On policy interactions among nations: when do cooperation and commitment matter ?*

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Abstract

This paper offers a comprehensive framework to study commitment and cooperation issues in games with multiple policymakers. To reconcile some puzzles in the recent literature on the effects of policy interactions among nations, we prove that games characterized by different commitment and cooperation schemes can have the same equilibrium outcome if certain spillover effects vanish at the common solution of these games. We provide a detailed discussion of these spillovers, showing that, in general, commitment and cooperation are non-trivial issues. Yet, models of the linear-quadratic variety with multiple policymakers can generate a 'symbiotic' result where commitment and cooperation issues are irrelevant, in the sense that the social optimum can be implemented under arbitrary commitment and cooperation schemes. The proposed framework can be extended to a stochastic environment and is sufficiently general to allow for a broad discussion of policy interactions, both within monetary unions and among fully sovereign nations.

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1 Introduction

The literature on the effects of policy interactions among nations proves to be quite puzzling. Paradoxes abound, and there exists an impressive range of different views on possible gains and costs from cooperation and commitment schemes. In a political context, these diverse views are a source of constant debate. Examples of controversially discussed cooperation and commitment schemes are, among many others, the Stability and Growth Pact of the European Monetary Union, international agreements on exchange rates or the adoption of currency boards.

These debates have clear counterparts at the academic level. A good example of the unsettling state of the discussion on policy interactions is provided by recent and conflicting analyses on policymaking in monetary unions. For example, Cooper and Kempf (2004) show that the ability of a central bank in a monetary union to commit with respect to national fiscal authorities affects the outcome of the policy-mix, as this device helps to prevent pressures to monetize national deficits. Leaning in the same direction, Chari and Kehoe (2004) consider a monetary union model which does not allow for any direct spillovers between countries and which has nevertheless the feature that equilibrium outcomes depend sensitively on patterns of cooperation and commitment, primarily driven by private sector coordination failures within countries and their relationship to the common monetary policy. However, in striking contrast to these findings, Dixit and Lambertini (2003) consider a monetary union model which allows for direct spillovers between countries and which has nevertheless the feature that policymakers attain the same equilibrium outcome, irrespective of whether policymakers cooperate or not and irrespective of the order of moves of players (i.e. commitment patterns).

Given these conflicting views, we are in need of a comprehensive analysis of policy interactions which would provide us with useful tools for the evaluation of various commitment and cooperation assumptions. In the present paper our goal is more modest: it is to provide some clues for understanding why some authors do or do not obtain an irrelevance result with respect to commitment and cooperations assumptions.

To this end, we set up a generic model for the analysis of policy interactions among independent but interdependent policymakers. We formalize cooperation among some players by means of coalitions. Hence, a given game is characterized by a commitment pattern and a coalition structure. We provide some propositions which develop special conditions under which commitment patterns and coalition structures do not matter. Specifically, we prove that games characterized by different commitment and cooperation schemes can have the same equilibrium outcomes if certain spillover effects vanish at the common solution of these games. We provide a detailed discussion of these spillovers, showing that, in general, commitment and cooperation are non-trivial issues. Yet, assuming consensus on the target values of all players, models of the linear-quadratic variety with multiple policymakers can generate a 'symbiotic' result where commitment and cooperation issues are irrelevant and where the unique equilibrium outcome of any game corresponds to the social optimum.

The proposed framework can be extended to a stochastic environment and is sufficiently general to resolve some of the puzzling findings on the (ir)relevance of cooperation and

commitment in the recent literature, both within monetary unions and among fully sovereign nations.
The remainder of the paper is structured as follows. Section 2 develops a general framework to study commitment and cooperation issues in games with multiple policymakers. We then offer a number of general propositions on the (ir)relevance of commitment patterns and coalition structures. In Section 3, we apply these propositions to discuss some recent contributions on policy interactions in monetary unions. In Section 4, extending our framework to the case of a stochastic economy, we look at some contributions in the literature on international policy coordination. Section 5 concludes. Proofs and some technical issues are delegated to the Appendix.

2 A unifying framework for policy analysis

2.1 Private agents and policymakers

There are N nations with index i. In each nation, there coexist private agents and policymakers. We refer to a generic player in this world economy, be it a private agent or a policymaker, as ξ, and the set of players as Ξ. A particular strategy of player ξ is denoted by x_ξ, while his payoff function is given by

$$V_\xi = V_\xi(\mathbf{x}),$$

where $\mathbf{x}$ denotes a strategy profile of all players, i.e. $\mathbf{x} = (x_\xi, \mathbf{x}_{-\xi})$.
Whenever we distinguish explicitly between private agents and policymakers, we adapt the general framework as follows. The world economy consists of a set of private agents $\mathcal{M}$, with M elements indexed by j. The private sector population of the $i-th$ nation is denoted by m_i. Agents are stacked according to their nationality. The m_1 first agents belong to nation 1, the next m_2 agents belong to nation 2 such that $M = \sum_{i=1}^{n} m_i$. Similarly, there exists a set of policymakers $\mathcal{P}$, with P elements indexed by q. A policymaker can be 'national' or 'international'. In principle, a policymaker may control a set of instruments. Here, for simplicity, we assume that a policymaker controls a unique instrument. There are p_i national policymakers in the $i-th$ nation. Moreover, there are p_{int} international policymakers, taking into consideration the payoffs of private agents belonging to different nations. A central bank in a monetary union formed by a subset of nations is an example of such an international policymaker. In sum, we have $P = p_{int} + \sum_{i=1}^{n} p_i$. Hence, $\Xi = \mathcal{M} \cup \mathcal{P}$, and the number of all players in the world economy is equal to $X = M + P$. Moreover, when we distinguish between strategy choices of the two types of players, we refer to a_{ij} as a strategy of private agent j in country i, to $\mathbf{a}_i$ as a strategy profile of all private agents in country i, to τ_{iq} as a strategy of policymaker q in country i, and to $\boldsymbol{\tau}_i$ as a strategy profile of all policymakers in country i. The payoff function of private agent j is denoted as follows:

$$U_j = U_j(\mathbf{a}_1, .., \mathbf{a}_i, .., \mathbf{a}_N, \boldsymbol{\tau}_1, .., \boldsymbol{\tau}_i, .., \boldsymbol{\tau}_N), \qquad \forall j \in \mathcal{M}.$$

The payoff function of policymaker p is denoted as follows:

$$V_q = V_q(\mathbf{a}_1, .., \mathbf{a}_i, .., \mathbf{a}_N, \boldsymbol{\tau}_1, .., \boldsymbol{\tau}_i, .., \boldsymbol{\tau}_N), \qquad \forall q \in \mathcal{P}.$$

The payoff function of a national policymaker may coincide with the payoff function of the representative private sector member of its own country. This assumption will become relevant in some applications discussed below.

2.2 Coalitions

Players may form coalitions. These coalitions may link players within a nation or cover players belonging to different nations. A coalition is a subset of players who cooperate. Any coalition C_θ is defined by two characteristics: i) it decides jointly over the strategies chosen by all its members, and ii) it maximizes the welfare W_θ of its members, with

$$W_\theta = \sum_{\xi \in C_\theta} \omega_\xi V_\xi(\mathbf{x}),$$

where ω_ξ denotes some weight attached to the welfare of player ξ. If a coalition covers players belonging to different nations, it is an international coalition. Notice that a membership to a coalition is different from the usual definition of membership, in the sense that it is assumed that all agents belong to one coalition only. The cardinal of a coalition C_θ is denoted by κ_θ^c. We denote by Θ the number of coalitions and define a coalition structure as a partition with the following features:

Definition 1 *A coalition structure* $\mathcal{C} = \{C_1, ..., C_\theta, ..., C_\Theta\}$ *is a set of coalitions such that:*
i) $C_\theta \cap C_{\theta'} = \emptyset$ *for all* $\theta \neq \theta'$ *and*
ii) $\overset{\Theta}{\underset{\theta=1}{\cup}} C_\theta = \Xi$.

When we distinguish explicitly between private agents and policymakers, coalitions may be formed either by private agents or by policymakers. For example, a trade-union is an agent-based coalition, and a fiscal cooperation scheme between national treasuries is a policymaker-based coalition. Mixed coalitions between private agents and policymakers are ruled out. For notational simplicity, we define coalitions in a broad sense so that they also include singletons (i.e. players acting in isolation) as special cases. An agent-based coalition C_θ is a subset of $\mathcal{M}$, leading to the restriction $1 \leq \kappa_\theta^c \leq M$, while a policymaker-based coalition is a subset of $\mathcal{P}$, implying $1 \leq \kappa_\theta^c \leq P$. If κ_θ^c is equal to M (P), then the grand agent-based (policymaker-based) international coalition forms. If the coalition covers all private agents (policymakers) in a nation, then it is an agent-based (policymaker-based) national coalition for the $i-th$ nation. The welfare for these two types of coalitions can be expressed as:

$$\begin{aligned} \text{Agent-based coalition} &: \quad W_\theta = \sum_{j \in C_\theta} \omega_j U_j(\mathbf{a}_1, .., \mathbf{a}_i, .., \mathbf{a}_N, \boldsymbol{\tau}_1, .., \boldsymbol{\tau}_i, .., \boldsymbol{\tau}_N) \\ \text{Policymaker-based coalition} &: \quad W_\theta = \sum_{q \in C_\theta} \omega_q V_q(\mathbf{a}_1, .., \mathbf{a}_i, .., \mathbf{a}_N, \boldsymbol{\tau}_1, .., \boldsymbol{\tau}_i, .., \boldsymbol{\tau}_N). \end{aligned}$$

where ω_j and ω_q denote the weights of agent j and policymaker q in their respective coalitions.

