.11	0.14
HP 100	-0.03	0.13	0.09	0.08	0.11	0.11	0.12
HP 400	-0.03	0.14	0.10	0.08	0.10	0.10	0.10
BP	-0.01	0.11	0.08	0.08	0.09	0.12	0.11
# obs	1468						
Start Years							
HP 6.25	-0.08	0.00	0.01	0.02	0.03	0.03	0.05
HP 100	-0.08	0.01	0.00	0.02	0.03	0.03	0.07
HP 400	-0.08	0.01	0.00	0.02	0.03	0.03	0.06
BP	-0.05	0.01	0.01	0.02	0.02	0.03	0.03

The second is that GDP data (or equivalents) are available only for a limited subset of countries. Of the 79 entities in our sample, only 48 have any broad national income aggregates data available for this period. Of these, a small number do not have complete data for the entire time period. We are currently working to extend the coverage of this data by using further country specific sources.

As a first cut, we examine the relationship between defaults and business cycles for those countries for which we have data. Business cycles are measured in a number of different ways.

Figure four presents a number of measures of this relationship for the entire sample of countries for which GDP data is available. As our data is annual, we follow Uhlig and Ravn (2002) in using a value of 6.25 for the smoothing parameter for the Hodrick-Prescott filter as our base measure of business cycles. The Figure shows that the relationship between defaults and economic activity is negative, but weak: the correlation is only -0.02. This finding is robust to using values of the smoothing parameter of 100 and 400, and to using a measure of business cycles constructed using the Band Pass filter of Baxter and King (2000).

One potential problem associated with looking at correlations is that defaults is a qualitative variable, whereas our measure of business cycles (percentage deviations from trend output) and qualitative variables. Another way of cutting this data is also presented in the table which sorts business cycle observations according to how far below trend they were, and looks at default incidence conditional upon these categories. The figure shows a similar picture using this method. Although defaults were slightly more likely when business cycles were most severe (in the bottom 5% of all observations) at 13% of the observations compares to just over 10% for this sample, the difference is small. Indeed, defaults occurred 10% of the time when output was in the top 10% of observations (that is, even when output was well above trend). Once again these results are robust to the filtering method chosen.

There are a number of potential reasons for this weak relationship. One is that, although defaults may begin when output is low, they may persist even after output has recovered. The second panel of the figure tests this hypothesis by restricting attention to the year in which a default began. As can be seen from the figure, the negative relationship does get stronger, suggesting that this phenomenon is present. However, the results are still quantitatively small.

There appear to be two reasons for this. The first is that there are many instances where output falls and countries doe not default. This is true even when the output declines are large. That is to say, a decline in output is not sufficient for defaults to occur. The second reasons is that there are also a modest number of occasions in which defaults occur when output is above trend. All these results are robust to excluding the UK and the Netherlands. When this analysis is repeated for the subsample of non-colonial borrowers the results are similar, largely due to the fact that there is little data available for the colonies during this period.

The fact that output often declines without a default, but that default is less likely in periods in which output did not decline, suggests that it may be the combination of an output decline in the defaulting country along with some other event that triggers a default. One possibility is that it is the combination of weak economic conditions at home, with tight credit market conditions abroad, that triggers a default. To test this hypothesis, Figure 6 presents data on defaults for all countries excluding the UK and the Netherlands according to the state of the business in the defaulting country (using HP 6.25), and the level of interest rates in the UK relative to trend. As can be seen the relationship is somewhat stronger. When interest rates in the UK are above trend at the same time as output is below trend in the borrower country, a default is about two and one half times as likely as when UK interest rates are below trend. We seize upon this fact in building our theory below.

Table Seven: Credit Market Conditions and Default

UK/Cty	>10%	>25%	<50%	<25%	<10%	<5%
>10%	0.02	0.02	0.10	0.16	0.22	0.36
>25%	0.02	0.06	0.12	0.18	0.24	0.26
<50%	0.06	0.06	0.19	0.15	0.14	0.28
<25%	0.01	0.07	0.03	0.04	0.06	0.07
<10%	0.02	0.01	0.06	0.03	0.05	0.05

A THEORY OF SOVEREIGN DEFAULT

Environment

Consider a world in which a developing country sovereign debtor (D) borrows from a single developed country creditor (C). Time evolves discretely, and at the start of each period $t = 0, 1, 2, ...$ the developing country has access to a productive opportunity that requires the input of foreign capital k. After investment is undertaken, production occurs yielding output, net of costs, of the single final consumption good of

$$Af(k) - Rk,$$

where f is a standard neoclassical production function, A is a random shock affecting the productivity of this investment, and R is the random opportunity cost of funds to the creditor. After production takes place, both the creditor and the debtor are able to make transfers of the consumption good to each other.

Both A and R are assumed to be unknown at the time investment takes place, and both the creditor and debtor are assumed to have private information about the likely outcomes of these variables which are indexed by the signals θ^C and θ^D. Obviously, it would be implausible to assume that a creditor is completely uninformed about a countries production possibilities. In order to capture the idea that both parties have some, but less than complete, information about the other party, we make three assumptions. First, we allow for an arbitrary correlation structure between signals in a time period. Second, we also assume that each agent's signal is also informative about the outcome for the other agent, writing the expectation of the country's productivity shock and the creditors opportunity cost of funds, conditional on these shocks, as $A\left(\theta^C, \theta^D\right)$ and $R\left(\theta^C, \theta^D\right)$ respectively. We order signals so that each agents return is increasing in its own signal, given the value of the other agents signal. Third, below we adopt a notion of incentive compatibility that is robust to differences in information and beliefs across parties.

More formally, the interaction between these agents is modelled as an infinitely repeated game. Each stage game begins with the realization of $\left(\theta^C, \theta^D\right) \equiv \theta \in \Theta \equiv \Theta_C \times \Theta_D$ according to some distribution π which is common knowledge. In much of what follows, we assume that Θ is a finite set, and assume that A and R take on values in a finite set. In order to prevent an agent from identifying the other agents signal with certainty, we assume that the probability of observing any of the finite number of combinations of values of R and A is positive for every possible combination of signals.

At the start of each period, each agent privately observes θ_i for $i = C, D$. Each agent then simultaneously sends a public announcement $\hat{\theta}_i \in \Theta_i$ (note that we are restricting messages to be these private signals). After observing all announcements, the creditor chooses a transfer of capital $k\left(\hat{\theta}\right)$ to the developing country borrower which is publicly observed. After this transfer, the costs of funds to the creditor $R(\theta)$ and output to the borrower in the developing country $A(\theta) f\left(k\left(\hat{\theta}\right)\right)$ are realized privately, and both parties can engage in transfers $\tau_i\left(\hat{\theta}\right)$ of the consumption good to each other. The production function f is assumed to be neoclassical with $f(0) = 0$. The period utility function (payoff functions) of the creditor and debtor thus take the form

$$\begin{aligned} u_C(k,\tau,\theta) &= -R(\theta)\,k + (-\tau_C + \tau_D)\,, \\ u_D(k,\tau,\theta) &= A(\theta)\,f(k) + (\tau_D - \tau_C)\,. \end{aligned}$$

Both the creditor and the debtor are risk neutral and discount the future using the common discount factor $\beta \in (0,1)$.

A strategy for the repeated game is, loosely speaking, a mapping from histories and private signals to announcements, a mapping from histories, private signals, and announcements to transfers of capital, and a mapping from histories, private signals, announcements and transfers of capital to transfers of the consumption good. We focus on perfect public equilibria (PPE), which are sequential equilibria in which players condition their strategies on only the public history and their private signal in the current period. We assume that after every period, agents observe the realization of some public randomization device and select continuation equilibria on this basis. This is a common assumption, and we do not introduce explicit notation for it.

Note that the stage game possesses a perfect Bayesian equilibrium in which no transfers of capital or the consumption good are made, and both agents truthfully announce their signal. We refer to this as autarky. Note also that repeated autarky attains the worst PPE, as any player can guarantee this payoff using a strategy that never transfers any goods and makes any arbitrary report. We have normalized our payoff functions so that, under autarky, the agents receive a payoff of zero.

If the signals, θ, were perfectly observable and there was no difficulty in enforcing contracts, then a first best allocation is achievable. All first best allocations involve capital being borrowed by the developing country up to the point where the marginal product of capital equals the cost of funds in the creditor country, or

$$k^*(\theta) = \arg\max_k A(\theta)\,f(k) - R(\theta)\,k$$

Transfers of the consumption good take care of distributional concerns. We will be interested in characterizing equilibria between these two extremes of the first best and autarky.

We focus on the special class of PPE's that can be represented as recursive mechanisms in which there is direct revelation. Recursive mechanisms use promised utility as a payoff-irrelevant state variable, and in each stage map promised utilities and vectors of public announcements into prescribed actions and promised continuation rewards. Specifically, a recursive mechanism is a set of continuation values V, a set of measurable functions mapping promised values and reports into transfers and continuation values, and an initial value in V. As long as it does not invite confusion, dependence on the initial value v is suppressed in what follows.

In order for a recursive mechanism to be equivalent to a PPE, it must be feasible, so that transfers are non-negative or

$$\tau_i\left(\hat{\theta}\right) \geq 0,$$

for all i, ensure that agents want to continue participating in the contract, and give agents an incentive to truthfully reveal their signals. Continuing participation is assured by requiring that the value of the mechanism, in each state of the world, is at least as large as what the agent would receive in autarky

$$(1-\beta)\left[-R(\theta)\,k\left(\hat{\theta}\right)+\left(-\tau_C\left(\hat{\theta}\right)+\tau_D\left(\hat{\theta}\right)\right)\right]+\beta w_C\left(\hat{\theta}\right)\geq 0,$$

and

$$\begin{aligned} & (1-\beta)\left[A(\theta)\,f\left(k\left(\hat{\theta}\right)\right)+\left(\tau_C\left(\hat{\theta}\right)-\tau_D\left(\hat{\theta}\right)\right)\right]+\beta w_D\left(\hat{\theta}\right) \\ \geq\ & (1-\beta)\left[A(\theta)\,f\left(k\left(\hat{\theta}\right)\right)+\tau_C\left(\hat{\theta}\right)\right], \end{aligned}$$

for all i, where the w_i are functions mapping signals into the equilibrium value set. This constraint says that, after having made announcements $\hat{\theta}$, no agent wants to transfer an amount other than that specified by the mechanism. It also corresponds to the usual "reputation" motive for repayment of sovereign debt[1].

In order to ensure that each agent prefers to make a truthful announcement about their type rather than deviate to some other announcement, we impose incentive compatibility constraints. One commonly used notion of incentive compatibility assumes that agents can commit to revealing their signals simultaneously. This leads to a notion of incentive compatibility in which an agent must prefer revealing the truth given the expected type of the other player, or

$$\begin{aligned} & E_\pi\left[-(1-\beta) R(\theta) k(\theta)-\tau_C(\theta)+\tau_D(\theta)+\beta w_C(\theta) \mid \theta_C\right] \\ \geq\ & E_\pi\left[-(1-\beta) R(\theta) k\left(\hat{\theta}_C, \theta_D\right)-\tau_C\left(\hat{\theta}_C, \theta_D\right)+\tau_D\left(\hat{\theta}_C, \theta_D\right)+\beta w_C\left(\hat{\theta}_C, \theta_D\right) \mid \theta_C\right], \end{aligned}$$

$$\begin{aligned} & E_\pi\left[(1-\beta) A(\theta) f(k(\theta))-\tau_D(\theta)+\tau_C(\theta)+\beta w_D(\theta) \mid \theta_D\right] \\ \geq\ & E_\pi\left[(1-\beta) A(\theta) f\left(k\left(\theta_C, \hat{\theta}_D\right)\right)-\tau_D\left(\theta_C, \hat{\theta}_D\right)+\tau_C\left(\theta_C, \hat{\theta}_D\right)+\beta w_D\left(\theta_C, \hat{\theta}_D\right) \mid \theta_D\right]. \end{aligned}$$

for all i, all $\hat{\theta}_i \in \Theta$ and all $\theta_i \in \Theta$.

