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Making Exchange Rates Sparkle: Restricting Its Present-Value Model with Common Trends and Common Cycles

James M. Nason
Research Department
Federal Reserve Bank of Atlanta
1000 Peachtree St., N.E.
Atlanta, GA 30309
email : jim.nason@atl.frb.org

John H. Rogers†
International Finance Division
Board of Governors of the
Federal Reserve System
Washington, D.C. 20551
email : John.H.Rogers@FRB.GOV

October 24, 2006

Abstract

Exchange rates have raised the ire of economists for more than 20 years. A problem is that there appears to be no exchange rate model that systematically beats a naive random walk in out of sample forecasts. Economists also find it irksome that theoretical models are unable to explain short-, medium-, and long-run exchange rate movements. Engel and West (2005) show that these failures can be explained by the present value model (PVM) because it predicts the exchange rate is a random walk if currency traders are highly interest sensitive and fundamentals have a unit root. This paper generalizes Engel and West (2005). We find that the PVM imposes *common trend* and *common cycle* restrictions on the exchange rate and its $I(1)$ fundamental. As the interest sensitivity of money demand grows large, the common cycle forces the exchange rate to approximate a martingale. A PVM of the exchange rates is also constructed from a dynamic stochastic general equilibrium (DSGE) open economy model. The DSGE-PVM predicts that the exchange rate is dominated by permanent monetary and productivity shocks, as the world real interest rate becomes small. Thus, our results complement and extend Engel and West (2005) to a larger class of models, while presenting a new challenge to future research.

JEL Classification Number : $E31$, $E37$, and $F41$

Key Words : Exchange Rates; Present-Value Model; Common Trends; Common Cycles.

†We wish to thank Menzie Chinn, Juan F. Rubio-Ramírez, Tom Sargent, Ellis Tallman, Noah Williams, and Tao Zha for useful discussions. The views in this paper represent those of the authors and are not those of either the Federal Reserve Bank of Atlanta, the Board of Governors of the Federal Reserve System, or any of its staff. Errors in this paper are the responsibility of the authors.

1. Introduction

The search for satisfactory exchange rate models continues to be elusive. Since the seminal work of Meese and Rogoff (1983a, 1983b), a variety of models have been tried in an effort to improve on naive random walk forecasts of exchange rates. These range from linear rational expectations models examined by Meese (1986) to nonlinear models proposed by Diebold and Nason (1990), Meese and Rose (1991), Gençay (1999), and Kilian and Taylor (2003).

The Journal of International Economics volume edited by Engel, Rogers, and Rose (2003) indicates that there has been a split between theoretical exchange rate models and what is considered a useful forecasting model. For example, Kilian and Taylor (2003) argue that there are specific nonlinear forecasting models that can vie with a naive random walk of exchange rates. However, their motivation is empirical only, bereft of theory. This approach maybe useful to obtain candidates for a forecast competition. Nonetheless, there are limits because, as Diebold and Nason (1990) note, the class of nonlinear exchange rate models might be infinite.

This paper takes a step back from the exchange rate forecasting problem to study a workhorse theory of exchange rate determination, the present-value model (PVM) of exchange rates. Actual data most often rejects the exchange rate PVM. Typical are tests Meese (1986) reported that are based on the first ten years of the floating rate regime. He finds that exchange rates are infected with persistent deviations from fundamentals, which reject the PVM. However, Meese is unable to uncover the source of the rejections. Rather than a condemnation of the PVM, we view results such as Meese's as a challenge to update and deepen its analysis.

A similar position is taken by Engel and West (2004, 2005). They explain the random walk behavior of exchange rates and the puzzle as to why alternative models have difficulty competing against it. Starting with the PVM and using uncontroversial assumptions about fundamentals and the discount factor, Engel and West (EW) show that the PVM predicts exchange rates approximate a random walk if currency traders are highly interest sensitive and fundamentals are $I(1)$. They also report empirical and simulation evidence consistent with their theoretical results.

This paper complements Engel and West (2005). The standard PVM predicts the exchange rate follows a random walk, but for reasons that differ from EW's. The random walk behavior of exchange rates is tied to the common cycle exchange rates and fundamentals share. We argue that neglect of this common cycle is a source of the failure of the PVM of the exchange rate. Thus, this paper takes up the challenge to update and deepen the PVM.