2.3 Commitment

We denote by Γ a multi-stage multi-coalitions extensive form game. There are T^{Γ} stages in this game, and we denote by $\mathcal{T}^{\Gamma}$ the set of stages: $\{1,..t,..,T^{\Gamma}\}$. We assume that each player is allocated to act at a particular stage. He plays only once in the entire game, at this particular stage, and he may form coalitions only with players who are also allocated to act at this particular stage. This is the standard assumption made in macroeconomic games, excluding repeated games. Hence, to describe a game it is convenient to distinguish between i) the order of moves of players (in the following for short: "commitment patterns"), determining at which stage every player acts, and ii) the "coalition structures" at each stage. In brief, a game is characterized by a commitment pattern $\mathfrak{C}$ and a coalition structure $\mathcal{C}$. Notice that different commitment patterns imply different coalition structures (since singletons are considered as special cases of coalitions), but not vice versa. Formally we use the following:

Definition 2 *A commitment pattern $\mathfrak{C}$ specifies an assignment for each player $\xi \in \Xi$ to act at one particular stage $t \in \mathcal{T}^{\Gamma}$.*

We denote by $\mathcal{X}_{\xi}$ the strategy space of player ξ, i.e. $x_{\xi} \in \mathcal{X}_{\xi}$ for all $\xi \in \Xi$. Moreover, we denote by $W(\Gamma)$ the payoff vector consisting of the payoffs of all players involved in the game:

$$W(\Gamma) = (V_1, .., V_{\xi}, .., V_X).$$

In short, a game Γ is characterized by the following elements:

- a coalition structure $\mathcal{C}$,
- a commitment pattern $\mathfrak{C}$,
- the strategy spaces $\mathcal{X}_{\xi}$ of all players $\xi \in \Xi$,
- the payoff vector $W(\Gamma)$.

2.4 Spillovers

This subsection offers a characterization of the marginal welfare effects of the actions of players. Given the existence of coalitions, it is important to distinguish between different types of spillovers. Generally speaking, the welfare effects of a particular strategy of any player can be decomposed into three distinct effects, namely the effects on his own welfare, the effects on the welfare of his coalition members (*within-coalition spillover effects*), and the effects on the welfare of players belonging to different coalitions (*between-coalition spillover effects*). In the context of multi-stage games these effects do not only include contemporaneous effects on players acting at the same stage, but also non-synchronous effects on players acting at a different stage.

Spillover effects between agents will play a crucial role in the rest of our analysis. Assuming that for all players the payoff function $V_{\xi}(\mathbf{x})$ is continuously differentiable in all arguments, we use the following definition.

Definition 3 *For a given commitment pattern and coalition structure* $(\mathfrak{C}, \mathcal{C})$ *and a given strategy profile* $\mathbf{x}$*, consider a representative player* ξ *with strategy* x_ξ *who belongs to coalition* C_θ*. Moreover, consider some other player* ξ'*. Then, we refer to* $\frac{\partial V_{\xi'}(\mathbf{x})}{\partial x_\xi}$ *as a within-coalition spillover effect if* ξ' *belongs to* C_θ *and as a between-coalition spillover effect if* ξ' *belongs to a coalition different from* C_θ*.*

With this definition, only between-coalition spillover effects may link two agents acting at different stages in the game. In this case, we shall refer to 'non-synchronous between-coalition spillover effects'. Moreover, since we rule out coalitions between policymakers and private agents, spillovers between these two types of players are necessarily between-coalition spillovers. Finally, notice that for the special case in which a particular coalition is made up of a singleton player within-coalition spillover effects are equal to zero.

2.5 Equivalence of Subgame perfect Nash equilibria

Any proper subgame of Γ played at t, $1 < t \leq T^\Gamma$, is denoted by G_t. G_t depends on H_t, where H_t denotes the history of actions decided before t. Let $\mathbf{x}\,|H_t$ denote the restriction on strategy profiles to be consistent with a particular history H_t. Then,

Definition 4 *A strategy profile* $\mathbf{x}$ *constitutes a subgame perfect Nash equilibrium of the game* Γ *if, for every proper subgame* G_t*, the restriction* $\mathbf{x}\,|H_t$ *induces a Nash equilibrium of* G_t*.*

Evidently, a large number of different games can be played in this multi-stage and multi-coalition economy, varying in terms of commitment patterns and coalition structures. In the following we establish some conditions which can be used to compare two different games Γ and Γ'. Any such comparison needs to keep track of both cooperation and commitment issues. A sufficient (but rather restrictive) condition for two apparently different games to admit nevertheless the same subgame perfect Nash equilibrium outcome, denoted by $\mathbf{z}$, is the following:

Proposition 1 *Consider two games* Γ *and* Γ'*, characterized by* $(\mathcal{C}, \mathfrak{C})$ *and* $(\mathcal{C}', \mathfrak{C}')$*, respectively. The two games admit the same subgame perfect Nash equilibrium outcome* $\mathbf{z}$ *if for the two games, at* $\mathbf{z}$*,*
i) there exist no within-coalition spillover effects between any pair of players (ξ, ξ') *belonging to a coalition which does not belong simultaneously to* $\mathcal{C}$ *and* $\mathcal{C}'$ *and if*
ii) there exist no between-coalition spillover effects between players in coalitions playing at different stages.

Proof: see appendix.

This proposition gives us conditions such that two different games can have the same subgame perfect Nash equilibrium outcome despite differences in terms of commitment or cooperation. These conditions are related to the absence of certain spillover effects at the equilibrium outcome $\mathbf{z}$. Notice that Proposition 1 does not require the absence of all spillover effects at $\mathbf{z}$. Such non-vanishing spillover effects can be of two varieties: they can be i) within-coalition spillover effects in coalitions which exist in both games or ii) between-coalition spillover effects existing between coalitions acting at the same stage. In other words, the common equilibrium outcome $\mathbf{z}$ is not necessarily the solution of the simultaneous Nash game, obtained when all players act as singletons.

To shed further light on the nature of Proposition 1 it is constructive to look into two special cases. First, we denote by Γ^{Nash} the game which is played by all players simultaneously (i.e. no commitment as there exist no sequential stages) and without any sort of coalition. We denote by $\mathbf{z}^{Nash}$ the equilibrium outcome of this 'no-commitment and no-cooperation' game, with $\mathbf{z}^{Nash}$ assumed to be unique. Then, Proposition 1 can be further extended as follows:

Corollary 1 *Any two extensive-form games Γ and Γ', characterized by arbitrary commitment patterns and coalition structures, admit the social planner's solution, identical to $\mathbf{z}^{Nash}$, if there are no marginal spillovers between any pair of players at $\mathbf{z}^{Nash}$.*

Remark: It is well-known that the Nash equilibrium outcome coincides with the social planner's solution when there are no spillover effects. The Corollary follows directly from the proof of Proposition 1. In sum, it states conditions under which cooperation and commitment are irrelevant. These conditions are quite stringent but they cannot be ruled out.

Second, in many applications differences in commitment patterns or coalition structures are restricted to subgames, while early stages are identical for the two games under considerations. It is straightforward to adapt the reasoning of Proposition 1 to such a special constellation. Consider two games Γ and Γ' with T^{Γ} and $T^{\Gamma'}$stages, respectively. Suppose that the two games have in common the first stages up to $\overline{T} - 1 < \min(T^{\Gamma}, T^{\Gamma'})$. $\overline{T}$ is a single node and proper subgames, $G_{\overline{T}}$ and $G'_{\overline{T}}$ may be defined at $\overline{T}$. By this we mean that the coalition structures, the commitment patterns and the information sets involved in the two games Γ and Γ' are identical up to $\overline{T} - 1$ but differ in the subsequent stages. Hence all differences between Γ and Γ' are captured by:

$$G_{\overline{T}} \neq G'_{\overline{T}}.$$

Given our assumptions that all players act only once and that the two games are identical up to stage $\overline{T} - 1$, the two subgames $G_{\overline{T}}$ and $G'_{\overline{T}}$ involve the same subset of players. Then we can offer the following:

Proposition 2 *Consider two games Γ and Γ' which are identical up to $\overline{T} - 1$ but have different commitment patterns or coalitions structures in the subsequent stages. If the two subgames $G_{\overline{T}}$ and $G'_{\overline{T}}$ satisfy the equivalence conditions of Proposition 1 then Γ and Γ' admit the same subgame perfect Nash equilibrium outcome $\mathbf{z}$.*

Proof: Starting from the proof of Proposition 1, Proposition 2 follows from backward induction.

This proposition tells us that for certain games not all the within-coalition and between-coalition spillovers listed in Proposition 1 need to vanish at the equilibrium outcome if one wants to establish the equivalence of games in line with the logic of the previous subsection. What matters are only those spillovers which occur in subgames which make the two games different.
Moreover, define $G_{\overline{T}}^{Nash}$ as the 'no-commitment and no-cooperation' one-stage Nash game involving the same players as $G_{\overline{T}}$ and $G'_{\overline{T}}$ and let $z_{G_{\overline{T}}}^{Nash}$ denote the subgame perfect Nash equilibrium outcome of the special game which is identical to Γ and Γ' up to $\overline{T}-1$ and exhibits the subgame $G_{\overline{T}}^{Nash}$ at $\overline{T}$. Then, similar to the logic underlying Corollary 1, Proposition 2 can be extended such that one can use $z_{G_{\overline{T}}}^{Nash}$ as a reference point for further comparisons:

Corollary 2 *Consider games which are identical up to $\overline{T}-1$. If there are at $z_{G_{\overline{T}}}^{Nash}$ no spillover effects between any players acting at stage $\overline{T}$ and later, then $z_{G_{\overline{T}}}^{Nash}$ is a subgame perfect Nash equilibrium outcome for all games Γ and Γ' which are identical up to $\overline{T}-1$ and which are characterized by arbitrary commitment patterns and coalition structures at stage $\overline{T}$ and later.*

Corollary 2 differs from the previous one because it does not imply that $z_{G_{\overline{T}}}^{Nash}$ is a subgame perfect Nash equilibrium outcome under arbitrary commitment patterns and coalition structures at all stages. Moreover, it is not necessarily true that $z_{G_{\overline{T}}}^{Nash}$ is equal to the social optimum. This feature results from the fact that there may well exist non-zero within-coalition and between-coalition spillover effects occurring prior to stage $\overline{T}$.