This notion of incentive compatibility is very sensitive to a players beliefs about the other players type. That is to say, mechanisms designed under this constraint typically lean heavily on the assumed form for π. In the context of sovereign debt, where agents negotiate face-to-face over a long period of time, may not be able to commit to reveal information simultaneously, and may have access to technologies for obtaining more information about economic conditions, this notion seems problematic. As a result, we focus our attention on a stronger notion of incentive compatibility

$$\begin{aligned} & -(1-\beta) R(\theta) k(\theta)-\tau_C(\theta)+\tau_D(\theta)+\beta w_C(\theta) \\ \geq\ & -(1-\beta) R(\theta) k\left(\hat{\theta}_C, \theta_D\right)-\tau_C\left(\hat{\theta}_C, \theta_D\right)+\tau_D\left(\hat{\theta}_C, \theta_D\right)+\beta w_C\left(\hat{\theta}_C, \theta_D\right), \end{aligned}$$

[1] Here we use the term "reputation" to refer to trigger strategy punishments as is common in the literature on sovereign debt. This contrasts with the usage of the term in, for example, Kreps and Wilson or Milgrom and Roberts.

$$
\begin{aligned}
& (1-\beta) A(\theta) f(k(\theta)) - \tau_D(\theta) + \tau_C(\theta) + \beta w_D(\theta) \\
\geq \; & (1-\beta) A(\theta) f\left(k\left(\theta_C, \hat{\theta}_D\right)\right) - \tau_D\left(\theta_C, \hat{\theta}_D\right) + \tau_C\left(\theta_C, \hat{\theta}_D\right) + \beta w_D\left(\theta_C, \hat{\theta}_D\right).
\end{aligned}
$$

This constraint says that each player must prefer to report the truth even after knowing the signal of the other player. The difference in the implicit timing assumptions underlying each of these notions of incentive compatibility make it natural to refer to the former as interim incentive compatibility, and the latter ex post incentive compatibility. Obviously, all ex post incentive compatible mechanisms are also interim incentive compatible. Moreover, they are interim incentive compatible for any arbitrary set of beliefs about the opponents type. The allocations they generate can be implemented as ex post perfect public equilibria, a concept introduced by Miller (2004b). It is in this sense that the incentive constraints ensure robustness to a range of information differences across agents.

A final requirement for a recursive mechanism to be equivalent to a PPE is that the set of continuation values be self generating. In the following section, we begin our characterization of the equilibrium value set V^* and use it to characterize optimal payments in equilibrium.

The following proposition, due to Miller (2004), establishes that it is without loss of generality to look at recursive mechanisms when agents are patient.

Proposition 1 *Let $w \in R^n$ be the payoffs of some perfect Bayesian equilibrium for the stage game. For any $\varepsilon > 0$, there exists a $\beta^* < 1$ such that if $\beta > \beta^*$*

1. *If a feasible and interim incentive compatible mechanism satisfies $v_i > w_i + \varepsilon$ for all i and all $v \in V$, then there exists a perfect public equilibrium that yields the same announcements, and actions along the equilibrium path;*

2. *If V is the set of payoffs along the equilibrium path of some perfect public equilibrium and $v_i > w_i + \varepsilon$ for all i and all $v \in V$, then there exists a feasible and*

interim incentive compatible mechanism with initial promised utility v.

Note that as every ex post incentive compatible mechanism is also interim incentive compatible, the theorem also applies to ex post incentive compatible mechanisms.

The advantage of focusing on recursive mechanisms is that they allow us to transform a potentially complicated dynamic mechanism design problem in a simpler static mechanism. Indeed given the quasi-linearity of our agents objective functions, we can view transfers of the consumption good and transfers of future utility as perfect substitutes for the purposes of providing incentives. Towards this, define transfers inclusive of the deviation of promised utility from its expected value as

$$t_i(\theta) = -\tau_i(\theta) + \tau_{-i}(\theta) + \frac{\beta}{1-\beta}\left[w_i(\theta) - E_\pi\left[w_i(\theta)\right]\right].$$

Note that by construction

$$E_\pi\left[\sum_i t_i(\theta)\right] = 0,$$

so that any equivalent static mechanism involves transfers that balance in expected value.

We assume that agents act as their own mechanism designers and play the best PPE which is associated with the highest value recursive mechanism. We are interested in a number of properties of this mechanism. In particular, we are concerned with whether or not the mechanism can attain the first best level of capital flows $k^*(\theta)$ in every period and, if not, whether the best recursive mechanism ex ante involves the usage of suboptimal continuation payments. If so, given our assumption that no goods are ever destroyed, it must be the case that continuation payments are inside the Pareto frontier. We interpret this as default. Given our revised definition of transfers, defaults will occur if for any θ, $t_C(\theta) + t_D(\theta) < 0$.

Equilibria: A first look

Examination of the incentive compatibility constraints alone is enough to partially characterize the optimal allocation. We start with the following standard Lemma.

Lemma 2 *If* $\theta_{C1} \leq \theta_{C2}$ *and* $\theta_{D1} \leq \theta_{D2}$, *then*

$$k(\theta_{C2}, \theta_{D1}) \leq k(\theta_{C1}, \theta_{D1}) \leq k(\theta_{C1}, \theta_{D2}),$$

$$t_C(\theta_{C1}, \theta_D) \geq t_C(\theta_{C2}, \theta_D) \text{ for all } \theta_D$$

and

$$t_D(\theta_C, \theta_{D1}) \geq t_D(\theta_C, \theta_{D2}) \text{ for all } \theta_C$$

Proof. Fix θ_D and let $\theta_{C1} \leq \theta_{C2}$. Then the creditors incentive compatibility constraint at θ_{C1} for θ_{C2} implies

$$-R(\theta_{C1}, \theta_D)[k(\theta_{C1}, \theta_D) - k(\theta_{C2}, \theta_D)] \geq t_C(\theta_{C2}, \theta_D) - t_C(\theta_{C1}, \theta_D),$$

while the incentive compatibility constraint at θ_{C2} for θ_{C1} implies

$$-R(\theta_{C2}, \theta_D)[k(\theta_{C2}, \theta_D) - k(\theta_{C1}, \theta_D)] \geq t_C(\theta_{C1}, \theta_D) - t_C(\theta_{C2}, \theta_D).$$

Summing yields

$$[R(\theta_{C2}, \theta_D) - R(\theta_{C1}, \theta_D)][k(\theta_{C1}, \theta_D) - k(\theta_{C2}, \theta_D)] \geq 0,$$

and so as R is non-decreasing in θ_C we must have

$$k(\theta_{C1}, \theta_D) \geq k(\theta_{C2}, \theta_D),$$

which proves the first inequality. Substituting this into the first equation above gives the result on creditor transfers

$$t_C(\theta_{C1}, \theta_D) \geq t_C(\theta_{C2}, \theta_D).$$

The rest of the inequalities are proven analogously. ■

The characterization of equilibria is complicated by the fact that there exists one incentive constraint for each agent and each pair of signals they might observe. That is, for each observed signal, the agent must prefer to report the truth rather than any other possible signal. The following standard Lemma establishes that it is sufficient to focus on neighboring signals in looking at the set of incentive compatible allocations. That is, if we place agent $i's$ signals in ascending order and index them by j, it is sufficient to ensure than an agent with signal j does not wish to report that they have either signal θ_{j-1} or signal θ_{j+1}. We refer to these as the "local" incentive compatibility constraints.

Lemma 3 *If a recursive mechanism satisfies the local incentive compatibility constraints, then it is incentive compatible.*

Proof. Let $\theta_j^D < \theta_k^D$ be two, non-consecutive, signals for the debtor. Our aim is to show that

$$\begin{aligned} & A\left(\theta^C, \theta_j^D\right) f\left(k\left(\theta^C, \theta_j^D\right)\right) + t^D\left(\theta^C, \theta_j^D\right) \\ \geq \; & A\left(\theta^C, \theta_j^D\right) f\left(k\left(\theta^C, \theta_k^D\right)\right) + t^D\left(\theta^C, \theta_k^D\right), \end{aligned}$$

and

$$\begin{aligned} & A\left(\theta^C, \theta_k^D\right) f\left(k\left(\theta^C, \theta_k^D\right)\right) + t^D\left(\theta^C, \theta_k^D\right) \\ \geq \; & A\left(\theta^C, \theta_k^D\right) f\left(k\left(\theta^C, \theta_j^D\right)\right) + t^D\left(\theta^C, \theta_j^D\right), \end{aligned}$$

are implied by the local incentive compatibility constraints. We prove the result for the upwards constraint for the developing country debtor. The result for other cases, and for the creditor, is proven analogously.

From the local upwards constraint at θ_j we have

$$\begin{aligned} & A\left(\theta^C,\theta_j^D\right) f\left(k\left(\theta^C,\theta_j^D\right)\right)+t^D\left(\theta^C,\theta_j^D\right) \\ \geq\ & A\left(\theta^C,\theta_j^D\right) f\left(k\left(\theta^C,\theta_{j+1}^D\right)\right)+t^D\left(\theta^C,\theta_{j+1}^D\right), \end{aligned}$$

while from the local upwards constraint at θ_{j+1} we have

$$\begin{aligned} & A\left(\theta^C,\theta_{j+1}^D\right) f\left(k\left(\theta^C,\theta_{j+1}^D\right)\right)+t^D\left(\theta^C,\theta_{j+1}^D\right) \\ \geq\ & A\left(\theta^C,\theta_{j+1}^D\right) f\left(k\left(\theta^C,\theta_{j+2}^D\right)\right)+t^D\left(\theta^C,\theta_{j+2}^D\right). \end{aligned}$$

Rearranging the second we get

$$A\left(\theta^C,\theta_{j+1}^D\right)\left[f\left(k\left(\theta^C,\theta_{j+1}^D\right)\right)-f\left(k\left(\theta^C,\theta_{j+2}^D\right)\right)\right]\geq t^D\left(\theta^C,\theta_{j+2}^D\right)-t^D\left(\theta^C,\theta_{j+1}^D\right).$$

By Lemma 2 above, k is weakly increasing in θ^D, and this implies

$$A\left(\theta^C,\theta_j^D\right)\left[f\left(k\left(\theta^C,\theta_{j+1}^D\right)\right)-f\left(k\left(\theta^C,\theta_{j+2}^D\right)\right)\right]\geq t^D\left(\theta^C,\theta_{j+2}^D\right)-t^D\left(\theta^C,\theta_{j+1}^D\right),$$

which can be rearranged to give

$$\begin{aligned} & A\left(\theta^C,\theta_j^D\right) f\left(k\left(\theta^C,\theta_{j+1}^D\right)\right)+t^D\left(\theta^C,\theta_{j+1}^D\right) \\ \geq\ & A\left(\theta^C,\theta_j^D\right) f\left(k\left(\theta^C,\theta_{j+2}^D\right)\right)+t^D\left(\theta^C,\theta_{j+2}^D\right). \end{aligned}$$