We extend the PVM of the exchange rate with nine propositions. Five propositions are constructed from the standard PVM, given fundamentals are $I(1)$ and fundamental growth has a Wold representation. The propositions are: (1) exchange rate and fundamentals cointegrate [Campbell and Shiller (1987)]; (2) the PVM cross-equation restrictions imply an error correction mechanism (ECM) for currency returns in which the lagged cointegrating relation is the only regressor; (3) if fundamental growth depends only on the lagged ECM, the exchange rate and fundamental share a common trend and a common cycle in the sense of Vahid and Engle (1993), (4) the PVM predicts a limiting economy (*i.e.*, the interest rate semi-elasticity of money demand becomes infinite) in which the exchange rate is a martingale, and (5) the EW random walk result can be interpreted as the limiting economy of (4) along with the restriction that the bivariate exchange rate-fundamental process fails to cointegrate, but shares a common cycle.

The four remaining propositions apply to an optimizing dynamic stochastic general equilibrium (DSGE) model of exchange rate determination. The four propositions the linearized DSGE-PVM yield are: (6) the exchange rate cointegrates with fundamentals, cross-country money, productivity, and consumption, when the latter are $I(1)$, (7) the exchange rate and fundamentals share a common cycle if the transitory component of cross-country money, productivity, and consumption are restricted to be white noise, (8) the exchange rate and fundamentals are co-dependent in the sense of Vahid and Engle (1997), and (9) an extension and generalization of Engel and West (2005). With the final proposition, we show that when the world real interest rate is small, the exchange rate is dominated by permanent monetary and productivity shocks. Thus, the DSGE-PVM connects the analysis of Engel and West to a wider class of models.

We begin to explore the predictions of the DSGE-PVM by casting it as an unobserved components (UC) model. This allows us to construct a state space model of the DSGE-PVM and form the Kalman filter to evaluate the likelihood. We adapt the Metropolis-Hastings simulator of Rabanal and Rubio-Ramírez (2005) to compute posterior distributions of the linearized the DSGE-PVM. Our estimates support the Engel and West (2005) hypothesis that the exchange rate approximates a random walk at reasonable estimates of the world real interest rate.

The outline of the paper follows. The next section solves the standard PVM of the exchange rate and presents its five propositions. Section 3 studies the DSGE-PVM and presents the remaining four propositions. Our econometric strategy is discussed in section 4. Section 5 presents preliminary empirical results. We conclude in section 6.

2. The Present-Value Model of Exchange Rates

The model of exchange rate determination combines a liquidity-money demand function, uncovered interest rate parity (UIRP), purchasing power parity (PPP), and flexible prices. This is a workhorse exchange rate model used by, among others, Dornbusch (1976), Frankel (1979), Bilson (1978), Frenkel (1979), Meese (1986), Mark (1995) and Engel and West (2004, 2005).

2a. The Model

Our analysis starts with the liquidity-money demand function

$$(1) \qquad m_{h,t} - p_{h,t} = \psi y_{h,t} - \phi r_{h,t}, \quad 0 < \psi, \phi,$$

where $m_{h,t}$ $p_{h,t}$, $y_{h,t}$, and $r_{h,t}$ denote the home country's money stock, aggregate price level, output, and the nominal interest rate. The first three variables are transformed by the natural logarithm. The parameter ψ measures the income elasticity of money demand. Since the nominal interest rate is in its level, ϕ is the interest rate semi-elasticity of money demand. Define the cross-country differentials $m_t = m_{h,t} - m_{f,t}$, $p_t = p_{h,t} - p_{f,t}$, $y_t = y_{h,t} - y_{f,t}$, $r_t = r_{h,t} - r_{f,t}$, where f denotes the foreign country. Assuming PPP holds, $e_t = p_t$, where e_t is the log of the (nominal) exchange rate in which the U.S dollar is the home country's currency.

Under UIRP, the law of motion of the exchange rate is approximately

$$(2) \qquad \mathbf{E}_t e_{t+1} - e_t = r_t.$$

Substitute for the nominal interest rate differential in the law of motion of the exchange rate (2) with the liquidity demand function (1) to produce the Euler equation

$$(3) \qquad \left[1 - \frac{\phi}{1+\phi}\mathbf{E}_t\mathbf{L}^{-1}\right] e_t = \frac{1}{1+\phi}[m_t - \psi y_t], \quad \mathbf{L}e_t = e_{t-1}.$$