2.6 The linear-quadratic model for policy analysis

The results presented in the previous section can be used to shed some light on the nature of policy interactions in a multi-player model where preferences are quadratic and constraints are linear. This approach to macroeconomic policy analysis has a long established tradition, dating back to Theil (1958), and our discussion of recent policy applications shows that this approach is, indeed, still very much in use.
Let us write such a model as follows, using our previous setting. Consider an economy with X players indexed by ξ. The state of the economy is described by a linear model, that is there exists a $(X \times 1)-$ vector $\mathbf{y}$, which depends linearly on the $(X \times 1)-$vector of strategies of all players $\mathbf{x}$:

$$\mathbf{y} = \overline{\mathbf{y}} + \mathbf{B}\mathbf{x}. \tag{1}$$

with $\overline{\mathbf{y}}$ being a vector of constants and the matrix $\mathbf{B}$ being invertible. The $\xi-th$ element of $\mathbf{y}$, y_ξ, characterizes the situation of agent ξ. Similarly, $\mathbf{y}^*$ is a $(X \times 1)$-vector of target values, with $\xi-th$ element y_ξ^*. Importantly, it is assumed that the target values are shared by all agents.
The payoff function corresponding to player ξ is a weighted sum of squared deviations of the elements of $\mathbf{y}$ from their target values, such that:

$$V_\xi = \frac{1}{2}\left[\omega_1^\xi(y_1^* - y_1)^2 + ... + \omega_\xi^\xi(y_\xi^* - y_\xi)^2 + ... + \omega_X^\xi(y_X^* - y_X)^2\right] \tag{2}$$

with $\omega_\xi^\xi > 0$ and $\boldsymbol{\omega}_{-\xi}^\xi \geq 0$. Notice that individual payoffs depend on the actions of other players through the model itself (the $\mathbf{B}$ matrix) and the specification of the payoff functions. These payoff functions may differ as the vector of weights $\boldsymbol{\omega}^\xi$ may be specific to agent ξ.

Proposition 3 *For an economy described by (1) and (2), the equilibrium outcome* $\mathbf{z}^{Nash} = \mathbf{B}^{-1}[\mathbf{y}^* - \overline{\mathbf{y}}]$ *of the 'no-commitment and no-cooperation' Nash game* Γ^{Nash} *is identical to the social planner's solution. Consequently,* $\mathbf{z}^{Nash}$ *is a subgame perfect Nash equilibrium outcome for any possible extensive-form game characterized by arbitrary commitment patterns and coalition structures.*

Proof: see appendix.

Proposition 3 states that in a linear-quadratic model under the assumptions made above neither commitment nor cooperation matter. The explanation directly derives from Corollary 1 to Proposition 1: in the linear-quadratic model, all spillover effects between all players are null at the Nash equilibrium. Hence, $\mathbf{z}^{Nash}$ is identical to the social planner's solution and therefore it is also a subgame perfect Nash equilibrium outcome for any possible extensive-form game characterized by arbitrary commitment patterns and coalition structures.

This proposition is reminiscent of the Tinbergen rule (Tinbergen, 1952). Actually it may be seen as a generalized Tinbergen rule in a game-theoretic environment, assuming that there is no disagreement about the target values of all players. Hence, it is central to stress that this result relies not only on the linear-quadratic nature of the problem, but also on the assumption that each player disposes of an instrument and that the number of independent instruments matches the number of objectives in the payoff functions of all players.[1] The proposition can be readily extended to the introduction of an aggregate variable characterizing the economy as a whole, entering each individual payoff function in the form of a squared gap to a common target value, as well as the introduction of an additional player, controlling an additional instrument.

3 Applications: Monetary Unions

The previous propositions can be used to shed light on a number of puzzling and seemingly contradictory results that have recently been established on the (ir-) relevance of cooperation and commitment between policymakers. These issues have been particularly controversially discussed in recent contributions which address the desirable design of policymaking in monetary unions. Therefore, we devote this section entirely to monetary union issues.

When do cooperation and commitment matter in a monetary union? In general, the possible existence of spillovers within countries (related to private actors), of spillovers

[1] For a recent discussion of linear-quadratic frameworks for policy purposes see, in particular, Woodford (2003). Yet, in his applications the Tinbergen criterion (of assuming an identical number of objectives and independent instruments) is typically not satisfied.

between countries (related to fiscal and private actors) and of a common monetary policy (affecting players in all countries) creates a number of channels which make this question non-trivial, i.e. it is clear that, in general, commitment and cooperation (i.e. coalition structures) do matter, within countries and between countries.

Against this general insight two recently established findings seem particularly puzzling. On the one hand, Dixit and Lambertini (2003) consider a model which allows for spillovers between players acting in different countries and which nevertheless has the feature that fiscal and monetary policymakers attain the same equilibrium outcome, irrespective of the commitment patterns of the policymakers and irrespective of whether policies are coordinated between countries or not. By contrast, Chari and Kehoe (2004) consider a model which does not allow for any spillovers between players acting in different countries and which nevertheless has the feature that equilibrium outcomes depend sensitively on commitment patterns and on whether policies are coordinated between countries or not.[2]

Within the general framework established above, however, it is straightforward to resolve this puzzle. To this end, let us consider a monetary union with N member countries, indexed by $i = 1, 2, .., N$. For each country there exists a single fiscal policymaker (i.e. $\tau_{iq} = \tau_i$). Moreover, there exists a single monetary policymaker (central bank) operating for the monetary union as a whole. Let π denote a strategy of the central bank. Recall from above that a_{ij} denotes a strategy of private agent j in country i. Then, a strategy profile all players is given by $\mathbf{x} = (\mathbf{a}, \boldsymbol{\tau}, \pi)$, with $\boldsymbol{\tau} = (\tau_i, \boldsymbol{\tau}_{-i})$ and $\mathbf{a} = (\mathbf{a}_i, \mathbf{a}_{-i})$, where $\mathbf{a}_i$ can be further decomposed into $\mathbf{a}_i = (a_{ij}, \mathbf{a}_{i,-j})$. These assumptions lead us to consider the following set of payoff functions:

- Payoff function of a (representative) private agent j in country i:

$$U_{ij} = U_{ij}(\mathbf{a}, \boldsymbol{\tau}, \pi).$$

- Payoff function of fiscal policymaker in country i :

$$V_i = V_i(\mathbf{a}, \boldsymbol{\tau}, \pi)$$

- Payoff function of cooperating fiscal policymakers:

$$V^{FC} = \sum_{i=1}^{n} \omega_i^F V_i = \sum_{i=1}^{n} \omega_i^F V_i(\mathbf{a}, \boldsymbol{\tau}, \pi)$$

  where ω_i^F denotes the fiscal weight attached to country i in the collective fiscal payoff function.

[2] Evidently, there exists a broad literature on strategic policy interactions in monetary unions going back at least to Mundell (1961). For recent contributions, using reduced-form one shot games similar to this paper, see, in particular, Beetsma and Bovenberg (1998, 2001), Calmfors (2001), Cukierman and Lippi (2001), and Uhlig (2003). For an example of a related second-generation model, as further discussed in Section 4.2. below, see Beetsma and Jensen (2005).

- Payoff function of the central bank:

$$V^M = \sum_{i=1}^{n} \omega_i^M V_i = \sum_{i=1}^{n} \omega_i^M V_i(\mathbf{a}, \boldsymbol{\tau}, \pi).$$

  where ω_i^M denotes the monetary weight attached to country i by the central bank.

Notice that by assuming $V_i^F = V_i^M = V_i$ we rule out disagreement about target values between policymakers, i.e. we restrict for the remainder of this section possible differences between monetary and fiscal policy objectives to the weighting factors ω_i^F and ω_i^M.[3]

3.1 The Dixit-Lambertini (2003) model

Using this notation, the Dixit-Lambertini model can be adapted to our set-up as follows. Two assumptions are particularly important. First, the payoff function V_i allows, in general, for fiscal spillover effects between countries. Second, there exists a uniform private sector throughout the monetary union such that there are no spillover effects within the private sector, be they within countries or between countries. Specifically, private sector behaviour reduces to $a_{ij} = a$ for all $i, j,$, leading to

$$\begin{aligned} U_{ij} &= U_i = U = U(a, \pi) \\ V_i &= V_i(a, \tau_i, \boldsymbol{\tau}_{-i}, \pi). \end{aligned}$$

Should policymakers care about commitment patterns? And should fiscal policymakers in the member countries of a monetary union cooperate or not? The framework of Dixit and Lambertini has the striking feature that it gives rise to a general irrelevance proposition of cooperation and commitment patterns which can be summarized within our set-up as follows:

Dixit-Lambertini (2003): *Assume there exist fiscal spillover effects between countries. Despite this feature, there are no benefits from fiscal cooperation, as long as there is agreement about all target values of all players. In fact, these target values can be attained under arbitrary coalition structures and commitment patterns of all players.*

Given our general discussion in Section 2, this result is at first sight puzzling for two reasons. First, as concerns the interaction between fiscal policymakers, Dixit and Lambertini allow, in principle, for the existence of fiscal *within-coalition spillover effects* in V_i, suggesting that there exist benefits from cooperation (i.e. the formation of a fiscal coalition). Second, as concerns the interaction between monetary policy, fiscal polices, and private sector behaviour, Dixit and Lambertini allow, in principle, for *between-coalition spillover effects*, making also commitment patterns non-trivial. Notwithstanding these two features, the driving force behind this strong result is easily identified if one recognizes that the

[3] Implications of the assumption $V_i^F \neq V_i^M$ are discussed, in particular, in Beetsma and Uhlig (1999) and Dixit and Lambertini (2001, 2003b).