Combining this with the local upwards constraint at θ_j we get

$$\begin{aligned} & A\left(\theta^C,\theta_j^D\right) f\left(k\left(\theta^C,\theta_j^D\right)\right)+t^D\left(\theta^C,\theta_j^D\right) \\ \geq\ & A\left(\theta^C,\theta_j^D\right) f\left(k\left(\theta^C,\theta_{j+2}^D\right)\right)+t^D\left(\theta^C,\theta_{j+2}^D\right), \end{aligned}$$

and iterating on this process gives the result. ■

Given an allocation of capital $k(\theta)$, the local incentive compatibility constraints are a system of linear inequalities that constrain the way transfers $t^i(\theta)$ can vary with a players signal. For the debtor, for a given θ^C, these inequalities imply

$$\begin{aligned} & A\left(\theta^C, \theta^D_{j-1}\right)\left[f\left(k\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(k\left(\theta^C, \theta^D_j\right)\right)\right] \\ \geq\ & t^D\left(\theta^C, \theta^D_j\right) - t^D\left(\theta^C, \theta^D_{j-1}\right) \\ \geq\ & A\left(\theta^C, \theta^D_j\right)\left[f\left(k\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(k\left(\theta^C, \theta^D_j\right)\right)\right] \end{aligned}$$

while for the creditor for a given θ_D they are given by

$$\begin{aligned} & -R\left(\theta^C_{j-1}, \theta^D\right)\left[k\left(\theta^C_{j-1}, \theta^D\right) - k\left(\theta^C_j, \theta^D\right)\right] \\ \geq\ & t^C\left(\theta^C_j, \theta^D\right) - t^C\left(\theta^C_{j-1}, \theta^D\right) \\ \geq\ & -R\left(\theta^C_j, \theta^D\right)\left[f\left(k\left(\theta^C_{j-1}, \theta^D\right)\right) - f\left(k\left(\theta^C_j, \theta^D\right)\right)\right]. \end{aligned}$$

For each θ^{-i}, it is possible to characterize the transfers made by player i in terms of a constant transfer $t^i\left(\theta^i_1, \theta^{-i}\right)$, and a sequence of changes in these transfers for $j \geq 2$

$$\Delta^i\left(\theta^i_j, \theta^{-i}\right) = t^i\left(\theta^i_j, \theta^{-i}\right) - t^i\left(\theta^i_{j-1}, \theta^{-i}\right).$$

Given a sequence of these changes in transfers for an agent satisfying the incentive compatibility constraints, we refer to their cumulative sum

$$r^i\left(\theta^i_j, \theta^{-i}\right) = \sum_{n=2}^{j} \Delta^i\left(\theta^i_j, \theta^{-i}\right),$$

written as a function of the $\theta' s$, as the *incentive compatible payment functions* of the agent.

It turns out that studying the properties of the incentive compatibility payments goes a long way towards determining the properties of equilibrium of this model. Recall that efficiency requires that efficient capital flows be attained in every period, which in turn requires that continuation rewards of both parties sum to the efficient level. Given our normalized definition of continuation payments, this implies that a necessary condition for efficiency is that $t^C(\theta) + t^D(\theta) = 0$ for all θ. Given a set of incentive payments that satisfy the above inequalities, this implies that we must be able to pick values for the constant transfers, which are functions only of the other players signal, such that continuation payments sum to zero in each state. This leads to the following proposition.

Proposition 4 *A recursive mechanism attains the efficient level of capital flows if and only if there exists incentive compatible payment functions r^C and r^D for $k^*(\theta)$, and there exists a pair of functions $f_1 : \Theta^C \to R$ and $f_2 : \Theta^D \to R$ such that*

$$r^C(\theta) + r^D(\theta) = f_1\left(\theta^C\right) + f_2\left(\theta^D\right).$$

Moreover, if the efficient level of capital flows is attainable, for any interior individually rational efficient allocation there exists a β^* such that if $\beta \geq \beta^*$ that allocation is attainable.

Proof. A recursive mechanism attains the efficient allocations if and only if $k(\theta) = k^*(\theta)$ in every period. Hence, given our definition of transfers, this requires that $t^C(\theta) + t^D(\theta) = 0$ for all θ. But then for any $\theta = \left(\theta^C, \theta^D\right)$,

$$\begin{aligned} r^C\left(\theta^C, \theta^D\right) + r^D\left(\theta^C, \theta^D\right) &= t^C\left(\theta^C, \theta^D\right) - t^C\left(\theta_1^C, \theta^D\right) + t^D\left(\theta^C, \theta^D\right) - t^D\left(\theta^C, \theta_1^D\right) \\ &= -t^C\left(\theta_1^C, \theta^D\right) - t^D\left(\theta^C, \theta_1^D\right). \end{aligned}$$

But then we can set $f_1\left(\theta^C\right) = -t^D\left(\theta^C, \theta_1^D\right)$ and $f_2\left(\theta^D\right) = -t^C\left(\theta_1^C, \theta^D\right)$.

To see the converse, suppose that the required functions exist. Let $t^D\left(\theta^C, \theta_1^D\right) = -f_1\left(\theta^C\right)$ and $t^C\left(\theta_1^C, \theta^D\right) = -f_2\left(\theta^D\right)$, and define $t^i\left(\theta^i, \theta^{-i}\right) = t^i\left(\theta_1^i, \theta^{-i}\right) + r\left(\theta^i, \theta^{-i}\right)$ for all i. These payments are incentive compatible for $k^*(\theta)$ by assumption. Moreover, for any θ

$$t^C(\theta) + t^D(\theta) = -f_1\left(\theta^C\right) - f_2\left(\theta^D\right) + r^C\left(\theta^C, \theta^D\right) + r^D\left(\theta^C, \theta^D\right) = 0,$$

which attains an efficient allocation.

Other individually rational efficient allocations can be attained by subtracting a constant from one of the fixed transfers and adding it to the other. For any interior individually rational allocation, this satisfies the continuing participation constraints as long as agents are sufficiently patient. ■

For relatively small numbers of possible values of the $\theta' s$, and hence also possible values for the $R' s$ and $A' s$, the inequalities defined by the incentive constraints may be quite loose, and functions of this sort may be easy to find.

Corollary 5 *If $\theta^C \in \Theta^C = \left\{\theta_L^C, \theta_H^C\right\}$ and $\theta^D \in \Theta^D = \left\{\theta_L^D, \theta_H^D\right\}$, the efficient level of capital flows is always attainable.*

Proof. Simply set $f_1\left(\theta^C\right) = r^D\left(\theta^C, \theta_H^D\right)$ and $f_2\left(\theta^D\right) = r^C\left(\theta_H^C, \theta^D\right)$. ■

As the number of points is increased, the constraints can be seen to tighten in the sense that makes functions that satisfy these properties more difficult to find. For example, with a continuous signal space and smooth distribution functions π, the envelope theorem holds so that the set of possible incentive compatible payment functions is a singleton. In general, the fact that the optimal allocation of capital depends upon the relative levels of A and R means that the incentive payments will not satisfy this property. In practice, such as in the numerical results below, despite the use of quite coarse grids, allocations are found to be quite different from the efficient level.

This proposition is important because it suggests that for most reasonable economies with imperfect information of the type specified above, it is not feasible for private agents interacting to attain a first best level of welfare. It does not, however, imply that we should ever observe default in equilibrium. The next section goes on to show, primarily through some numerical examples, that this is in fact the case. That is, even when agents acting privately design contracts that are ex ante efficient, they find it optimal to allow for defaults to occur ex post.

COMPUTATION

In this section, we show how to compute optimal recursive mechanisms. Recall that an optimal mechanism satisfies

$$V = \max_{x,\tau,w} \sum_{i=D,C} E_\pi \left\{ (1-\beta) \left[A(\theta) f(k(\theta)) - R(\theta) k(\theta) \right] + \beta w_i(\theta) \right\},$$

which follows from the resource constraint on transfers. The following standard lemma, stated without proof, is useful in characterizing the solution to this optimum problem.

Lemma 6 *The value of an optimal EPPPE satisfies*

$$V = \max_{\theta} \sum_{i=D,C} w_i(\theta).$$

The proof of this result is notation intensive, but simple to understand. If the value of the mechanism was smaller than the largest continuation value, it would be preferable to start the mechanism at this higher value. Conversely, if the value was larger, it is possible to construct a better mechanism which raises the maximum continuation value.

Rearranging the expression for an optimal mechanism, and substituting for our revised definition of transfers, we get

$$\begin{aligned} V &= \max_{x,w} (1-\beta) E_\pi \left[A(\theta) f(k(\theta)) - R(\theta) k(\theta) \right] \\ &\quad +\beta \left\{ \max_\theta \sum_{i=D,C} w_i(\theta) - \max_\theta \sum_{i=D,C} w_i(\theta) + E_\pi \sum_{i=D,C} w_i(\theta) \right\} \\ &= \max_{x,w} E_\pi \left[A(\theta) f(k(\theta)) - R(\theta) k(\theta) \right] - \left\{ \max_\theta \sum_{i=D,C} w_i(\theta) - E_\phi \sum_{i=D,C} w_i(\theta) \right\} \\ &= \max_{x,w} E_\pi \left[A(\theta) f(k(\theta)) - R(\theta) k(\theta) \right] - \max_\theta \left\{ \sum_{i=D,C} t_i(\theta) \right\}, \end{aligned}$$

where we have used the result of the preceding Lemma. This transformation of the problem also points to why we defined the equivalent static mechanism described above.

All that remains is to maximize this social welfare function subject to the constraints of ex ante budget balance (which holds by construction of the equivalent static mechanism), the ex post incentive compatibility constraints, and the individual rationality constraints. As discussed above, we ignore the latter under the assumption that agents are sufficiently patient.

Note also that this reformulation of the problem allows us to avoid the necessity of computing the entire equilibrium value set.

Default

To study the constrained efficient level of capital flows, we continue to assume that agents are patient so that we can ignore the continuing participation constraints. It is well known that in incentive problems of this kind it may be necessary to burn resources to give both agents an incentive to report correctly their type. Intuitively, if both agents need to be punished, then this cannot be done while preserving efficiency: if allocations are efficient, then punishing one agent necessarily rewards the other.

There are many ways in which resources can be wasted or burnt. In some cases, it may be reasonable to assume that the country and the creditor waste the consumption good. In the context of sovereign debt negotiations this does not seem politically feasible for the country, or acceptable to the shareholders of the creditor organization. Another possibility is that the country and the creditor structure their relationship in such a way as to waste surplus after some states of nature by not trading. The absence of capital flows and the wasting of potential gains from trade resembles a default, and we identify it as such in what follows. The public randomization device can be used to ensure that the right amount of resources are burnt in expected value.

Then the problem of identifying and characterizing defaults becomes one of understanding when the parties would benefit by burning resources. The first obvious question that needs to be asked is whether or not defaults of this kind will ever necessarily occur? Given our earlier result that full efficiency can be acheived with a sufficiently coarse shock distribution, it is not surprising that this depends on the coarsity of the shock space. In particular, for the case of a continuum of signal types and a smooth distribution function π, the following Proposition and its associated Lemma establish conditions under which default always occurs. We verify that this result holds for our discrete signal model, even with quite coarse signal spaces, in our numerical simulations below.