Iterate on Euler equation (3) through date T, recognize the transversality condition

$$lim_{T\to\infty}\left[\frac{\phi}{1+\phi}\right]^{T+1} \mathbf{E}_t e_{t+T} = 0$$

and obtain the present-value relation

$$(4) \qquad e_t = \frac{1}{1+\phi}\sum_{j=0}^{\infty}\left[\frac{\phi}{1+\phi}\right]^j \mathbf{E}_t z_{t+j},$$

where the (log) of the exchange rate equals the annuity value of the (log) level of the fundamentals, $z_t \equiv m_t - \psi y_t$. In the PVM, the fundamental z_t is the cross-country money stock differential netted for its income demand component. Also, note that the present-value relation

(4) yields the weak prediction that the exchange rate Granger-causes the fundamental $m - \psi y$, a prediction that is explored by Engel and West (2005).

2b. Cointegration Restrictions

The present-value relation (4) provides several predictions given

Assumption 1: $z_t \sim I(1)$.

Assumption 2: $(1-\mathbf{L})z_t$ has a Wold representation, $(1-\mathbf{L})z_t = \Delta z^* + \zeta(\mathbf{L})\upsilon_t$.[1]

Given Assumptions 1 and 2, the first prediction is that e_t and z_t share a common trend. This follows from subtracting the latter from both sides of the equality of the present-value relation (4) and combining terms to produce the ECM

(5) $$e_t - z_t = \sum_{j=1}^{\infty}\left[\frac{\phi}{1+\phi}\right]^j \mathbf{E}_t\Delta z_{t+j}, \quad \Delta \equiv (1-\mathbf{L}).$$

The ECM reflects the forces that push the exchange rate toward long-run PPP.

Proposition 1: *If z_t satisfies Assumptions 1 and 2, $\mathcal{X}_t = \beta' q_t$ forms a cointegrating relation with cointegrating vector $\beta' = [1 \quad -1]$, where $q_t \equiv [e_t \quad z_t]'$.*

The proposition is a variation of results found in Campbell and Shiller (1987). Note that the cointegrating relation becomes $\mathcal{X}_t = \zeta\left(\frac{\phi}{1+\phi}\right)\upsilon_t$, under Assumptions 1 and 2.

The cointegrating relation $\mathcal{X}_t$ equals the expected present discounted value of Δm_t minus $\psi\Delta y_t$. Thus, $\mathcal{X}_t$ is stationary, given Assumption 1 (*i.e.*, m_t and y_t are $I(1)$ and fail to share a common trend). We interpret $\mathcal{X}_t$ as the 'adjusted' exchange rate because it eliminates cross-country money stock movements netted for its income demand. The 'adjusted' exchange

[1] The restrictions on the moving average are Δz^* is linearly deterministic, $\zeta_0 = 1$, $\zeta(\mathbf{L})$ is an infinite order lag polynominal with roots outside the unit circle, the ζ_is are square summable, and υ_t is mean zero, homoskedastic, linearly independent given history, and is serially uncorrelated with itself and the past of Δz_t. Assumption 2 places more restrictions on fundamental growth than Engel and West (2005) require.

rate is a forward-looking function of the expected path of fundamental growth. This suggests the cointegrating relation is a "*cycle generator*", as described by Engle and Issler (1995), with the serial correlation of fundamental growth its source.

2c. Equilibrium Currency Return Dynamics

The second PVM prediction begins by writing the present-value relation (4) as

$$e_t - \frac{1}{1+\phi} z_t = \frac{1}{1+\phi} \sum_{j=1}^{\infty} \left[\frac{\phi}{1+\phi} \right]^j \mathbf{E}_t z_{t+j}.$$

Next, difference this equation,

$$\Delta e_t - \frac{1}{1+\phi} \Delta z_t = \frac{1}{1+\phi} \sum_{j=1}^{\infty} \left[\frac{\phi}{1+\phi} \right]^j \left[\mathbf{E}_t z_{t+j} - \mathbf{E}_{t-1} z_{t+j-1} \right],$$

add and subtract $\mathbf{E}_t z_{t+j-1}$ inside the brackets, and use the present-value relation (5) to find

$$\Delta e_t - \frac{1}{\phi} \mathcal{X}_{t-1} = \frac{1}{1+\phi} \sum_{j=0}^{\infty} \left[\frac{\phi}{1+\phi} \right]^j \left[\mathbf{E}_t - \mathbf{E}_{t-1} \right] z_{t+j}. \tag{6}$$

Equilibrium currency return persistence is tied to the ECM, which acts to restore long-run PPP.