analysis is conducted within a linear-quadratic framework in line with the discussion in Section 2.6. Specifically, Dixit and Lambertini reserve the scalar a, summarizing union-wide private sector strategies, for private sector inflation expectations, i.e. $a \equiv \pi^e$, and all equilibria satisfy the assumption of rational expectations such that $\pi^e = \pi$. This feature can be recovered from writing U as

$$U = U(a, \pi) = \frac{1}{2}(\pi - \pi^e)^2,$$

i.e. $\pi^e = \pi$ results from a minimization of the squared inflation forecast error. Moreover, the policy objective V_i represents a weighted sum of squared deviations of (country-specific) output (y_i) and (union-wide) inflation values from target values, denoted by y_i^* and π^* = 0, respectively, such that

$$V_i = \frac{1}{2}\left[\omega_i(y_i^* - y_i)^2 + \pi^2\right],$$

while the output levels depend linearly on the vector of actions $\mathbf{x} = (\pi^e, \tau_i, \boldsymbol{\tau}_{-i}, \pi)$[4]:

$$y_i = \overline{y_i} + \sum_{k=1}^{n} b_{ik}\tau_k + b_i(\pi - \pi^e).$$

By construction of U, V_i, and V^M, there is consensus on the target values between all players under all conceivable cooperation and commitment schemes. Hence, as shown in the Appendix, the economy satisfies all the requirements of Proposition 3, i.e. all spillover effects, including those between private agents and policymakers, vanish at the unique equilibrium outcome of the 'no-commitment and no-cooperation' Nash game Γ^{Nash}. Because of this feature, this outcome is identical to the social optimum (i.e. all players always attain their target values), and the irrelevance proposition extends to all possible commitment patterns not only of policymakers, but also of the private sector.[5]

The key result of Dixit and Lambertini is refreshing and provocative at the same time since it challenges the conventional wisdom that the existence of spillovers should create

[4] Our representation abstracts from two features of the original Dixit-Lambertini model which are, however, inconsequential for the key result and our discussion of it, also in Section 3.3 below. First, the original model decomposes inflation into a part controlled by the central bank and a contribution related to fiscal policies. Second, the original model has a certain stochastic flavour, in the sense that the variables $\overline{y_i}$, b_{ik}, and b_i are stochastic. Yet, since players react after the realizations of these variables, the resulting ex post game is in line with the set-up of Section 2.6, where without loss of generality $\overline{\mathbf{y}}$ and $\mathbf{B}$ may also be seen as predetermined rather than as constant variables. This assessment covers also the final scenario in the original paper of so-called 'discretionary monetary leadership' where fiscal policy is strong enough to prevent genuine (ex ante) uncertainty.

[5] The corresponding summary in Dixit and Lambertini (2003, p. 245) is as follows: "If the monetary and fiscal authorities in a monetary union have identical output and inflation goals, those goals can be achieved without the need for fiscal coordination, without the need for monetary commitment, irrespective of which authority moves first and despite any disagreement about the relative weights of the two sets of objectives." Under the particular assumption of reducing private sector behaviour to forecasting inflation, the notion of 'arbitrary' timing protocols of private sector activities is not meaningful. Yet, in refined models with richer private sector strategies this would be different.

meaningful commitment and cooperation problems. Certainly, the model is special in many ways. For example, private sector actions are restricted to the assumption of rational inflation expectations at the union-wide level. Similarly, there is no role for country-specific inflation effects on national output levels, i.e. possible tensions between such effects and policy reactions of the central bank to union-wide inflation developments are ruled out. However, it would be possible to introduce refinements of the model along these lines which respect the crucial characteristics of the linear-quadratic set-up such that the irrelevance proposition would not be challenged.[6] Hence, the evident limitations of this proposition are linked to more fundamental concerns. First, it is clear that linear-quadratic frameworks, while being convenient and widely used approximations, are, by construction, not generic. Second, the irrelevance proposition requires that the number of policy objectives matches the number of independent instruments available to all policymakers. Moreover, when using these instruments, monetary and fiscal policymakers face systematically different constraints in their effectiveness vis-à-vis the private sector. The consequences of these two features are addressed in turn in the following two subsections.

3.2 The Chari-Kehoe (2004) model

Results very different from Dixit and Lambertini are obtained by Chari and Kehoe. In a sense, this is not surprising since they study a general class of economies in which the very special features of linear-quadratic economies do not apply. Yet, the striking feature of their analysis is that they manage to establish non-trivial results on cooperation and commitment in a framework which deliberately rules out any spillovers between players acting in different countries.

To reconstruct this reasoning in terms of our broad framework, the two crucial assumptions invoked by Chari and Kehoe can be summarized as follows. First, the payoff functions U_{ij} and V_i rule out, in general, any spillover effects between players acting in different countries, be they private or fiscal. Second, the payoff function U_{ij} exhibits spillovers between private agents within any country $i = 1, 2, .., N$, leading to

$$\begin{aligned} U_{ij} &= U_{ij}(a_{ij}, \mathbf{a}_{i,-j}, \tau_i, \pi) \\ V_i &= \sum_{j \in \mathcal{M}_i} U_{ij} = \sum_{j \in \mathcal{M}_i} U_{ij}(a_{ij}, \mathbf{a}_{i,-j}, \tau_i, \pi). \end{aligned}$$

The main proposition of Chari and Kehoe, adopted to our framework, can be summarized as follows:

Chari-Kehoe (2004): *Assume there are no spillovers between any players acting in different countries. Then, fiscal cooperation is still relevant under certain commitment patterns. Specifically, i) if the monetary authority can commit (moves first) and fiscal policymakers move last, the equilibrium outcomes of fiscal cooperation and fiscal non-cooperation*

[6] There exist hybrid monetary unions models, like Calmfors (2001), which respect for some, but not all reduced form equations, the linear-quadratic structure. Yet, to use them as counterexamples to the reasoning of Dixit and Lambertini is not entirely satisfactory.

are identical. However, ii) if the monetary authority cannot commit (moves last) and fiscal policymakers move first, the two equilibrium outcomes differ because of a time consistency problem of monetary policy, related to the interaction with the private sector.

Notice that part i) of this result follows directly from Proposition 2. If fiscal policy moves last and if, by assumption, there are no fiscal within-coalition spillovers it is evident that fiscal cooperation becomes irrelevant. Correspondingly, part ii) of the result reflects, broadly speaking, that this same reasoning does not go through if fiscal policy moves first and if there exist non-synchronous between-coalition spillovers related to monetary policy (which moves last), in line with the logic of Proposition 1.[7] The main contribution of Chari and Kehoe is to discuss thoroughly the role of private sector behaviour in this context. Specifically, it is well-known that monetary policy, if it cannot credibly move prior to the other actors, may be a source of non-synchronous spillovers in a monetary union, reflecting the logic of a last-round bailout motive of monetary policy. However, as discussed in detail by Chari and Kehoe, for this argument to prevail under their rather stringent assumptions it is crucial that non-cooperative private sector behaviour reinforces these spillovers such that monetary policy cannot undo them at the margin by means of a simple envelope theorem argument. Economically speaking, if private sector agents expect a monetary reaction to earlier fiscal decisions, but the private sector itself suffers within each country from a coordination problem, then this creates a fiscal cooperation problem in the first place which cannot be undone by monetary policy at a later stage. Hence, for a fiscal cooperation problem to exist under the assumptions maintained by Chari and Kehoe it is not enough that fiscal policy moves prior to monetary policy. Instead, this constellation needs to be enriched by a (plausible) lack of private sector coordination which breaks the logic of the envelope theorem.[8]
Notice, however that in the general class of economies studied by Chari and Kehoe, fiscal cooperation is always relevant if one allows within V_i for fiscal spillover effects between countries, irrespective of whether the lack of commitment of monetary policy may induce an additional fiscal cooperation problem.

3.3 Tinbergen (1956) vs. Barro-Gordon (1983): what are the relevant constraints?

Apart from the special characteristics of a linear quadratic set-up, there is a second, separate feature of the Dixit-Lambertini model which drives their irrelevance proposition. Specifically, there exist, using our notation, $N+1$ policymakers with the same number of policy objectives, as captured by the intention to close N (country-specific) output gaps $(y_i^* - y_i)$ and the single (and union-wide) inflation gap $\pi^* - \pi$. Moreover, policymakers have access to $N+1$ instruments, as embodied in the vector of strategies $(\boldsymbol{\tau}, \pi)$, which can

[7] More specifically, part i) of the result is obtained under the sequence of moves: 1) monetary policy, 2) private sector, 3) fiscal policy. By contrast, part ii) of the result is obtained under the sequence of moves: 1) fiscal policy, 2) private sector, 3) monetary policy.

[8] In the Appendix we summarize this reasoning in a more detailed way by adapting the main insights of the Chari-Kehoe model to our framework.

be both independently and very effectively chosen, resembling the analysis of Tinbergen (1956). Both aspects give rise to a number of questions.