Proposition 7 *Suppose that θ is distributed continuously on Θ and that $\partial A\left(\theta^C, \theta^D\right)/\partial\theta^C > 0$ and $\partial R\left(\theta^C, \theta^D\right)/\partial\theta^D > 0$ for almost all θ. Then if $t_C(\theta) + t_D(\theta) = 0$ for all θ, $k(\theta) = k$ for all non-isolated θ.*

Proof. Take $\theta^1 \equiv \left(\theta^1_C, \theta^1_D\right) \in int(\Theta)$ and let $\theta^2 = \left(\theta^1_C + \delta_C, \theta^1_D - \delta_D\right)$ for some

$\delta_C, \delta_D > 0$. Now consider comparing the difference in the valuations of allocations

$$\begin{aligned}
&\left[-R\left(\theta_C^1+\delta_C,\theta_D^1-\delta_D\right)k\left(\theta_C^1+\delta_C,\theta_D^1-\delta_D\right)+R\left(\theta_C^1+\delta_C,\theta_D^1\right)k\left(\theta_C^1+\delta_C,\theta_D^1\right)\right]\\
&-\left[-R\left(\theta_C^1,\theta_D^1-\delta_D\right)k\left(\theta_C^1,\theta_D^1-\delta_D\right)+R\left(\theta_C^1,\theta_D^1\right)k\left(\theta_C^1,\theta_D^1\right)\right]\\
&+\left[A\left(\theta_C^1+\delta_C,\theta_D^1-\delta_D\right)f\left(k\left(\theta_C^1+\delta_C,\theta_D^1-\delta_D\right)\right)-A\left(\theta_C^1,\theta_D^1-\delta_D\right)k\left(\theta_C^1,\theta_D^1-\delta_D\right)\right]\\
&-\left[A\left(\theta_C^1+\delta_C,\theta_D^1\right)f\left(k\left(\theta_C^1+\delta_C,\theta_D^1\right)\right)-A\left(\theta_C^1,\theta_D^1\right)k\left(\theta_C^1,\theta_D^1\right)\right]\\
&+\int_{\theta_C^1}^{\theta_C^1+\delta_C}\left(\frac{\partial R\left(s_C,\theta_D^1-\delta_D\right)}{\partial s_C}k\left(s_C,\theta_D^1-\delta_D\right)-\frac{\partial R\left(s_C,\theta_D^1\right)}{\partial s_C}k\left(s_C,\theta_D^1\right)\right)ds_C\\
&-\int_{\theta_D^1}^{\theta_D^1-\delta_D}\left(\frac{\partial A\left(\theta_C^1+\delta_C,s_D\right)}{\partial s_D}f\left(k\left(\theta_C^1+\delta_C,s_D\right)\right)-\frac{\partial A\left(\theta_C^1,s_D\right)}{\partial s_D}f\left(k\left(\theta_C^1,s_D\right)\right)\right)ds_D
\end{aligned}$$

Take first order Taylor series approximations to the functions R, A and f

$$\begin{aligned}
R\left(\theta_C,\theta_D\right) &= r_0+r_C\theta_C+r_D\theta_D\\
A\left(\theta_C,\theta_D\right) &= a_0+a_C\theta_C+a_D\theta_D\\
f\left(k\right) &= f_0+f_kk.
\end{aligned}$$

Substituting and rearranging gives

$$\begin{aligned}
&\left[-\left(r_0+r_C\left(\theta_C^1+\delta_C\right)+r_D\left(\theta_D^1-\delta_D\right)\right)+f_k\left(a_0+a_C\left(\theta_C^1+\delta_C\right)+a_D\left(\theta_D^1-\delta_D\right)\right)\right]\\
&\times k\left(\theta_C^1+\delta_C,\theta_D^1-\delta_D\right)+\left[-\left(r_0+r_C\theta_C^1+r_D\theta_D^1\right)+f_k\left(a_0+a_C\theta_C^1+a_D\theta_D^1\right)\right]k\left(\theta_C^1,\theta_D^1\right)\\
&-\left[-\left(r_0+r_C\left(\theta_C^1+\delta_C\right)+r_D\theta_D^1\right)+f_k\left(a_0+a_C\left(\theta_C^1+\delta_C\right)+a_D\theta_D^1\right)\right]k\left(\theta_C^1+\delta_C,\theta_D^1\right)\\
&-\left[-\left(r_0+r_C\theta_C^1+r_D\left(\theta_D^1-\delta_D\right)\right)+f_k\left(a_0+a_C\theta_C^1+a_D\left(\theta_D^1-\delta_D\right)\right)\right]k\left(\theta_C^1,\theta_D^1-\delta_D\right)\\
&+\int_{\theta_C^1}^{\theta_C^1+\delta_C}\left(\frac{\partial R\left(s_C,\theta_D^1-\delta_D\right)}{\partial s_C}k\left(s_C,\theta_D^1-\delta_D\right)-\frac{\partial R\left(s_C,\theta_D^1\right)}{\partial s_C}k\left(s_C,\theta_D^1\right)\right)ds_C\\
&-\int_{\theta_D^1}^{\theta_D^1-\delta_D}\left(\frac{\partial A\left(\theta_C^1+\delta_C,s_D\right)}{\partial s_D}f\left(k\left(\theta_C^1+\delta_C,s_D\right)\right)-\frac{\partial A\left(\theta_C^1,s_D\right)}{\partial s_D}f\left(k\left(\theta_C^1,s_D\right)\right)\right)ds_D
\end{aligned}$$

For simplicity of notation let

$$\begin{aligned}
r_1 &= r_0+r_C\theta_C^1+r_D\theta_D^1,\\
a_1 &= a_0+a_C\theta_C^1+a_D\theta_D^1.
\end{aligned}$$

Then we can regarrange to get

$$
\begin{aligned}
&[-(r_1 + r_C\delta_C - r_D\delta_D) + f_k(a_1 + a_C\delta_C - a_D\delta_D)]\, k\left(\theta_C^1 + \delta_C, \theta_D^1 - \delta_D\right) \\
&+ [-r_1 + f_k a_1]\, k\left(\theta_C^1, \theta_D^1\right) - [-(r_1 + r_C\delta_C) + f_k(a_1 + a_C\delta_C)]\, k\left(\theta_C^1 + \delta_C, \theta_D^1\right) \\
&- [-(r_1 - r_D\delta_D) + f_k(a_1 - a_D\delta_D)]\, k\left(\theta_C^1, \theta_D^1 - \delta_D\right) \\
&+ \int_{\theta_C^1}^{\theta_C^1+\delta_C} \left(\frac{\partial R\left(s_C, \theta_D^1 - \delta_D\right)}{\partial s_C} k\left(s_C, \theta_D^1 - \delta_D\right) - \frac{\partial R\left(s_C, \theta_D^1\right)}{\partial s_C} k\left(s_C, \theta_D^1\right) \right) ds_C \\
&- \int_{\theta_D^1}^{\theta_D^1-\delta_C} \left(\frac{\partial A\left(\theta_C^1 + \delta_C, s_D\right)}{\partial s_D} f\left(k\left(\theta_C^1 + \delta_C, s_D\right)\right) - \frac{\partial A\left(\theta_C^1, s_D\right)}{\partial s_D} f\left(k\left(\theta_C^1, s_D\right)\right) \right) ds_D
\end{aligned}
$$

or

$$
\begin{aligned}
&[f_k a_1 - r_1]\left[k\left(\theta_C^1 + \delta_C, \theta_D^1 - \delta_D\right) + k\left(\theta_C^1, \theta_D^1\right) - k\left(\theta_C^1 + \delta_C, \theta_D^1\right) - k\left(\theta_C^1, \theta_D^1 - \delta_D\right)\right] \\
&+\delta_C [f_k a_C - r_C]\left[k\left(\theta_C^1 + \delta_C, \theta_D^1 - \delta_D\right) - k\left(\theta_C^1 + \delta_C, \theta_D^1\right)\right] \\
&-\delta_D [f_k a_D - r_D]\left[k\left(\theta_C^1 + \delta_C, \theta_D^1 - \delta_D\right) - k\left(\theta_C^1, \theta_D^1 - \delta_D\right)\right] \\
&+r_C \int_{\theta_C^1}^{\theta_C^1+\delta_C} \left(k\left(s_C, \theta_D^1 - \delta_D\right) - k\left(s_C, \theta_D^1\right)\right) ds_C \\
&-a_D f_k \int_{\theta_D^1}^{\theta_D^1-\delta_D} \left(k\left(\theta_C^1 + \delta_C, s_D\right) - k\left(\theta_C^1, s_D\right)\right) ds_D.
\end{aligned}
$$

From the Lemma in the text, we know that the equilibrium choice of k is increasing in θ_D and decreasing in θ_C. It need not, however, be continuous, but it is bounded. Let any line of discontinuity have slope locally of $\bar{k} \in (1, \infty)$ (the alternate case follows symmetrically) with the size of the discontinuity between θ^1 and $\left(\theta_C^1 + \delta_C, \theta_D^1\right)$ given by K, and let

$$\begin{aligned}
k_C\left(\theta^i\right) &= \frac{\partial k\left(\theta\right)}{\partial\theta_C}|_{\theta^i} < 0, \\
k_D\left(\theta^i\right) &= \frac{\partial k\left(\theta\right)}{\partial\theta_D}|_{\theta^i} > 0, \\
k_{CC}\left(\theta^i\right) &= \frac{\partial^2 k\left(\theta\right)}{\partial\left(\theta_C\right)^2}|_{\theta^i}, \\
k_{DD}\left(\theta^i\right) &= \frac{\partial^2 k\left(\theta\right)}{\partial\left(\theta_D\right)^2}|_{\theta^i}, \\
k_{CD}\left(\theta^i\right) &= \frac{\partial^2 k\left(\theta\right)}{\partial\theta_C\partial\theta_D}|_{\theta^i},
\end{aligned}$$

for $i = 1, 2$. Then choose δ_C and δ_D so that the line of non-differentiability passes first between θ^1 and $\left(\theta_C^1 + \delta_C, \theta_D^1\right)$ and then $\left(\theta_C^1, \theta_D^1 - \delta_D\right)$ and $\left(\theta_C^1 + \delta_C, \theta_D^1 - \delta_D\right)$. Then

$$\begin{aligned}
k\left(\theta_C^1 + \delta_C, \theta_D^1\right) &\approx k\left(\theta_C^1, \theta_D^1\right) + \int_0^{\delta_C - \frac{\delta_C}{2k}} \left(k_C\left(\theta^1\right) + k_{CC}\left(\theta^1\right) s_C\right) ds_C \\
&\quad + \int_0^{\delta_C + \frac{\delta_C}{2k}} \left(k_C\left(\theta^2\right) - k_{CD}\left(\theta^2\right)\delta + k_{CC}\left(\theta^2\right) s_C\right) ds_C \\
k\left(\theta_C^1, \theta_D^1 - \delta_D\right) &\approx k\left(\theta_C^1, \theta_D^1\right) + \int_0^{\delta_D} \left(k_D\left(\theta^1\right) + k_{DD}\left(\theta^1\right) s_D\right) ds_D \\
k\left(\theta_C^1 + \delta_C, \theta_D^1 - \delta_D\right) &\approx k\left(\theta_C^1 + \delta_C, \theta_D^1\right) + \int_0^{\delta_C} \left(k_D\left(\theta^2\right) + k_{DD}\left(\theta^2\right) s_D\right) ds_D
\end{aligned}$$