Proposition 2: *The Present-Value Model predicts that the equilibrium generating equation of currency returns is an ECM(0), assuming Proposition 1 holds.*

The ECM(0) of currency returns is $\Delta e_t = \vartheta \mathcal{X}_{t-1} + u_t$, where $\vartheta = \frac{1}{\phi}$ and $u_t = \frac{\vartheta}{1+\vartheta} \zeta\left(\frac{1}{1+\vartheta}\right) \upsilon_t$ because the present-value term of equation (6) is $\frac{1}{1+\phi} \zeta\left(\frac{\phi}{1+\phi}\right) \upsilon_t$ under assumption 2.[2]

2d. The Common Trend and Common Cycle of Exchange Rates and Fundamentals

Proposition 2 provides an easy method to compute a BNSW common trend-common cycle decomposition for q_t, if Δz_t is also an ECM(0). The decomposition relies on the cointegrating relation $\mathcal{X}_t = e_t - z_t$ and the relationship between currency returns and fundamental growth.

Proposition 3: *Assume fundamental growth follows the ECM(0) $\Delta z_t = \eta \mathcal{X}_{t-1} + \varpi_t$, where ϖ_t is Gaussian. Given Proposition 2, q_t has a common feature, $\mathcal{F}_t = \overline{\beta}' \Delta q_t$, in the sense of Engle*

[2]The error u_t is also justified if the econometrician's information set is strictly within that of currency traders.

and Kozicki (1993), where $\overline{\beta}' = [1 \quad -\frac{\vartheta}{\eta}]$. *The cointegrating and common feature vectors* β *and* $\overline{\beta}$ *restrict the trend-cycle decomposition of* q_t, *as described by Vahid and Engle (1993).*

The currency return-fundamental growth common feature is apparent in the VECM(0)

$$\begin{bmatrix} \Delta e_t \\ \Delta z_t \end{bmatrix} = \begin{bmatrix} \vartheta \\ \eta \end{bmatrix} x_{t-1} + \begin{bmatrix} u_t \\ \varpi_t \end{bmatrix}.$$

Pre-multiply the bivariate ECM(0) by $\overline{\beta}'$ to obtain the common feature vector $\mathcal{F}_t$. According to Engle and Kozicki (1993), $\overline{\beta}$ creates a common feature in Δq_t because a linear combination of currency returns and fundamental growth are unpredictable based on the relevant history (*i.e.*, u_t and ϖ_t are uncorrelated at all non-zero leads and lags). Hecq, Palm, and Urbain (2006) note that $\mathcal{F}_t$ restricts the spectra of Δq_t to be flat. This explains Hecq, Palm, and Urbain (2000, 2003, 2005) calling $\mathcal{F}_t (= \overline{\beta}'\Delta q_t)$ a strong form common feature.

Proposition 3 predicts $q_t = [e_t \;\; z_t]'$ has a BNSW decomposition with one common trend and one common feature. This mimics a result in Vahid and Engle (1993), which sets the trend of q_t to $\mathbf{I}_2 - \overline{\beta}(\beta'\overline{\beta})^{-1}\beta'$.[3] These restrictions decompose e_t into trend and cycle components $\frac{-\phi\eta}{1-\phi\eta}\overline{\beta}'q_t$ and $\frac{1}{1-\phi\eta}\beta' q_t$, respectively. Since the cycles common to currency returns and fundamental growth arise in the short-, medium-, and long-run, no long-run predictability exists in the exchange rate. A prediction at odds with the empirical evidence of Mark (1995).

2e. *A Limiting Model of Exchange Rate Determination*

Proposition 2 relies on $\phi < \infty$ to define short- to medium-run currency return dynamics. This raises the question of the impact of relaxing this bound.

Proposition 4: *The exchange rate approaches a martingale (in the strict sense) as* $\frac{1}{\phi} \longrightarrow 0$, *according to the present-value relation* (6) *and Proposition 2.*

[3]Vahid and Engle show a n–dimension VAR(1) with d cointegrating relations has $n - d$ common feature relations.

Proposition 4 suggests an equilibrium path for e_{t+1} in which its best forecast is e_t, given relevant information.[4] The hypothesis of Proposition 4 drives the error u_t and slope coefficient ϑ of the ECM(0) regression to $u_t \xrightarrow{p} 0$ and $\vartheta \xrightarrow{p} 0$, which implies $\mathbf{E}_t e_{t+1} = e_t$.[5] The martingale implies the exchange rate can be a random walk.