First, the 'independence' assumption may not be seen as entirely convincing if one interprets the reduced-form equations as an approximation to a model in which all policy instruments are assumed to have budgetary implications. Under this assumption, one rather needs to respect that all instruments are jointly tied together by some version of a consolidated budget constraint of the public sector, as stressed, in particular, by Cooper and Kempf (2004).[9] For the sake of illustration, let us assume that the linearized budget constraint can be represented as

$$\sum_{i=1}^{n} \alpha_i \tau_i + \alpha_\pi \pi = 0. \tag{3}$$

Hence, for (3) to be satisfied in equilibrium, not all $N+1$ instruments can be chosen independently. Instead, any feasible commitment pattern needs to respect that there exists at least one player who adjusts his instrument passively to satisfy (3). For a given set of coefficients $\boldsymbol{\alpha}$, this implies that, in general, it will not be possible to simultaneously achieve all $N+1$ policy objectives. Alternatively, this reasoning relates to the insight that for genuine policy trade-offs to exist the number of independent instruments needs to be smaller than the number of objectives.[10]

Second, as concerns the 'effectiveness' of the instruments, the ability of policymakers to implement the social optimum under arbitrary coalition structures and commitment patters reflects that monetary and fiscal policymakers face systematically different constraints vis-à-vis the private sector. Specifically, monetary policy suffers from the well-known time consistency problem, i.e. in a rational expectations equilibrium ($\pi = \pi^e$) monetary policy cannot close the (structural) output gap ($y_i^* - \overline{y_i}$), while fiscal polices do not face such a restriction. How plausible is it to postulate that the time consistency problem is systematically different for monetary and fiscal policymaking? This fundamental difference between the two types of policymakers is remarkably different from the otherwise symmetric treatment of all policymakers in the spirit of Tinbergen. To see this striking feature more clearly, it is constructive to entirely shut down fiscal policy within the closed-economy counterpart of the model discussed in Section 3.1. Then, the analysis collapses to the standard monetary policy model of Barro and Gordon (1983) where the relevance of monetary commitment is well-known, reflecting the trade-off faced by monetary policy to meet output and inflation objectives with a single instrument.

[9] Otherwise the model of Cooper and Kempf (2004) is very different from Dixit and Lambertini (2003). In particular, it is not of the linear-quadratic variety.

[10] However, from a broader perspective, the neglect of the budget constraint could be rationalized if one thinks about policy actions without (direct) budgetary incidence, like reform measures which affect the competitiveness of industries etc. Moreover, in a narrow fiscal context, one could assume that national treasuries have access to (lump-sum) balancing items which are not related to the spillovers between countries.

4 Applications: International monetary policy cooperation

The purpose of this concluding section is to show that the literature on international monetary policy cooperation among fully sovereign nations offers clear analytical counterparts to our discussion of the (ir)-relevance of cooperation and commitment in monetary unions. This assessment hold true for so-called first-generation models (with ad-hoc payoff functions similar to the economies covered so far) as well as for the by now widely used second-generation models (where the payoff functions of policymakers are made fully consistent from first principles with the welfare objectives of private agents). For either type of model 'irrelevance' results obtain under particular assumptions. As we show in the remainder of this section, key features of these assumptions can be reproduced within our general framework. Moreover, by doing so we also point out an important qualitative difference between first- and second generation models. Our key references for first-generation models are Rogoff (1985) and the summery treatment by Canzoneri and Henderson (1991), while our discussion of second-generation models is primarily based on Obstfeld and Rogoff (2002) and the synthesis paper by Canzoneri et al. (2005). In line with other core contributions in this field, it should be emphasized early on that all these studies abstract from fiscal policy issues.

4.1 First-generation models: Rogoff (1985) and Canzoneri-Henderson (1991)

Rogoff (1985) and Canzoneri and Henderson (1991) offer widely cited contributions of the first-generation type which give clear insights about the nature of cooperation and commitment problems in international monetary policymaking. The Canzoneri-Henderson (benchmark-) model is often referred to because it can be used to see the existence of generic benefits from cooperation between policymakers, while the analysis of Rogoff (1985) gives rise to the insight that such benefits can well be elusive if commitment patterns (particularly the timing of private sector actions) are not considered as well.
Both contributions study symmetric two-country set-ups, leading to reduced forms which duplicate the Barro-Gordon trade-offs. These trade-offs, however, are enriched with monetary spillovers between the two countries. Moreover, both studies use linear-quadratic set-ups. We offer a simplified representation which, while capturing key insights from the two studies, is kept deliberately similar to the exposition of the Dixit-Lambertini model discussed above.

There exist two countries. In each country, the monetary policymaker controls domestic inflation (i.e. π_i is the single instrument of the monetary policymaker in country i) and he faces an output and an inflation objective. It is assumed that the output levels in the two countries do not respond to the monetary policy instrument of the other country. However, the inflation objective creates monetary spillover effects between countries. Specifically, the inflation objective is defined in terms of CPI-inflation which, because of trade linkages, depends on both domestic and foreign inflation. Let private sector payoffs in the two countries be denoted by

$$U = \frac{1}{2}(\pi_i - \pi_i^e)^2, \quad i = 1, 2$$

The policy objective V_i in country i is defined as

$$V_i = \frac{1}{2}\left[\omega_i(y_i^* - y_i)^2 + (\pi_i^{CPI})^2\right], \quad i = 1, 2$$

where π_i^{CPI} denotes the CPI-inflation of country i. Output y_i in country i is given by

$$y_i = \overline{y_i} + b_i^y(\pi_i - \pi_i^e), \quad i = 1, 2,$$

while π_i^{CPI} depends linearly on the inflation rates of the two countries

$$\pi_1^{CPI} = b^{\pi,h}\pi_1 - b^{\pi,f}\pi_2 \text{ and } \pi_2^{CPI} = -b^{\pi,f}\pi_1 + b^{\pi,h}\pi_2,$$

where $0 < b^{\pi,h} < 1$ and $b^{\pi,f} \neq 0$ summarize how domestic and foreign inflation feed into the respective CPI-inflation objectives of the two countries. Using this simplified representation, the key result can be summarized as:

Rogoff (1985) and Canzoneri-Henderson (1991): *Assume there exist monetary spillover effects between countries. Then, coalition structures and commitment patterns, in general, are not irrelevant, notwithstanding agreement about all target values of all players.*

Why is cooperation, generically, relevant?[11] And why do the gains and benefits from cooperation depend on the assumed commitment pattern? Generally speaking, the above reduced-form equations exhibit both between-coalition spillover effects (between private agents and the monetary policymakers) and within-coalition spillover effects (between the monetary policymakers). To undo these spillovers in line with Proposition 3 would require that the number of objectives matches the number of independent instruments. However, this is not the case, since there are altogether six gaps to be closed (two output gaps, two CPI-inflation gaps, and two gaps related to inflation forecast errors), while there are only four players (one policymaker and one private sector agent in each of the two countries, controlling the four instruments π_1, π_2, π_1^e, π_2^e). This mismatch between instruments and objectives rules out that the spillovers between all players vanish at the simultaneous move Nash game Γ^{Nash}. Hence, these spillovers gives rise not only to a strategic conflict over the choice of the two policy instruments, but they also ensure that commitment patterns are 'relevant'.

For a simple illustration of this broad result, consider a two-stage game in which the two private agents move first, while the two policymakers move last. As it is well-known, this timing gives rise to a time inconsistency problem of monetary policy and, as pointed out by Rogoff (1985), a potential cooperation between the two policymakers may either mitigate or reinforce the magnitude of this problem. To see why, it is instructive to consider a

[11] The term 'generic' is used to indicate that there exists in this literature a separate discussion, not addressed in this paper, about the quantitative (ir)relevance of international monetary spillovers in modelling frameworks which, in principle, give cooperation issues a role.

completely symmetric version of the above set-up and to compute the time consistent (i.e. subgame perfect Nash) equilibrium under cooperation and non-cooperation of the two policymakers. With $\pi_1 = \pi_1^e = \pi_2 = \pi_2^e = \pi$, one obtains under non-cooperation

$$\pi^{NC} = \frac{\omega b^y (y^* - \overline{y})}{(b^{\pi,h})^2 - b^{\pi,h} b^{\pi,f}} > 0,$$

while the cooperative outcome is given by

$$\pi^C = \frac{\omega b^y (y^* - \overline{y})}{(b^{\pi,h} - b^{\pi,f})^2} = \frac{\omega b^y (y^* - \overline{y})}{(b^{\pi,h})^2 - b^{\pi,h} b^{\pi,f} + (b^{\pi,f})^2 - b^{\pi,h} b^{\pi,f}} > 0$$

Then, to obtain a constellation which is conducive to gains from cooperation (by mitigating the time inconsistency problem), assume $b^{\pi,f} < 0 < b^{\pi,h} < 1$, implying $\pi^{NC} > \pi^C$. This particular sign pattern can be made consistent with a nominal exchange rate (a variable which is specified in the underlying model to make home and foreign inflation choices comparable) which depreciates in equilibrium by less than one-to-one in response to a one-sided increase in home inflation, holding foreign inflation constant. In other words, market clearing in response to a one-sided increase in home inflation generates a real appreciation of the home currency against the foreign currency. Under this assumption, it is ensured that under non-cooperation the two policymakers do not internalize that 'higher inflation at home also increases the inflation objective in the other country'. Collectively, such a scenario generates an inflation bias under non-cooperation, implying that there are gains from cooperation, in line with the overall thrust of the analysis of Canzoneri and Henderson (1991).[12] Alternatively, a constellation conducive to gains from non-cooperation can be obtained if one assumes $0 < b^{\pi,f} < b^{\pi,h} < 1$, implying $\pi^{NC} < \pi^C$. In this scenario, assuming an opposite pattern of exchange rate adjustments in the underlying economy, under non-cooperation the two policymakers do not internalize that 'higher inflation at home decreases the inflation objective in the other country'. Hence, in this scenario non-cooperation of policymakers has the benign feature to mitigate the inflation bias, as stressed by Rogoff (1985). Finally, it is clear that commitment patterns are in this context, indeed, 'relevant', i.e. the sketched inflationary bias under either scenario entirely disappears if the order of moves is reversed, i.e. if one considers a two-stage game in which the two policymakers move first, while the private agents move last.