and hence

$$\begin{aligned}
&\int_{\theta_C^1}^{\theta_C^1+\delta_C} \left(k\left(s_C, \theta_D^1 - \delta_D\right) - k\left(s_C, \theta_D^1\right)\right) ds_C \\
\approx\ &\int_0^{\delta_C + \frac{\delta_C}{2k}} \left(k\left(\theta_C^1, \theta_D - \delta_D\right) - k\left(\theta_C^1 + \delta_C, \theta_D\right)\right. \\
&+ \left(k_C\left(\theta^1\right) + k_C\left(\theta^2\right) + \left(k_{CD}\left(\theta^1\right) - k_{CD}\left(\theta^2\right)\right)\delta_C\right) s_C\big) ds_C \\
&+ \int_0^{\delta_C + \frac{\delta_C}{2k}} \frac{\delta_C}{\delta_D}\left(k_{CC}\left(\theta^1\right) + k_{CC}\left(\theta^2\right)\right)\left(s_C\right)^2 ds_C \\
&+ \int_0^{\delta_C - \frac{\delta_C}{2k}} \left(k\left(\theta_C^1 + \delta_C, \theta_D^1 - \delta_D\right) - k\left(\theta_C^1, \theta_D^1\right)\right. \\
&- \left.\left(k_C\left(\theta^1\right) + k_C\left(\theta^2\right) + \frac{\delta_C}{\delta_D}\left(k_{CC}\left(\theta^1\right) + k_{CC}\left(\theta^2\right)\right) s_C\right) s_C\right) ds_C
\end{aligned}$$

and

$$\begin{aligned}
&\int_{\theta_D^1}^{\theta_D^1-\delta_D} \left(k\left(\theta_C^1 + \delta_C, s_D\right) - k\left(\theta_C^1, s_D\right)\right) ds_D \\
\approx\ &\int_0^{\delta_D} \left(k\left(\theta_C^1 + \delta_C, \theta_D\right) - k\left(\theta_C^1, \theta_D^1\right)\right. \\
&+ \left.\left(k_D\left(\theta^2\right) - k_D\left(\theta^1\right) - \frac{\delta_C}{\delta_D}\left(k_{DD}\left(\theta^2\right) + k_{DD}\left(\theta^1\right)\right) s_D\right) s_D\right) ds_D
\end{aligned}$$

Now divide by δ_C and take limits as $\delta_C, \delta_D \to 0$ keeping $\delta_D/\delta_C = m$ constant, we get

$$K\left(r_D + \frac{1}{\bar{k}} r_C\right) + \left(k_D\left(\theta^2\right) - k_D\left(\theta^1\right)\right)\left[f_k a_1 - r_1\right].$$

For there to be no surplus burning, this must equal zero. This leaves two cases. First, if $K = 0$, then either $r_1 = f_k a_1$ or $k_D\left(\theta^2\right) = k_D\left(\theta^1\right)$. Monotonicity requires that

$$\lim_{\delta_C \to 0}\left(k_D\left(\theta^2\right) - k_D\left(\theta^1\right)\right) = \lim_{\delta_C \to 0}\left(k_{DD}\left(\theta^2\right) - k_{DD}\left(\theta^1\right)\right) = 0.$$

Hence divide by δ_C^2 and take limits to get

$$-\frac{1}{2}\left(4 a_C k_D\left(\theta^1\right) - 2 r_D\left(k_C\left(\theta^2\right) + k_C\left(\theta^1\right)\right)\right).$$

For this to equal zero, as a_C and r_D are non-zero by the assumption of interdependence, we must have $k_C\left(\theta^2\right) = k_C\left(\theta^1\right) = 0$, and $k_D\left(\theta^1\right) = 0$ which implies $k_D\left(\theta^2\right) = 0$ by monotonicity. Hence k must be constant everwhere it is continuous. Second, if $K > 0$, as $r_D > 0$ we must have

$$K\left(r_D + \frac{1}{\overline{k}} r_C\right) = \left(k_D\left(\theta^2\right) - k_D\left(\theta^1\right)\right)\left[r_1 - f_k a_1\right].$$

As the left hand side is positive, so must be the right hand side. But then $r_1 \neq f_k a_1$ and $k_D\left(\theta^2\right) \neq k_D\left(\theta^1\right)$ which is incompatible with the first case holding on both sides of the discontinuity. Hence with interdependent valuations, it must be that the allocation is continuous except possibly at isolated points. Hence the allocation must be constant except possibly at isolated points. ■

Proposition 8 *(Default in Equilibrium) Suppose that θ is distributed continuously on Θ and that $\partial A\left(\theta^C, \theta^D\right)/\partial\theta^C > 0$ and $\partial R\left(\theta^C, \theta^D\right)/\partial\theta^D > 0$ for almost all θ. Then there exists a positive measure of non-isolated θ such that $t_C(\theta) + t_D(\theta) < 0$.*

Proof. Assume not. As default does not occur, it must be the case that $k(\theta) = k$ for all θ non-isolated. Suppose k is such that $A(\min\theta_C, \max\theta_D) f(k) > R(\min\theta_C, \max\theta_D) k$ (the opposite case proceeds symmetrically). Towards a contradiction, for some $\varepsilon > 0$ small, consider the subset of Θ defined by $E = [a, \varepsilon] \times [b - \varepsilon, b]$, and the new allocation function defined such that $k(\theta) = k + \delta$ for all $\theta \in E$ and $k(\theta) = k$ otherwise, for some δ satisfying $A(\min\theta_C, \max\theta_D) f(k+\delta) > R(\min\theta_C, \max\theta_D)(k+\delta)$. This increases welfare by

$$\int_E \left[A(\theta) f(k+\delta) - R(\theta)(k+\delta)\right] d\phi(\theta) > 0.$$

Now choose fixed transfer functions for the creditor identically equal to zero, and for the debtor equal to

$$A\left(\theta^C, 1-\varepsilon\right)\left[f(k+\delta) - f(k)\right] + c$$

for $\theta^C < \varepsilon$ and c otherwise. As the ex post incentive compatible payments for the debtor are given by

$$r_D(\theta) = A(\theta) f(k(\theta)) - A(\theta^C, a) f(k(\theta^C, a)) - \int_a^{\theta_D} \frac{\partial A(\theta^C, s_D)}{\partial s_D} f(k(\theta^C, s_D)) \, ds_D,$$

for all $\theta \in E^C$ they are zero, while for $\theta \in E$ we have

$$\begin{aligned} r_D(\theta) &= A(\theta) f(k+\delta) - A(\theta^C, a) f(k) - \\ &\quad \left\{ f(k) \left[A(\theta^C, 1-\varepsilon) - A(\theta^C, a) \right] + f(k+\delta) \left[A(\theta) - A(\theta^C, 1-\varepsilon) \right] \right\} \\ &= A(\theta^C, 1-\varepsilon) \left[f(k+\delta) - f(k) \right], \end{aligned}$$

while for the creditor they are zero on E^C and are

$$r_C(\theta) = \delta R(\varepsilon, \theta^D),$$

we have that the surplus gap is given by

$$\max_\theta Y(\theta) = c,$$

where c must be chosen to ensure $E[Y(\theta)] = 0$, or

$$\int_E \left\{ A(\theta^C, b-\varepsilon) \left[f(k+\delta) - f(k) \right] - \delta R(\varepsilon, \theta^D) \right\} d\phi(\theta).$$

But then the net benefit of this change to society is given by

$$\begin{aligned} &\int_E \left[A(\theta) f(k+\delta) - R(\theta)(k+\delta) \right] d\phi(\theta) \\ &- \int_E \left\{ A(\theta^C, b-\varepsilon) \left[f(k+\delta) - f(k) \right] - \delta R(\varepsilon, \theta^D) \right\} d\phi(\theta) \\ = \; & f(k+\delta) \int_E \left\{ A(\theta) - A(\theta^C, b-\varepsilon) \right\} d\phi \\ &(\theta) - \delta \int_E \left\{ R(\theta) - R(\varepsilon, \theta^D) \right\} d\phi(\theta) + \int_E \left\{ A(\theta^C, b-\varepsilon) f(k) - R(\theta) k \right\} d\phi(\theta) \end{aligned}$$

The first two terms here are positive, while the third is positive for suitably small ε. Hence net welfare is increased, a contradiction of optimality. ■

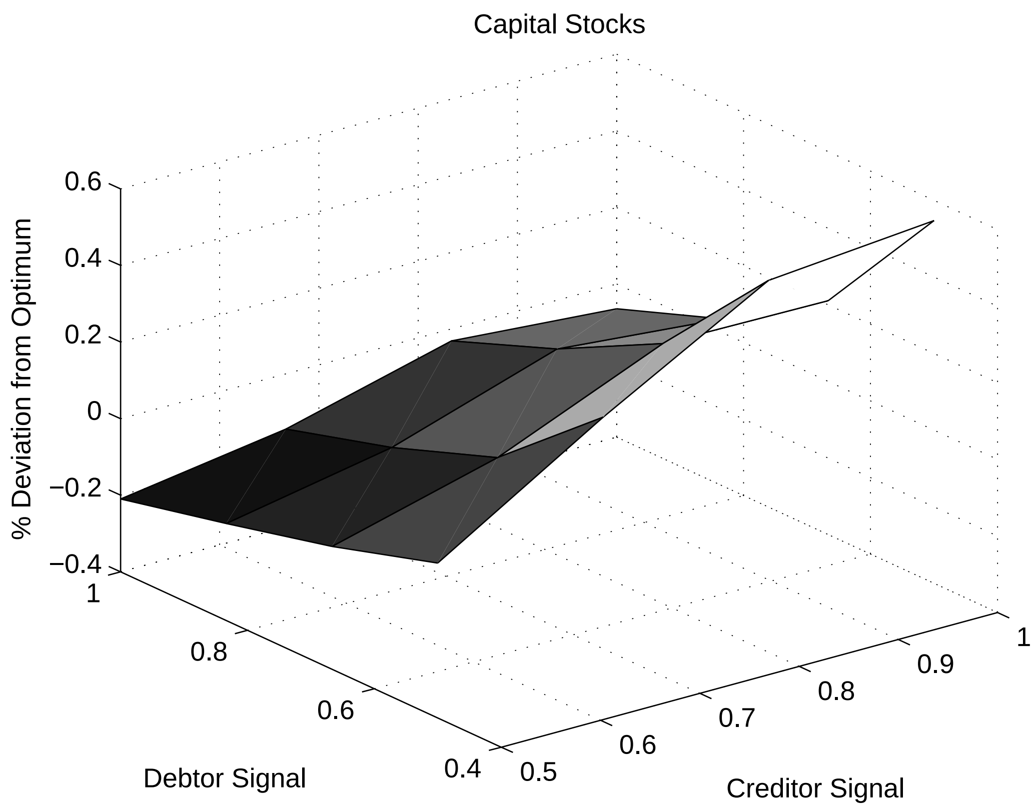
A Numerical Example

To assess the performance of this framework in both restricting capital flows away from their optimal level, and in producing defaults, we solve a simple version of the above framework. The aim of this numerical example is to illustrate the sorts of qualitative behaviors that can be obtained. Consequently, we do not calibrate to any empirical moments.