2f. PVM Exchange Rate Dynamics Redux

Engel and West (2005) show that the PVM of the exchange rate yields an approximate random walk as ϕ grows large. Their result does not rely on Proposition 2, unlike Proposition 3. Rather, EW invoke Assumptions 1 and 2, the present-value relation (4), the Weiner-Kolmogorov prediction formula, and the *conjecture* $e_t = \mathtt{a} z_t$ to find currency returns are unpredictable.

The EW hypothesis is $plim_{\vartheta \longrightarrow 0}[\Delta e_t - \mathtt{a}\zeta(\mathbf{1})\upsilon_t] = 0$. Its hypothesis test begins with

$$e_t = z_{t-1} + \sum_{j=0}^{\infty} \left[\frac{\phi}{1+\phi}\right]^j \mathbf{E}_t \Delta z_{t+j},$$

which is developed from the present-value relation (4). EW use this equation to construct

$$\Delta e_t - \mathbf{E}_{t-1}\Delta e_t = \zeta\left(\frac{\phi}{1+\phi}\right)\upsilon_t,$$

given Assumptions 1 and 2 and the Weiner-Kolmogorov prediction formula. Note that the last equation implies currency returns equal the annuity value of fundamental growth

$$\Delta e_t = \frac{1}{1+\phi} \sum_{j=0}^{\infty} \left[\frac{\phi}{1+\phi}\right]^j \mathbf{E}_t \Delta z_{t+j}.$$

The last two equations yield

$$\Delta e_t = \zeta\left(\frac{\phi}{1+\phi}\right)\upsilon_t + \frac{1}{1+\phi} \sum_{j=0}^{\infty} \left[\frac{\phi}{1+\phi}\right]^j \mathbf{E}_{t-1} \Delta z_{t+j}.$$

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[4] Hansen, Roberds, and Sargent (1991) study linear rational expectations models that anticipate Proposition 4.

[5] Maheswaran and Sims (1993) show that the martingale restriction has little empirical content for tests of asset pricing models when data is sampled at discrete moments in time.

By letting $\vartheta \xrightarrow{p} 0$, the random walk hypothesis of EW is verified independent of the ECM(0) of Proposition 2 (and cointegration restriction of Proposition 1).[6]

EW employ Assumptions 1 and 2 to show the exchange rate approaches a random walk as interest sensitivity becomes large. We obtain their result by exploiting the common feature implication of the PVM for currency returns and fundamental growth. We connect this common feature to the EW result with the assumption that Δq_t is $I(0)$ and has a Wold representation, $\Delta q_t = \lambda(\mathbf{L})\xi_t$. When q_t lacks common trends, the exchange rate and fundamental possess a multivariate BN decomposition, $q_t = \lambda(\mathbf{1})\Xi_t + \Lambda(\mathbf{L})\xi_t$, where $\lambda(\mathbf{1})$ has full rank, $\Lambda(\mathbf{L}) = \sum_{i=0}^{\infty} \Lambda_i$, $\Lambda_i = -\sum_{j=i+1}^{\infty} \lambda_j$, and $\Xi_t = \sum_{j=0}^{\infty} \xi_{t-j}$. Since the multivariate BN decomposition in growth rates is

$$\Delta q_t = \lambda(\mathbf{1})\xi_t + \Delta\Lambda(\mathbf{L})\xi_t, \tag{7}$$

we have

Proposition 5: *The exchange rate-random walk hypothesis of Engel and West (2005) requires that currency returns and fundamental growth share a common feature, as well as* $\frac{1}{\phi} \longrightarrow 0$.

The EW hypothesis eliminates the BN cycle, $\Lambda(\mathbf{L})\xi_t$, from equation (7). All that remains to drive Δq_t is $\lambda(\mathbf{1})\xi_t$. Thus, Proposition 5 predicts the exchange rate and fundamental are random walks because serially correlated common cycles are annihilated.

Propositions 3, 4, and 5 shape the restrictions that affirm the EW hypothesis. Serial correlation is eliminated from Δq_t by the common feature vector $\overline{\beta}'$, which for the multivariate BN growth rates representation (7) sets $\overline{\beta}'\Delta q_t = \overline{\beta}'\lambda(\mathbf{1})\xi_t$. When $\overline{\beta}' \xrightarrow{p} [1 \;\; 0]$, Proposition 5 predicts the limiting behavior of the exchange rate is a random walk independent of fundamentals. Thus, the EW hypothesis is consistent with a common feature restriction on short-, medium-,

[6] This analysis matches equations $A.3 - A.11$ and the surrounding discussion of Engel and West (2005).

and long-run movements in the exchange rate and fundamentals.