In sum, the two models are canonical examples of frameworks in which cooperation and commitment are not irrelevant. Given the close relationship of the reduced forms to the Dixit-Lambertini framework, this result may be at first sight somewhat surprising. Yet, the discrepancy simply reflects that the two models, deliberately, allow for a mismatch

[12] The original Canzoneri-Henderson model differs in one important aspect, since it allows for shocks (a symmetric global shock and an asymmetric country-specific shock, both affecting only the inflation objectives of the two countries), leading to an 'ex post' game of inflationary surprises in which the monetary instruments are chosen after the shocks have been realized. Yet, since the policymakers move after the uncertainty has been resolved, the logic of this ex-post game is not different form the deterministic framework underlying Proposition 3. On this, see also footnote 4 above. Moreover, the gaps in V_i differ from our representation because of the role of the inflationary shocks and the assumption that there are no tensions between private agents and policymakers about the desired output level.

between instruments and objectives which is absent in the Dixit-Lambertini analysis. To put it differently, Rogoff (1985) and Canzoneri and Henderson (1991) are in the tradition of Barro and Gordon (1983), and not of Tinbergen (1956).
In line with the discussion of Section 3, however, it would be straightforward to recover an irrelevance result on international policy cooperation within the above synthesis framework, covering, in fact, both monetary and fiscal policy, if one introduced in each country one additional (fiscal) player with an additional and independent instrument that relates linearly to output and CPI-inflation. Moreover, under this assumption, not only cooperation but also commitment would become irrelevant, in the spirit of Proposition 3. Alternatively, rather than to increase the number of instruments, with the same effect a reduction of objectives could be considered: for example, if the two countries were to focus solely on the inflation objective (i.e. $\omega_i = 0$), the desirability of cooperation, once more, would disappear.

4.2 Second-generation models

Since second-generation models consider genuine stochastic set-ups we offer first a self-standing extension of Proposition 3 to a stochastic environment.

4.2.1 Stochastic extension of the linear-quadratic set-up

In the deterministic version of the linear-quadratic set-up underlying Proposition 3, it has been argued that the number of independent instruments needs to match the number of (squared) gaps in the payoff function of all players for cooperation and commitment to become irrelevant. This reasoning can be extended to a special stochastic environment where the relevant gaps reappear as entries of a variance-covariance matrix.
To this end, let the world economy consist of N nations with index i. In each nation, there exists exactly one policymaker who controls one (monetary) instrument. The world economy is subject to S shocks, summarized in the vector $\boldsymbol{\varepsilon}$, and it is assumed that $S = N$, i.e. the number of shocks is identical to the number of players (instruments). The state of the world economy is described by a linear model, i.e. there exists a $(N \times 1)-$ vector $\mathbf{y}$, which depends linearly on the $(N \times 1)$ vector $\boldsymbol{\varepsilon}$ of shocks with mean zero and variance-covariance matrix Ω_ε, as well as on the $(N \times 1)$-vector $\mathbf{x}$ of actions of all players:

$$\mathbf{y} = \mathbf{B}\mathbf{x} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim (\mathbf{0}, \boldsymbol{\Omega}_\varepsilon) \tag{4}$$

with the $(N \times N)$ matrix $\mathbf{B}$ being invertible. All players choose ex ante non-cooperatively policy rules which are linear in $\boldsymbol{\varepsilon}$, i.e.

$$\mathbf{x} = \mathbf{R}\boldsymbol{\varepsilon}, \tag{5}$$

with the matrix $\mathbf{R}$ being $(N \times N)$.[13] The matrix $\mathbf{R}$ summarizes the strategy profiles of all players. Specifically, let $\mathbf{r}_i$ denote the (row)-vector of response coefficients chosen by

[13] We deliberately use this loose wording (rather than to say that players 'commit' via rules) in order to avoid misunderstandings with our usage of the term commitment (i.e. the 'order of moves of players'), as described in Definition 2.

player i, while $\mathbf{R}_{-i}$ contains the reactions of all other players. The payoff function of player i is described by an autonomous component $\widetilde{V}_i$ and a quadratic form which describes a weighted sum of the variance and covariance terms associated with $\mathbf{y}$, such that

$$V_i = \widetilde{V}_i + \boldsymbol{\omega}_i' \Omega_{\mathbf{y}} \boldsymbol{\omega}_i. \tag{6}$$

Assuming non-cooperative behaviour of all N players, the representative player i has the objective to choose optimally the elements of the vector $\mathbf{r}_i$ to minimize V_i, taking as given $\mathbf{R}_{-i}$. One easily verifies that the matrix $\mathbf{R}^{Nash} = -\mathbf{B}^{-1}$ satisfies the best-response property for all players in the simultaneous Nash game. Given the linear structure of $\mathbf{y}$, the solution $\mathbf{R}^{Nash} = -\mathbf{B}^{-1}$ is unique for non-degenerate $\boldsymbol{\omega}_i$ and $\Omega_{\boldsymbol{\varepsilon}}$. Moreover, let V_i^* denote the welfare of player i under the solution of the social planner and assume

$$\widetilde{V}_i = V_i^*. \tag{7}$$

Then, since $\mathbf{R}^{Nash} = -\mathbf{B}^{-1}$ ensures that $V_i - \widetilde{V}_i = 0$ for all i, V_i must coincide with the solution of the social planner. In sum, this leads to the Proposition:

Proposition 4 *For an economy described by (4), assuming that players choose policy rules which are linear in the shocks so as to minimize (6) with $\widetilde{V}_i$ being given by (7), the equilibrium outcome $\mathbf{R}^{Nash} = -\mathbf{B}^{-1}$ of the simultaneous Nash game is identical to the social planner's solution. Consequently, it is also a subgame perfect Nash equilibrium outcome of any possible extensive-form game characterized by different commitment patterns and coalition structures.*

Proof: see appendix.

Proposition 4 has a clear analogy to Proposition 3. Specifically, the last statement of Proposition 4 captures the idea that situations can be imagined where players, when ex ante writing down their rules, know that, once uncertainty has been resolved, they will be called to implement their actions not simultaneously, but in a certain order and as a member of a certain coalition. Then, within the stochastic extension of the linear-quadratic model described by (4)-(7), ex ante knowledge about any such situation, when shared by all players, would not lead to strategy choices different from R^{Nash}, assuming that the number of independent instruments matches the number of shocks.

4.2.2 The models of Obstfeld-Rogoff (2002) and Canzoneri et al. (2005)

In second-generation models, the sources of potential strategic conflict between countries are similar to first-generation models. Yet, what makes the analysis conceptually different is that second-generation models typically consider stochastic ex ante games in which the objectives of policymakers correspond to the *expected* welfare of private agents. This welfare measure (i.e. V_i) is consistent with optimizing private sector behaviour in a general equilibrium setting and depends on the properties of the underlying exogenous shocks and the parameters of the policy rules, i.e. the instruments over which policymakers optimize. The range of different policy games that can be studied within such a set-up is typically large, reflecting the richness of the underlying general equilibrium specification which invites for variations.

The model of Obstfeld-Rogoff (2002) as well as the closely related synthesis model of Canzoneri et al. (2005) are representative of the paradigm of 'New open economy macroeconomics'. In line with this paradigm, both studies consider a two-country open economy extension of a New Keynesian framework with sticky wages, thereby ensuring that monetary policy has an effective stabilization role. Specifically, while the private sector sets nominal wages before the uncertainty is resolved, monetary policymakers can credibly use rules which make their actions contingent on the realizations of the shocks.

To establish a natural benchmark for the assessment of the welfare effects of monetary policy, Obstfeld and Rogoff (2002) decompose the objective (V_i) into a flexible-wage component ($\widetilde{V}_i$) and a residual component which captures the additional effects coming from the existence of sticky wages ($\boldsymbol{\omega}_i' \Omega_{\mathbf{y}} \boldsymbol{\omega}_i$). Then, based on this decomposition, a two-step procedure is invoked to check the (ir)relevance of monetary policy cooperation between the two countries.

First, it is checked whether the flexible wage solution around which the monetary stabilization takes place is 'constrained Pareto efficient ex ante'. Broadly speaking, this criterion will be satisfied if the sticky wage distortion is the only general equilibrium distortion which is affected by monetary policy. By contrast, if the set-up allows for further (and genuine open-economy) imperfections that can be affected by monetary policy, like exchange rate externalities and risk-sharing concerns under imperfect capital markets, this criterion is typically no longer satisfied.[14]

Second, assuming that the flexible wage solution is 'constrained Pareto efficient ex ante', it needs to be established whether the structure of the shocks is such that this solution can be implemented by monetary policy. In terms of Proposition 4, this amounts to verify that the number of policymakers (more generally speaking, the number of independent policy instruments) matches the number of shocks.

In sum, this logic extends the reasoning of the first-generation models by adding a more stringent welfare foundation of policy objectives. Adapted to our framework, it can be summarized as follows:

Obstfeld-Rogoff (2002) and Canzoneri et al. (2005): *Assume there exist monetary spillover effects between countries. Then, coalition structures and commitment patterns between policymakers, in general, are not irrelevant. However, if the flexible wage solution is constrained Pareto efficient ex ante and if there are sufficient instruments to stabilize the economies at this solution in line with Proposition 4, coalition structures and commitment patterns between policymakers become irrelevant.*

In order to obtain an irrelevance result along these lines, both criteria need to be satisfied and the two papers offer distinct examples in which at least one criterion is not satisfied.[15]

[14] For systematic discussions of interactions between closed-economy and open-economy distortions in closely related models, see, among others, Corsetti and Pesenti (2001) and Benigno (2002).

[15] The cited literature focuses on the (ir)relevance of policy cooperation. However, whenever the irrelevance of policy cooperation is established by means of Proposition 4, this also implies the irrelevance of

Obstfeld and Rogoff (2002), consider a class of preferences which do not guarantee efficient risk-sharing under country-specific, asymmetric shocks. Under this assumption, the validity of the '$S = N-$stabilization criterion' is not sufficient to ensure that non-cooperative ('self-oriented') Nash policies attain the cooperative outcome. Intuitively, the risk-sharing criterion comes in as an additional welfare objective which will not be addressed if non-cooperative stabilization policies successfully undo the distortions related to sticky wages. Alternatively, Canzoneri et al. (2005) focus more narrowly on logarithmic preferences which ensure that for all conceivable shocks efficient risk-sharing will always be ensured. Hence, in their model the validity of the $S = N-$stabilization criterion would be sufficient to ensure that the Nash policies attain the cooperative outcome. Yet, their model allows within each country for a number of sector-specific shocks, implying, in general, that the two monetary policymakers have not sufficient instruments to offset all shocks ($N < S$). Because of this feature, cooperation is generically relevant. Finally, it needs to be emphasized that often the two criteria cannot be independently assessed. In particular, Obstfeld and Rogoff point out that in their model global shocks, which affect both countries in exactly the same manner, do not create a risk-sharing problem, differently from country-specific shocks.