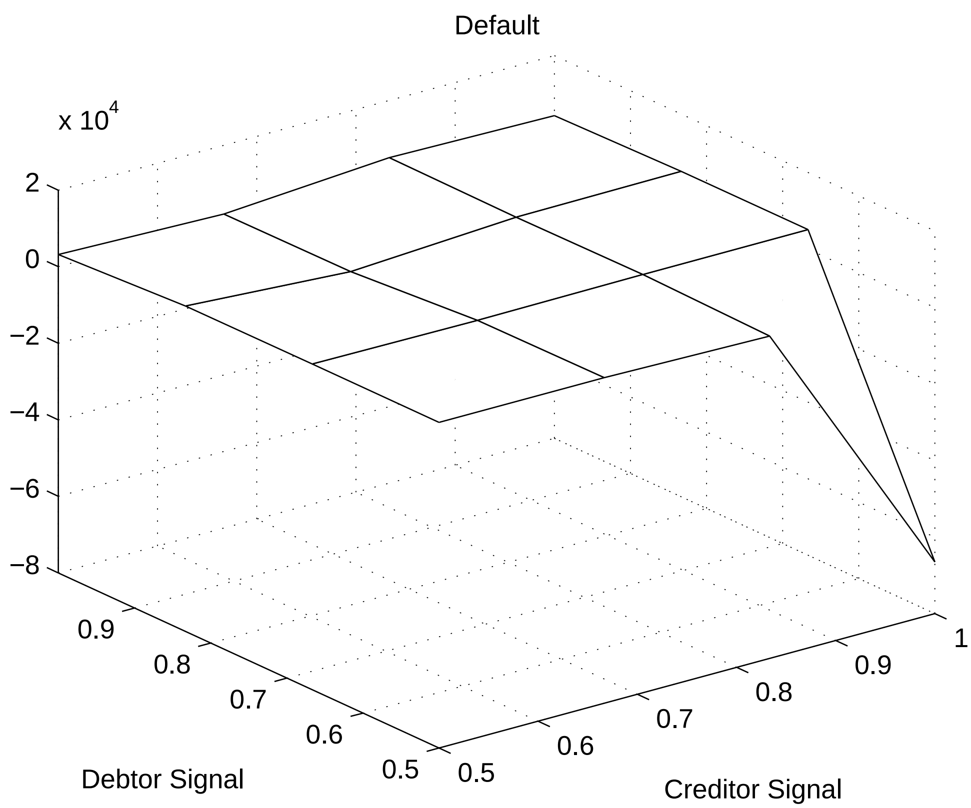
Towards this, we solve a version of the computational problem outlines above. This problem is non-standard in that the objective function is not differentiable in general, and the constraint set is not convex. The non-convexity implies that there may be multiple solutions to this problem. Moreover, from the result of Proposition One above we know that for arbitrary choices of the state space, there may be non-generic solutions that attain full efficiency.

In practice, commercial maximization algorithms often found multiple local maxima. To address this issue, we did three things. First, we assumed that the production function in the developing country is Cobb-Douglas with a capital share of 2/3. For this choice of capital share (larger than typically used) the numerical algorithm was better behaved. We conjecture that this is because the constraint set is convex with a linear production function, and so by setting the capital share large, we get closer to a linear production function. Note that, for a small open economy, the assumption of a linear production function may be justified by its price taking behavior on world markets as in Ventura (1999). Second, we compute the solution to the problem multiple times from different initial conditions, and compare the value of the objective function at each local maximum, selecting the largest. Third, as shown in the appendix, we experimented with a number of alternative formulations of the problem that were potentially behaved numerically. We then compared the results of using these different numerical approaches.

We discretize the signal space to have 16 points, uniformly distributed over the interval $[0.5, 1]$. We assume that returns to both the creditor and debtor are positively correlated by making the expected rates of return, conditional upon the true signals, equal to a constant times a weighted arithmetic average of the signals, with a weight of two-thirds on the agents own signal.



The results for the constrained optimal capital stocks are presented in Figure 6, which graphs the percentage deviation of the constrained efficient capital stock from the efficient capital stock. This illustrates one feature of the solution that appears to be robust: the presence of private information makes the capital flows less responsive to relative economic conditions than in the optimum. This can be seem by the fact that the plot is upwards sloping in the space of signals going from left to right. Recall that the larger is the debtors signal, relative to the creditors, the larger the optimal investment in the debtor country. The fact that the constrained optimal capital flow are less than the optimal means not enough capital is being allocated here. Similarly, in the opposite quadrant of the diagram – where capital flows should be smaller – the constrained optimal allocation gives the country more capital than in the efficient allocation. This also illustrates the point that capital flows in aggregate need not be smaller with asymmetric information.



The results for our equivalent static transfers are presented in Figure 7. Recall that the transfers must equal zero in expected value, so if there is any combination of signals for which the transfers sum to a negative number, there must also be some signals that lead to a positive number. This plot shows that over almost all of the state space, the sum of transfers across both agents is close to constant. This indicates that the optimal mechanism is stationary in those states: the debtor and creditor simply resume trading the next period on the same terms as before and there are no defaults.

Of most interest for our purposes is that there is one very negative entry (and two negative entries total) in this figure corresponding to the highest value of the creditors signal, and the lowest for the debtor. This corresponds to a situation in which interest rates are high in the creditor country, and productivity is low in the debtor country. If we interpret this phenomenon as default, this suggests that defaults will occur after a period in which output is below trend in the debtor country and credit markets are tight.

To see why it is natural to think of this event as a default, recall that to obtain equivalent static transfers that sum to a negative number it is necessary that promised utilities sum to be something less than the maximum promised expected utilities for the next period. That is, we must be moving inside the utility possibility frontier. This is because we have ruled out the possibility of consumption burning: we do not allow the creditor and debtor to waste the consumption good. We justify this assumption by the idea that it is very difficult for a country to waste the resources of its citizens in response to dealings with creditors in certain states of the world. Similarly, it is hard for a creditor to justify wasting resources to its shareholders. However, it seems more realistic for each agent to (falsely) blame the other for a breakdown in negotiations that leads to production opportunities being wasted (and indeed, we seem to observe politicians in developing countries blaming international financial institutions often in practice).

Having said that, there are still many ways for the parties to waste surplus. There is nothing in the model per se which tells us which way this must occur.[2] However, we can show that one way of wasting surplus is to behave in a way that mimics the default properties we observed above in the data. That is, we show that this model *can* produce the properties from the data qualitatively, and not that it must inevitably do so.

To see that we can generate these properties, we focus on showing that these ncontinuation payoffs can be attained by strategies that involve playing the autarkic staregy for a finite number of periods. Note that under the autarkic strategy in any period, both parties receive a zero payoff that period. This means that we can construct an increasing series of promised utilities during a default along which no capital or transfers are made in each period, and where $w_{it} = \beta w_{it+1}$. In the last period, the public randomization device can be used to ensure the terminal promises sum to the efficient surplus level. Agents are arbitrarily assumed to report their true signal. It can be verified that this constitutes a PPE supported by trigger strategies in which the parties revert to autarky, as long as β is sufficiently large.

BAILOUTS

In the section above, we established that with even small amounts of imperfect information, it was in general not possible for private agents acting alone to attain the first best level of capital flows. Moreover, the best that private agents acting alone can achieve often requires that they design contracts that allow for the possibility of default to occur ex post in equilibrium. That is, private agents acting alone often do

[2]This contrasts with the extremum results of Abreu, Pearce and Stachetti (1993).

not achieve even the constrained efficient level of capital flows in every period. Both of these results beg the question of whether it is possible for some form of supranational intervention to effect welfare increases.

One obvious concern might be that attempts to reduce the costs of default would lower the incentive for agents to interact optimally ex ante and would reduce the efficiency of capital flows ex ante. In the following, we model a benevolent supranational institution with the ability to transfer resources to and from both the creditor and country. That is, it is feasible for the supranational to engage in bailouts. Our only constraint on the supranational institution is that it be required to expect to make zero profits. That is, we allow the supranational institution to bailout both the debtor and creditors in particular states of the world, as long as it does not expect itself make losses on average in the long run. It turns out that the utility of such a policy, as well as the subtleties of its design, depend very much on the information available to the supranational institution.

Uninformed Supranational Institution

To begin, we assume that the supranational institution is uninformed in the sense of having no private information about economic conditions in the borrower country or the opportunity cost of funds to the creditor. In this case, it turns out that it is always possible to attain fully efficient capital flows, as long as agents are sufficiently patient.

Proposition 9 *For any interior individually rational efficient allocation* (v_C, v_D)*, there exists a set of transfers for the supranational* $t^I(\theta)$ *and a* β^* *such that if* $\beta \geq \beta^*$ *the efficient allocation can be attained.*

Proof. Let $k(\theta) = k^*(\theta)$. If there exists an additively separable selection as in Proposition 1 above, then we are done. Suppose this is not the case, and let $\hat{t}_i(\theta)$ for $i = C, D$ be some set of transfers that satisfies the incentive compatibility constraints for the creditor and debtor for $k^*(\theta)$. Let

$$\hat{t}^I(\theta) = -\hat{t}_C(\theta) - \hat{t}_D(\theta),$$

which are then not all zero by assumption, and compute

$$x = E_\pi t^I(\theta).$$

The efficient allocation is stationary, and so let x_C and x_D satisfy

$$\begin{aligned} x_C &= v_C - E_\pi\left[-R(\theta)\,k^*(\theta) + \hat{t}_C(\theta)\right], \\ x_D &= v_D - E_\pi\left[A(\theta)\,f(k^*(\theta)) + \hat{t}_D(\theta)\right]. \end{aligned}$$

Then x_C and x_D are two numbers satisfying $x_C + x_D = x$. Setting $t_C(\theta) = \hat{t}_C(\theta) + x_C$, $t_D(\theta) = \hat{t}_D(\theta) + x_D$ and $t^I(\theta) = -t_C(\theta) - t_D(\theta)$ gives us our transfers.

Finally, noting that the allocations are interior in the space of utilities, there exists a β_i^* for $i = C, D$ such that if $\beta \geq \beta_i^*$ the allocation satisfies the continuing participation constraints for agent i. Let β^* be the maximum over these. ■

We have endowed the supranational institution with the ability to transfer resources to and from both the creditor and the debtor. As a result, it is feasible for the supranational to engage in a policy of "bailing out" the private sector. However, the form of these transfers is quite unlike any bailout observed in practice. First, and most obviously, with optimally designed transfer schemes from the supranational, no default ever occurs in equilibrium. Nonetheless, transfers occur in equilibrium. That is to say, even though defaults have been abolished, the necessity for a supranational institution to be involved in transferring resources to and from debtors and creditors remains.

Second, the size of the transfers required can be quite large in any one period. Moreover, although the supranational expects to make zero profits from these transfers, along any given sample path it is possible for profits to become arbitrarily large, both positively and negatively. This obviously begs the question of whether it is feasible for any supranational institution to implement such a scheme in practice both financially, in the sense of occasionally needing to sustain large losses, and politically, in these sense of occasionally making large profits.

Perfectly Informed Supranational

Suppose that the supranational now observes both of the signals of the creditor and debtor. In this case, it is obvious that a benevolent supranational could implement the level of fully efficient capital flows simply by revealing the true signals. It is also straightforward to show that fully efficient capital flows are attainable with a selfish supranational without any transfers to the creditor and debtor as long as agents are sufficiently patient. The proof is immediate and relies on the fact that any constant transfers to the supranational leave the supranational indifferent to reporting the truth. Once the true signals have been reported, trigger strategies can be used to support efficient borrowing as long as the players are sufficiently patient.

Proposition 10 *If the supranational observes* (θ_C, θ_D), *there exists a* β^* *such that if* $\beta > \beta^*$ *fully efficient capital flows are attainable.*

CONCLUSIONS

Defaults appear to be very costly, to both creditors and debtors. This begs the question of whether or not supranational organizations can intervene in international financial markets to reduce the costs of default while at the same time still promoting efficient capital flows?

Towards an answer to this question, this paper began by examining a range of data sources on defaults and sovereign borrowing that suggest that a default is best viewed as a temporary suspension of sovereign capital flows, following by the resumption of lending on similar terms to pre-existing loans. This led to the design of a new theory of borrowing and default based on the interaction between limited enforcement possibilities and asymmetric information. We showed that under these conditions, private capital markets were always inefficient, and that even the second best level of capital flows required the potential for ex post defaults: that is, the model produced defaults in equilibrium.

Using this model, we were able to show that government intervention in the form of bailouts could not only eliminate defaults, but also do so in a way that promoted the first best level of capital flows. However, the form of these bailouts was counter-intuitive: supranational institutions need to tax countries in periods in which they would otherwise default, and lend money when output was high. Intuitively, these taxes substitute for defaults in providing incentives for countries to truthfully report their output.

In addition to making these substantive points in the context of this one model, one aim of this paper was to outline a methodological approach to understanding the sovereign default phenomenon. Ultimately, our interest in defaults as a phenomenon must be driven by the hope that economic analysis can provide proscriptions for policy makers that raise international welfare. But to evaluate potential policy options requires a laboratory – an economic model – within which we can assess alternative policy options. Moreover, it is necessary that this model treat defaults as arising endogenously out of the private decisions of creditors and debtors who respond optimally to explicit aspects of the environment they face. For if not, policy prescriptions are liable to turn out to be infeasible, or undesirable, for exactly the same reasons that markets failed to perform well in the first place. The model of the current paper illustrates one approach to providing such a model; other approaches are no doubt possible.

In assessing between possible alternative models, it will be necessary to examine the implications of these models for a wider array of data. It will thus be necessary to document an array of other facts about default. For example, it would be of interest to examine the relationship between default and other measures of economic activity. Although GDP is the broadest measure of a nations income, it is available only for a restricted subset of countries. Moreover, it is data that was not available to market participants throughout much of the history of sovereign lending. Our ongoing research aims to collect data on trade and government revenue to establish the link between these variables and default.

DATA SOURCES

Potential Defaulters

Creditors do not default. As a result, in examining the relationship between default and economic activity, we restricted our sample to countries that were debtors, at least in gross terms. Three criteria were used to assess whether or not a country had gross debts during the period and was therefore a potential defaulter. First, if the country was in default during the period according to the list in Suter (1992), that country was includes. Second, if that country was listed in Stone (1965) and Clemens and Williamsons (2002) studies of capital importers, they were included. Third and finally, if a country had securities listed on the London Stock Exchange during this period, they were included. With regard to the latter, issues of the Investors Monthly Manual was examined for all years between 1870 and 1914. This data source also provided the dates of access for all countries that began accessing capital markets during this period.

Default

The classification of Suter (1992), as adapted by Standard and Poors, was used as our measure of default. Default is coded as a 0,1 variable.

Economic Activity

The primary sources for data on broad measures of economic activity, like GDP, were the compendia by Maddison (2002) and Mitchell (2001). These were extended in a number of directions using the country specific sources listed in the references. Given our interest in business cycles, in a small number of cases, missing values for output were substituted using data on primary energy consumption taken from the

Correlates of War database. All data is real in units of local currency.

COMPUTATION

Recall that the optimal mechanism solves

$$V = \max_{x,t} E_{\pi}\left[A(\theta) f(k(\theta)) - R(\theta) k(\theta)\right] - \max_{\theta}\left\{\sum_{i=D,C} t^{i}(\theta)\right\},$$

subject to the constraints implied by incentive compatibility for the debtor

$$\begin{aligned} & A\left(\theta^{C},\theta_{j-1}^{D}\right)\left[f\left(k\left(\theta^{C},\theta_{j-1}^{D}\right)\right) - f\left(k\left(\theta^{C},\theta_{j}^{D}\right)\right)\right] \\ \geq\ & t^{D}\left(\theta^{C},\theta_{j}^{D}\right) - t^{D}\left(\theta^{C},\theta_{j-1}^{D}\right) \geq A\left(\theta^{C},\theta_{j}^{D}\right)\left[f\left(k\left(\theta^{C},\theta_{j-1}^{D}\right)\right) - f\left(k\left(\theta^{C},\theta_{j}^{D}\right)\right)\right] \end{aligned}$$

and creditor

$$\begin{aligned} & -R\left(\theta_{j-1}^{C},\theta^{D}\right)\left[k\left(\theta_{j-1}^{C},\theta^{D}\right) - k\left(\theta_{j}^{C},\theta^{D}\right)\right] \\ \geq\ & t^{C}\left(\theta_{j}^{C},\theta^{D}\right) - t^{C}\left(\theta_{j-1}^{C},\theta^{D}\right) \geq -R\left(\theta_{j}^{C},\theta^{D}\right)\left[f\left(k\left(\theta_{j-1}^{C},\theta^{D}\right)\right) - f\left(k\left(\theta_{j}^{C},\theta^{D}\right)\right)\right]. \end{aligned}$$

and the budget balance equation

$$\sum_{\theta} \pi(\theta)\left[t^{D}(\theta) + t^{C}(\theta)\right] = 0.$$

Given level of capital flows $\hat{k}(\theta)$, the problem above can also be solved for the transfers that support it while also satisfying ex ante budget balance. Specifically, consider the problem of solving for the optimal transfers conditional upon some $\hat{k}(\theta)$ as follows.

Problem 11 *(Conditionally Optimal Transfers)*

$$\min_{t} \max_{\theta}\left\{\sum_{i=D,C} t^{i}(\theta)\right\},$$

subject to the constraints implied by incentive compatibility for the debtor

$$\begin{aligned} & A\left(\theta^C, \theta^D_{j-1}\right)\left[f\left(\hat{k}\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(\hat{k}\left(\theta^C, \theta^D_j\right)\right)\right] \\ \geq \; & t^D\left(\theta^C, \theta^D_j\right) - t^D\left(\theta^C, \theta^D_{j-1}\right) \geq A\left(\theta^C, \theta^D_j\right)\left[f\left(\hat{k}\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(\hat{k}\left(\theta^C, \theta^D_j\right)\right)\right] \end{aligned}$$

and creditor

$$\begin{aligned} & -R\left(\theta^C_{j-1}, \theta^D\right)\left[\hat{k}\left(\theta^C_{j-1}, \theta^D\right) - \hat{k}\left(\theta^C_j, \theta^D\right)\right] \\ \geq \; & t^C\left(\theta^C_j, \theta^D\right) - t^C\left(\theta^C_{j-1}, \theta^D\right) \geq -R\left(\theta^C_j, \theta^D\right)\left[\hat{k}\left(\theta^C_{j-1}, \theta^D\right) - \hat{k}\left(\theta^C_j, \theta^D\right)\right]. \end{aligned}$$

and the budget balance equation

$$\sum_\theta \pi(\theta)\left[t^D(\theta) + t^C(\theta)\right] = 0.$$

This algorithm can then be used to solve for transfers to a third party, like a supranational, that guarantee budget balance in each state. Fortunately, with $k(\theta)$ fixed, the programming problem can be simplified substantially. In particular, the solution to the conditionally optimal transfer problem can be solved by instead solving the following linear programming problem.

Problem 12 *(Auxiliary Conditionally Optimal Transfers Problem)*

$$\max_t \sum_\theta \pi(\theta)\left[t^D(\theta) + t^C(\theta)\right],$$

subject to the constraints implied by incentive compatibility for the debtor

$$\begin{aligned} & A\left(\theta^C, \theta^D_{j-1}\right)\left[f\left(\hat{k}\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(\hat{k}\left(\theta^C, \theta^D_j\right)\right)\right] \\ \geq \; & t^D\left(\theta^C, \theta^D_j\right) - t^D\left(\theta^C, \theta^D_{j-1}\right) \geq A\left(\theta^C, \theta^D_j\right)\left[f\left(\hat{k}\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(\hat{k}\left(\theta^C, \theta^D_j\right)\right)\right] \end{aligned}$$

and creditor

$$\begin{aligned} & -R\left(\theta^C_{j-1}, \theta^D\right)\left[\hat{k}\left(\theta^C_{j-1}, \theta^D\right) - \hat{k}\left(\theta^C_j, \theta^D\right)\right] \\ \geq \; & t^C\left(\theta^C_j, \theta^D\right) - t^C\left(\theta^C_{j-1}, \theta^D\right) \geq -R\left(\theta^C_j, \theta^D\right)\left[\hat{k}\left(\theta^C_{j-1}, \theta^D\right) - \hat{k}\left(\theta^C_j, \theta^D\right)\right]. \end{aligned}$$

and

$$t^D(\theta) + t^C(\theta) \leq 0.$$

The equivalence, up to a constant, is proven in the following.

Proposition 13 *Let* $\left\{\hat{t}^i(\theta)\right\}$ *solve the auxiliary conditionally optimal transfer problem, and let*

$$\tilde{t}^i(\theta) = \hat{t}^i(\theta) - \frac{1}{2}\sum_{\theta}\pi(\theta)\left[\hat{t}^D(\theta) + \hat{t}^C(\theta)\right].$$

Then $\left\{\tilde{t}^i(\theta)\right\}$ *solves the conditionally optimal transfer problem.*

Proof. Let $\{t^{*i}(\theta)\}$ solve the conditionally optimal transfer problem attaining value V_1, and let $\left\{\hat{t}^i(\theta)\right\}$ solve the auxiliary conditionally optimal transfer problem attaining value V_2.