In sum, because of the endogenous, model specific foundation of the relevant policy objectives, it seems safe to conclude that second-generation models are even less of the Tinbergen-type than first-generation models. Or, to put it differently, because of the less mechanical link between instruments and welfare objectives, it becomes analytically much harder to check whether the Tinbergen criterion is satisfied.

5 Conclusion

In this paper, we set up a general framework to address the importance of commitment patterns and cooperation schemes in policy games between various policymakers. We prove that the nature of spillover effects between agents is of key relevance to answer this issue. To this end, we offer a simple classification of spillover effects between agents which distinguishes between within-coalition and between-coalition spillover effects. Based on this classification, we provide general propositions which prove that under some conditions, linked to these spillover effects, commitment and cooperation schemes do not matter. In particular, linear-quadratic models can well lead to the conclusion that commitment and cooperation issues are entirely irrelevant. Yet, the conditions which are responsible for this puzzling result are shown to be rather restrictive and, more importantly, these conditions have no longer any bite in a generic, non-linear environment. We then apply these theoretical results to review a number of recent, seemingly contradictory contributions on policy interactions within monetary unions and among fully sovereign nations.

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commitment patterns, i.e. policymakers may be called to implement their actions under arbitrary orders of moves.

6 Appendix

6.1 Proof of Proposition 1

We consider two-stage games Γ and Γ', allowing for coalitions among subsets of agents. It is easy to generalize the proof to games with more stages. Consider first a game Γ. We partition the set of players into two subsets Ξ_1 and Ξ_2. Ξ_1 (Ξ_2) is formed of players making their decision at stage 1 (2). At each stage, coalitions may be active. There are K (L) coalitions at stage 1 (2), denoted by C_k (C_l). We denote by $\mathcal{C}_1$ ($\mathcal{C}_2$) the set of coalitions formed in stage 1 (2). For a given structure of coalitions, the game is solved by subgame perfection. A subgame perfect Nash equilibrium outcome $\mathbf{z}$ of the two-stage game satisfies the following conditions:
At stage 2,

$$\omega_\xi \frac{dV_\xi(\mathbf{z})}{dx_\xi} + \sum_{\xi' \in C_l,\, \xi' \neq \xi} \omega_{\xi'} \frac{dV_{\xi'}(\mathbf{z})}{dx_\xi} = 0, \qquad \forall \xi \in C_l, \forall C_l \in \mathcal{C}_2 \tag{8}$$

where the first term captures the effect of the action of player ξ on its own welfare, while the second term describes the within-coalition spillover effects on the coalition members in the coalition C_l. Since stage 2 is the final stage of the game, there are by construction no non-synchronous effects on players of coalitions acting at subsequent coalitions.
At stage 1,

$$\begin{aligned} & \omega_\xi \left[\frac{\partial V_\xi(\mathbf{z})}{\partial x_\xi} + \left[\sum_{C_l \in \mathcal{C}_2} \sum_{\xi'' \in C_l} \frac{\partial V_\xi(\mathbf{z})}{\partial x_{\xi''}} \frac{\partial x_{\xi''}}{\partial x_\xi} \right] \right] \\ & + \sum_{\xi' \in C_k,\, \xi' \neq \xi} \omega_{\xi'} \left[\frac{\partial V_{\xi'}(\mathbf{z})}{\partial x_\xi} + \left[\sum_{C_l \in \mathcal{C}_2} \sum_{\xi'' \in C_l} \frac{\partial V_{\xi'}(\mathbf{z})}{\partial x_{\xi''}} \frac{\partial x_{\xi''}}{\partial x_\xi} \right] \right] \\ = \; & 0, \; \forall \xi \in C_k, \forall C_k \in \mathcal{C}_1. \end{aligned} \tag{9}$$

To describe stage 1 interactions, four effects can be distinguished, in line with the classification of Definition 3. The first term captures the direct effect of the action of player ξ on his own welfare, while the second term describes the indirect effect on his own welfare through actions taken by players in coalitions formed in the second period. For this second term to be non-zero it is necessary that there exist non-synchronous between-coalition spillover effects between ξ and at least one player ξ'' acting at stage 2, i.e. $\frac{\partial V_\xi(\mathbf{z})}{\partial x_{\xi''}}$ and $\frac{\partial x_{\xi''}}{\partial x_\xi}$ must be non-zero for at least one pair ξ and ξ''. The third term describes the within-coalition spillover effects of the action x_ξ on the coalition members in the coalition C_k. Finally, the fourth term captures the indirect welfare effect on the coalition members in the coalition C_k through actions taken by players in coalitions formed at stage 2. For this fourth term to be non-zero it is necessary that there exist non-synchronous between-coalition spillover effects between at least one other member of C_k and at least one player ξ'' acting at stage 2.

Correspondingly, one can derive the set of conditions applying to Γ'.

To ensure that (8) and (9) admit the same equilibrium outcome for two games Γ and Γ', the set of sufficient conditions summarized in Proposition 1 are derived from the following two-step procedure. First, to undo the effects of different commitment patterns in Γ and Γ', all non-synchronous between-coalition spillover effects are required to be zero at $\mathbf{z}$. This requirement ensures that the second and fourth term discussed above vanish in equilibrium. Second, a condition is needed which addresses the effects of different coalitions structures in Γ and Γ'. Certainly, a sufficient condition would be to require that the within-coalition spillover effects for all coalitions formed in Γ and Γ' vanish at $\mathbf{z}$, implying that (8) and (9) reduce for both games to $\omega_\xi \frac{\partial V_\xi(\mathbf{z})}{\partial x_\xi} = 0$, $\forall \xi \in \Xi$. Yet, having controlled for possible differences in commitment patterns already in the first step, (8) and (9) admit for Γ and Γ' the same equilibrium outcome also under the weaker condition that the within-coalition spillover effects need to vanish at $\mathbf{z}$ only for those coalitions which are formed in Γ', but not in Γ, and vice versa.
This reasoning can be generalized to games with more than 2 stages, as any $h - stage$ extensive form game can be restated as a sequence of 2-stage extensive-form games. □

Remark at Corollary 1**:** If at $\mathbf{z}^{Nash}$ all spillovers between any pair of players vanish it is clear from (8) and (9) that $\mathbf{z}^{Nash}$ is a subgame perfect Nash equilibrium outcome for any possible extensive-form game characterized by arbitrary commitment patterns and coalition structures.

6.2 Proof of Proposition 3

Using (1) in (2), we can express V_ξ as:

$$\begin{aligned} V_\xi &= \frac{1}{2}[\omega_1^\xi(y_1^* - \overline{y_1} - \sum_{j=1}^{X} b_{1j}x_j)^2 + ... + \omega_\xi^\xi(y_\xi^* - \overline{y_\xi} - \sum_{j=1}^{X} b_{\xi j}x_j)^2 + ... \\ &\quad + \omega_X^\xi(y_X^* - \overline{y_X} - \sum_{j=1}^{X} b_{Xj}x_j)^2] \end{aligned}$$

In general, any Nash equilibrium outcome $\mathbf{z}^{Nash}$of the simultaneous "no-commitment and no-cooperation" Nash game Γ^{Nash} played by the X players satisfies the set of conditions:

$$\frac{\partial V_\xi(\mathbf{z}^{Nash})}{\partial x_\xi} = 0, \forall \xi \in \Xi.$$

In the linear-quadratic model, this set of equations can be expressed as follows:

$$\frac{\partial V_\xi(\mathbf{z}^{Nash})}{\partial x_\xi} = \sum_{k=1}^{X} \omega_k^\xi b_{k\xi} \left[y_k^* - \overline{y_k} - \sum_{j=1}^{X} b_{kj}x_j \right] = 0, \forall \xi \in \Xi \tag{10}$$

The spillover effect of ξ' on agent ξ's payoff is given by the expression:

$$\frac{\partial V_\xi(\mathbf{z}^{Nash})}{\partial x_{\xi'}} = \sum_{k=1}^{X} \omega_k^\xi b_{k\xi'} \left[y_k^* - \overline{y_k} - \sum_{j=1}^{X} b_{kj}x_j \right] \tag{11}$$

Let $\mathbf{z}^{Nash} = \mathbf{B}^{-1}[\mathbf{y}^* - \overline{\mathbf{y}}]$. This vector satisfies (10), as required for a Nash-equilibrium. Moreover, at $\mathbf{z}^{Nash}$ for any pair (ξ, ξ') equation (11) will then also be zero. Hence, the linear-quadratic case satisfies Corollary 1 to Proposition 1. □

6.3 The model of Dixit and Lambertini: a special case of Proposition 3

The representation of the model of Dixit and Lambertini described in Section 3.1 can be rewritten as follows such that it satisfies (1) and (2). First, define the inflation forecast error such that $\pi^{fe} \equiv \pi - \pi^e$. Then, introduce a new vector $\widetilde{\mathbf{y}} = (\pi^{fe}, \mathbf{y}, \pi)$, with $\widetilde{\mathbf{y}}$ relating to the states of the three groups of agents: single private sector actor, country-specific fiscal policymakers, single monetary policymaker. Since

$$\begin{aligned} \pi^{fe} &= \pi - \pi^e \\ y_i &= \overline{y_i} + \sum_{k=1}^{n} b_{ik}\tau_k + b_i(\pi - \pi^e) \\ \pi &= \pi, \end{aligned}$$