Construct $\left\{\tilde{t}^i(\theta)\right\}$ as shown, then these transfers are incentive compatible and satsify ex post budget balance. Hence they are feasible for the conditionally optimal transfer problem and we have

$$V_1 \leq \max_{\theta}\left\{\tilde{t}^C(\theta) + \tilde{t}^D(\theta)\right\} = -V_2.$$

Construct the transfers $\{t^{*i}(\theta) - V_1/2\}$. These satsify incentive compatibility and the sum over agents is non-positive for all θ. Hence, they are feasible for the auxiliary problem and therefore

$$V_2 \geq \sum_{\theta}\pi(\theta)\left\{t^{*C}(\theta) + t^{*D}(\theta)\right\} - V_1 = -V_1.$$

But then

$$-V_1 \geq V_2 \geq -V_1,$$

and the result is proven. ■

In general, we are interested in the solution of the problem inclusive of the choice of the allocation of the capital good. That is, we are interested in Program One:

$$V^1 = \max_{x,t} \left\{ E_\pi \left[A(\theta) f(k(\theta)) - R(\theta) k(\theta) \right] - \max_\theta \left\{ \sum_{i=D,C} t_i(\theta) \right\} \right\},$$

subject to incentive compatibility constraints which constrain the change in transfers for the debtor

$$\begin{aligned} & A\left(\theta^C, \theta^D_{j-1}\right) \left[f\left(k\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(k\left(\theta^C, \theta^D_j\right)\right) \right] \\ \geq\ & t^D\left(\theta^C, \theta^D_j\right) - t^D\left(\theta^C, \theta^D_{j-1}\right) \\ \geq\ & A\left(\theta^C, \theta^D_j\right) \left[f\left(k\left(\theta^C, \theta^D_{j-1}\right)\right) - f\left(k\left(\theta^C, \theta^D_j\right)\right) \right] \end{aligned}$$

and creditor

$$\begin{aligned} & -R\left(\theta^C_{j-1}, \theta^D\right) \left[k\left(\theta^C_{j-1}, \theta^D\right) - k\left(\theta^C_j, \theta^D\right) \right] \\ \geq\ & t^C\left(\theta^C_j, \theta^D\right) - t^C\left(\theta^C_{j-1}, \theta^D\right) \\ \geq\ & -R\left(\theta^C_j, \theta^D\right) \left[k\left(\theta^C_{j-1}, \theta^D\right) - k\left(\theta^C_j, \theta^D\right) \right], \end{aligned}$$

and ex ante budget balance

$$E_\pi \left[\sum_i t_i(\theta) \right] = 0.$$

This program is complicated by two concerns. The first is the non-differentiability of the objective function, while the second is the non-convexity of the constraint set implied by the incentive compatibility constraints. In order to simplify the computations, it is useful to transform this problem in two ways.

First, we relax the problem slightly by allowing free disposal of the capital good. If we let $y^D(\theta)$ and $y^C(\theta)$ denote the amount of output, before productivity shock, of the debtor and creditor respectively, then feasibility then requires

$$\left(y^D(\theta), y^C(\theta)\right) \in C \equiv \left\{ (y_1, y_2) \in R^2_+ : y_1 \leq f(y_2) \right\},$$

which is a convex set. We will then reformulate the choice of $k(\theta)$ as the choice of $y^D(\theta)$ and $y^C(\theta)$ subject to the requirement that they lie in C. This replaces the non-convexity in the constraint set, at the cost of relaxing the problem.

Second, we replace the minimisation of the maximum transfer subject to ex ante budget balance with a term that rewards violating ex post budget balance and some constraints on the size of the fixed transfers. As long as the solution to this new reformulated Program 2 is on the boundary of C, it turns out that it returns values that, but for a constant, attain the solution to the original problem after scaling.

In particular, we will say that $\{y,t\}$ solves program 2 if there exists $\{y,\tilde{t}\}$

$$V^2 = \max_{y,\tilde{t}} E_\pi \left[A(\theta)\, y^D(\theta) - R(\theta)\, y^C(\theta) + \sum_{i=D,C} \tilde{t}_i(\theta) \right],$$

subject to $y \in C$, the incentive compatibility constraints which constrain the change in transfers for the debtor

$$\begin{aligned} & A\left(\theta^C, \theta^D_{j-1}\right)\left[y^D\left(\theta^C, \theta^D_{j-1}\right) - y^D\left(\theta^C, \theta^D_j\right)\right] \\ \geq\ & t^D\left(\theta^C, \theta^D_j\right) - t^D\left(\theta^C, \theta^D_{j-1}\right) \\ \geq\ & A\left(\theta^C, \theta^D_j\right)\left[y^D\left(\theta^C, \theta^D_{j-1}\right) - y^D\left(\theta^C, \theta^D_j\right)\right] \end{aligned}$$

and creditor

$$\begin{aligned} & -R\left(\theta^C_{j-1}, \theta^D\right)\left[y^C\left(\theta^C_{j-1}, \theta^D\right) - y^C\left(\theta^C_j, \theta^D\right)\right] \\ \geq\ & t^C\left(\theta^C_j, \theta^D\right) - t^C\left(\theta^C_{j-1}, \theta^D\right) \\ \geq\ & -R\left(\theta^C_j, \theta^D\right)\left[y^C\left(\theta^C_{j-1}, \theta^D\right) - y^C\left(\theta^C_j, \theta^D\right)\right], \end{aligned}$$

extra constraints that ensure the choices of the $y's$ are incentive compatible,

$$\begin{aligned} y^D(\theta_{C1}, \theta_{D1}) &\leq y^D(\theta_{C1}, \theta_{D2}), \\ y^C(\theta_{C2}, \theta_{D1}) &\leq y^C(\theta_{C1}, \theta_{D1}), \end{aligned}$$

for all $\theta_{C1} \leq \theta_{C2}$ and $\theta_{D1} \leq \theta_{D2}$, and

$$\sum_i \tilde{t}_i(\theta) \leq 0,$$

for all θ. Then let

$$t_i(\theta) = \tilde{t}_i(\theta) - \frac{1}{2} E_\pi \left[\sum_{i=D,C} \tilde{t}_i(\theta) \right].$$

It is easy to see that Program 2 is convex, and hence should be robust in numerical simulation. The following Proposition shows that solutions to Program 2 without free dispoals also attain the solution to Program 1.

Proposition 14 *Any solution to program two* $\{y_i^*(\theta), t_i^*(\theta)\}_{i=1,2,\theta\in\Theta}$ *for which* $y^{D*}(\theta) = f\left(y^{C*}(\theta)\right)$, *also solves program one.*

Proof. Let $\{y_i^*(\theta), t_i^*(\theta)\}_{i=1,2,\theta\in\Theta}$ solve Program 2 with $y^{D*}(\theta) = f\left(y^{C*}(\theta)\right)$. First, note that this is feasible for Program 1: it satisfies the technology by virtue of the auxiliary assumption and we can let $k^*(\theta) = y^{C*}(\theta)$. It satisfies ex post incentive compatibility by virtue of Lemma 1 and the incentive constraints on the problem. To see that it satisfies ex ante budget balance, note that

$$E_\pi\left[\sum_i t_i^*(\theta)\right] = E_\pi\left[\sum_i \left(\tilde{t}_i(\theta) - \frac{1}{2} E_\pi\left[\sum_{i=D,C} \tilde{t}_i(\theta)\right]\right)\right] = 0.$$

As this is feasible for Program 1, then

$$\begin{aligned} V^1 &\geq E_\pi\left[A(\theta) f(k^*(\theta)) - R(\theta) k^*(\theta)\right] - \max_\theta \left\{\sum_{i=D,C} \tilde{t}_i(\theta)\right\} + E_\pi\left[\sum_{i=D,C} \tilde{t}_i(\theta)\right] \\ &= V^2, \end{aligned}$$

by virtue of the fact that at least one of the inequality constraints on the scaled transfers binds.

Now let $\left\{\hat{k}(\theta), \hat{t}_i(\theta)\right\}_{i=1,2,\theta\in\Theta}$ solve program 1, and define

$$\phi = \max_{\theta}\left\{\sum_{i=D,C} \hat{t}_i(\theta)\right\}$$

Then $\left\{\hat{k}(\theta), \hat{t}_i(\theta) - \phi/2\right\}_{i=1,2,\theta\in\Theta}$ is feasible for program 2 by Lemma 1, the incentive compatibility constraints, and the fact that

$$\sum_i \hat{t}_i(\theta) - \max_{\theta}\left\{\sum_{i=D,C} \hat{t}_i(\theta)\right\} \leq 0.$$

But then

$$\begin{aligned} V^2 &\geq E_\pi\left[A(\theta) f\left(\hat{k}(\theta)\right) - R(\theta)\hat{k}(\theta) + \sum_{i=D,C} \hat{t}_i(\theta) - \max_{\theta}\left\{\sum_{i=D,C} \hat{t}_i(\theta)\right\}\right] \\ &= E_\pi\left[A(\theta) f\left(\hat{k}(\theta)\right) - R(\theta)\hat{k}(\theta)\right] - \max_{\theta}\left\{\sum_{i=D,C} \hat{t}_i(\theta)\right\} = V^1, \end{aligned}$$

where the last line follows from the constraint on the choice of the $\hat{t}$. But then $V^1 = V^2$ and we are done. ■

REFERENCES

[1] Athey, Susan and Kyle Bagwell. 2001. "Optimal Collusion with Private Information." RAND Journal of Economics. 32 (3). 428-465.

[2] Athey, Susan, and David A. Miller. 2003. Efficiency in Repeated Trade with Hidden Valuations. Stanford University Working Paper.

[3] Bulow, J., and K. Rogoff. 1989a. "A Constant Recontracting Model of Sovereign Debt." Journal of Political Economy 97: 155–78.

[4] Bulow, J., and K. Rogoff. 1989b. "Sovereign Debt: Is to Forgive to Forget?" American Economic Review 79, no. 1: 43–50.

[5] Clemens, Michael A., and Jefferey G. Williamson. 2004. Wealth Bias in the First Global Capital Market Boom, 1870-1913. Economic Journal 114:304-337.

[6] Eaton, J., and M. Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." Review of Economic Studies 48: 289–309.

[7] Finnemore, Martha. 2003. The Purpose of Intervention: Changing Beliefs about the Use of Force. Ithaca, NY: Cornell University Press, 2003.

[8] Kletzer, K.M., and B.D. Wright. 2000. "Sovereign Debt as Intertemporal Barter." American Economic Review 90, no. 3: 621–39.

[9] Maddison, Angus. 2003. The World Economy: Historical Statistics. OECD Development Centre

[10] Marshall, Monty G. and Keith Jaggers. 2003. "Polity IV Project: Political Regime Characteristics and Transitions, 1800-2002." http://www.cidcm.umd.edu/inscr/polity/

[11] Miller, David A. 2004. Attainable Payoffs in Repeated Games with Interdependent Private Information. University of California, San Diego Working Paper.

[12] Miller, David A. 2004. The Dynamic Cost of Ex Post Incentive Compatibility in Repeated Games of Private Information. Stanford University Working Paper.

[13] Reinhart, C. K. Rogoff and M. Savastano (2003).

[14] Rose, A. K. and M.M. Spiegel. Forthcoming. "A Gravity Model of Sovereign Lending: Trade, Default, and Credit." IMF Staff Papers.

[15] Rubin, Donald. 1987. Multiple Imputation for Nonresponse in Surveys. New York: Wiley.

[16] Schafer, Joseph L. 1997. Analysis of Incomplete Multiariate Data. London: Chapman and Hall.

[17] Stone, Irving. 1999. The Global Export of Capital from Great Britain, 1865-1914. New York: St. Martin's Press.

[18] Suter, Christian. 1992. Debt Cycles in the World Economy: Foreign Loans, Financial Crises, and Debt Settlements, 1820–1990. Boulder, Colorado: Westview Press.

[19] Tomz, Michael. 2004a. Sovereign Debt and International Cooperation: Reputational Reasons for Lending and Repayment. Book manuscript. Palo Alto: Stanford University.

[20] Tomz, Michael. 2004b. "Voter Sophistication and Domestic Preferences Regarding Debt Default." Working Paper. Palo Alto: Stanford University.

[21] Wang, Cheng. 1995. Dynamic Insurance with Private Information and Balanced Budgets. Review of Economic Studies 62 (4):577-595.

[22] Wright, Mark L.J. 2003a. "Reputations and Sovereign Debt." Working Paper. Palo Alto: Stanford University.

[23] Wright, Mark L.J. 2003b. "Creditor Coordination and Sovereign Debt." Working Paper. Palo Alto: Stanford University.