$\widetilde{\mathbf{y}}$ can be linearly linked to the instruments $\mathbf{x} = (\pi^e, \boldsymbol{\tau}, \pi)$ in line with (1), i.e.

$$\widetilde{\mathbf{y}} = \overline{\widetilde{\mathbf{y}}} + \widetilde{\mathbf{B}}\mathbf{x},$$

with $\overline{\widetilde{\mathbf{y}}} = (0, \overline{\mathbf{y}}, 0)$. Moreover, the target value of the inflation forecast error satisfies $\pi^{fe^*} = 0$. Hence, the payoff function of all three types of (non-cooperative) players

$$\begin{aligned} U &= \frac{1}{2}(\pi^{fe^*} - \pi^{fe})^2 = \frac{1}{2}(\pi - \pi^e)^2 \\ V_i &= \frac{1}{2}\left[\omega_i(y_i^* - y_i)^2 + (\pi^* - \pi)^2\right] = \frac{1}{2}\left[\omega_i(y_i^* - y_i)^2 + \pi^2\right] \\ V^M &= \sum_{i=1}^{n}\omega_i^M V_i = \frac{1}{2}\left[\sum_{i=1}^{n}\left[\omega_i^M \omega_i(y_i^* - y_i)^2\right] + \pi^2\right] \end{aligned}$$

are in line with (2). □

6.4 The model of Chari and Kehoe: main results *[to be polished]*

Consider the following four games discussed by Chari and Kehoe which we adapt to our notation. In all games the non-cooperative private sector players always move after the fiscal players.

1. Game CK 1: There is *no cooperation* between fiscal policymakers and *no commitment* of monetary policy: *i/* the fiscal authorities set τ_i non cooperatively, *ii/* the private agents set a_{ij} non cooperatively, *iii/* the central bank sets π.

2. Game CK 2: There is *cooperation* between fiscal policymakers and *no commitment* of monetary policy: *i/* the fiscal authorities jointly set the vector of fiscal instruments $\boldsymbol{\tau}$ with the aim to maximize V^{FC}, *ii/* the private agents set a_{ij} non cooperatively, *iii/* the central bank sets π.

3. Game CK 3: There is *no cooperation* between fiscal policymakers, and there is *commitment* of monetary policy: *i/* the central bank sets π, *ii/* the fiscal authorities set τ_i non cooperatively, *iii/* the private agents set a_{ij} non cooperatively.

4. Game CK 4: There is *cooperation* between fiscal policymakers, and there is *commitment* of monetary policy: *i/* the central bank sets π, *ii/* the fiscal authorities jointly set the vector of fiscal instruments $\boldsymbol{\tau}$ with the aim to maximize V^{FC}, *iii/* the private agents set a_{ij} non cooperatively.

The logic behind *i*) and *ii*) can be inferred from the following sketch, i.e. we do not reproduce the detailed proof from Chari and Kehoe. Consider a perfectly symmetric set-up and let $\mathbf{z}_g = (\mathbf{a}, \boldsymbol{\tau}, \pi)$, $g = 1, 2, 3, 4$ denote the solution vectors to the four games.

Result ii) obtains from comparing CK 1 and CK 2:
Consider CK 1: By backward induction, stage 3 gives rise to the first-order condition $\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} = 0$, leading to a solution for π such that $\pi = \pi(\mathbf{a}, \boldsymbol{\tau})$. Stage 2 gives rise to a first-order condition $\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ij}} = 0$ (where we use the envelope theorem which ensures $\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial a_{ij}} = 0$), leading to a solution for a_{ij} such that $a_{ij} = \pi(\boldsymbol{\tau})$. Stage 3 gives rise to a first-order condition

$$\sum_{k \in \mathcal{M}_i, k \neq j} \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ik}} \frac{\partial a_{ik}}{\partial \tau_i} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \tau_i} = 0, \tag{12}$$

where we use the envelope theorem which ensures $\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial \boldsymbol{\tau}_i} = \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \boldsymbol{\tau}_i} = 0$. Two elements are crucial for the following analysis: i) since the common monetary policy moves last, this makes private sector actions in stage 2 dependent on the entire vector of fiscal actions $\boldsymbol{\tau}$. ii) The non-cooperative behaviour of private sector players within countries creates spillovers which become relevant at stage 1 for the fiscal players, i.e. there exist within-coalition spillover effects between fiscal players which are entirely related to the commitment of fiscal policy with respect to all other players. Under the assumption of non-cooperative fiscal policy in CK 1, these effects are not internalized.
Consider CK 2: By backward induction, stage 3 and 2 are identical to CK 1. Stage 3, again using the envelope theorem, gives rise to a first-order condition

$$\sum_{k \in \mathcal{M}_i, k \neq j} \frac{\partial U_{ij}(\mathbf{z}_2)}{\partial a_{ik}} \sum_{l=1}^{n} \frac{\partial a_{ik}}{\partial \tau_l} + \frac{\partial U_{ij}(\mathbf{z}_2)}{\partial \tau_i} = 0, \tag{13}$$

i.e. within-coalition spillover effects between fiscal players are internalized, making the solutions to CK 1 and CK 2 generically different.

Result i) obtains from comparing CK 3 and CK 4.
Consider CK 3. By backward induction, stage 3 gives rise to the first-order condition $\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}} = 0$, leading to a solution for a_{ij} such that $a_{ij} = (\mathbf{a}, \tau_i)$. Stage 2 gives rise to a first-order condition

$$\sum_{k \in \mathcal{M}_i, k \neq j} \frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ik}} \frac{\partial a_{ik}}{\partial \tau_i} + \frac{\partial U_{ij}(\mathbf{z}_3)}{\partial \tau_i} = 0, \tag{14}$$

(where we use the envelope theorem which ensures $\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \tau_i} = 0$), leading to a solution for τ_i such that $\tau_i = \tau_i(\pi)$. Stage 3 gives rise to a first-order condition $\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial \pi} = 0$, where we use the envelope theorem which ensures $\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \pi} = \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \boldsymbol{\tau}_i} \frac{\partial \boldsymbol{\tau}_i}{\partial \pi} = 0$.
Consider CK 4: The solution of CK 4 satisfies the same first-order conditions as CK 3, since there are no within-coalition spillover effects between fiscal players at stage 2.

6.5 Proof of Proposition 4

Consider equations (4)-(7) used in the main text

$$\begin{aligned} \mathbf{y} &= \mathbf{Bx} + \boldsymbol{\varepsilon}, \;\; \boldsymbol{\varepsilon} \sim (\mathbf{0}, \boldsymbol{\Omega}_{\varepsilon}) \\ \mathbf{x} &= \mathbf{R}\boldsymbol{\varepsilon} \\ V_i &= \widetilde{V}_i + \boldsymbol{\omega}_i' \Omega_{\mathbf{y}} \boldsymbol{\omega}_i \\ \widetilde{V}_i &= V_i^*. \end{aligned}$$

Combining the first two equations yields

$$\mathbf{y} = [\mathbf{I} + \mathbf{BR}]\boldsymbol{\varepsilon},$$

implying $\mathbf{y} \sim (\mathbf{0}, [\mathbf{I} + \mathbf{BR}]\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}[\mathbf{I} + \mathbf{BR}]')$, which in turn leads to

$$V_i = \widetilde{V}_i + \boldsymbol{\omega}_i'[\mathbf{I} + \mathbf{BR}]\Omega_{\boldsymbol{\varepsilon}}[\mathbf{I} + \mathbf{BR}]'\boldsymbol{\omega}_i.$$

Assuming non-cooperative behaviour of all N players, player i has the objective to choose optimally the vector of response coefficients $\mathbf{r}_i$, i.e.

$$\min_{\mathbf{r}_i} V_i = \min_{\mathbf{r}_i} \widetilde{V}_i + \boldsymbol{\omega}_i'[\mathbf{I} + \mathbf{BR}]\Omega_{\boldsymbol{\varepsilon}}[\mathbf{I} + \mathbf{BR}]'\boldsymbol{\omega}_i,$$

taking as given $\mathbf{R}_{-i}$. For sure, if $\mathbf{R} = -\mathbf{B}^{-1} \Rightarrow \mathbf{y} = \mathbf{0}$, implying $V_i = \widetilde{V}_i$ for all realizations of $\boldsymbol{\varepsilon}$ and for all player-specific weights $\boldsymbol{\omega}_i$.
Consider first player 1. Then, with $\mathbf{r}_1$ denoting the row-vector of response coefficients of player, $\mathbf{R}$ can be decomposed as

$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{R}_{-1} \end{bmatrix}.$$

Moreover, define $\mathbf{B^{-1}} = \begin{bmatrix} \widetilde{\mathbf{b}_1} \\ \widetilde{\mathbf{B}_{-1}} \end{bmatrix}$, i.e. $\widetilde{\mathbf{b}_1}$ denotes the first row vector of $\mathbf{B^{-1}}$. Suppose that $\mathbf{R}_{-1} = -\widetilde{\mathbf{B}_{-1}}$. Then $V_1 = \widetilde{V_1}$ if $\mathbf{r}_1 = -\widetilde{\mathbf{b}_1}$. Hence, $\mathbf{r}_1 = -\widetilde{\mathbf{b}_1}$ is a best response to $\mathbf{R}_{-1}$. Repeating the same argument for all $i = 2, ...N$ it is clear that the rows of the matrix $\mathbf{R^{Nash}} = -\mathbf{B}^{-1}$ satisfy the best-response property for all players. Given the linear structure of $\mathbf{y}$, this solution is unique for non-degenerate $\boldsymbol{\omega}_i$ and $\Omega_{\boldsymbol{\varepsilon}}$. Moreover, since $\widetilde{V}_i = V_i^*$, $\mathbf{R^{Nash}} = -\mathbf{B}^{-1}$ ensures that $V_i = V_i^*$ for all players, leading directly to Proposition 4. □

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