2g. Tests of the PVM of the Exchange Rate

Propositions 1, 3, and 5 yield testable restrictions on exchange rates and fundamentals. If the lag length of the levels VAR of the exchange rate and fundamental exceeds one, the VECM(0) required by Proposition 1 is rejected. Cointegration tests suffice to examine Proposition 1. Vahid and Engel (1993) and Engel and Issler (1995) provide common feature tests that yield information about the EW hypothesis and Proposition 5. Table 2 summarizes the results of these tests.

We estimate VARs of foreign currency-U.S. dollar exchange rates and fundamentals using Canadian, Japanese, U.K., and U.S. data on a $1976Q1$ – $2004Q4$ sample.[7] VAR lag lengths are chosen using likelihood ratio (LR) statistics, given a VAR(8), ..., VAR(1).[8] The Canadian-U.S., Japanese-U.S., and U.K.-U.S. samples yield a VAR(8), VAR(5), and VAR(4), respectively.[9] Thus, the Canadian, Japanese, U.K., and U.S. data reject a weak implication of Proposition 3.

Engel and West (2005) argue there is little evidence for cointegration between exchange rates and fundamentals. Table 2 contains the trace and $\lambda-$max statistics of Johansen (1991, 1994) for the case $2*$ (Canada-U.S. sample) and case 1 (Japan-U.S. and U.K.-U.S. samples) models defined by Ostwerwald-Lenum (1992), along with critical values provided by MacKinnon, Haug, and Michelis (1999). These tests fail to support cointegration for the exchange rate and the fundamental and Proposition 3, which favors the views of Engel and West.

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[7]Fundamentals equal cross-country money minus cross-country output, which implies an income elasticity of money demand, ψ, calibrated to one. This calibration is consistent with estimated reported by Mark and Sul (2003). The money stocks (outputs) are measured in current (constant) local currency units and per capita terms.

[8]The VARs include a constant and linear time trend. The LR statistics employ the Sims (1980) correction and have standard asymptotic distribution according to results in Sims, Stock, and Watson (1990).

[9]The Canadian-U.S. and Japanese-U.S. VARs are selected when the $p-$value of the LR test is five percent or less. Since the U.K.-U.S. VAR offers ambiguous results, we settle on a VAR(4).

Table 2 includes squared canonical correlations of currency returns and fundamental growth. The common feature null is that the smallest correlation equals zero. We use a χ^2 statistic found in Vahid and Engle (1993) and a $F-$statistic suggested by Rao (1973) to test this null. The tests reject the null for the largest canonical correlation, but not for the smaller one on the three data sets. This evidence affirms a common feature in currency returns and fundamental growth, which supports Proposition 5 and our interpretation of the EW hypothesis.

3. A DSGE Based Present-Value Model of the Exchange Rate

Propositions 1 – 5 place different restrictions on the joint behavior of exchange rates and fundamentals. The standard PVM of the exchange rate is the source of these restrictions. For example, the present-value relation (6) is the basis of Propositions 2, 3, and 4. These propositions require currency returns and fundamental growth to form a VECM(0). However, the Canadian dollar-U.S. dollar, Yen-U.S. dollar, and Pound-U.S. dollar exchange rates and relevant fundamentals reject a VECM(0) as indicated by the LR tests of table 2. Table 2 also includes cointegration tests that reject the standard PVM common trend restriction of Proposition 1.

Rejection of the PVM is often given as a reason to discard linear rational expectations models of exchange rates. This paper does not. In this section, we develop a PVM model of the exchange rate derived from a canonical optimizing two-country monetary DSGE model. Our aim is to construct an equilibrium exchange rate model whose short-run and long-run behavior better reflects dynamics in actual data. We address the empirical implications of the DSGE-PVM below.

3a. The DSGE Model

The optimizing monetary DSGE model consists of the preferences of domestic and foreign economies and their resource constraints. For the home ($i = h$) and foreign ($i = f$) countries, the former objects take the form

$$(8)\qquad \mathcal{U}\left(C_{i,t},\ \frac{M_{i,t}}{P_{i,t}}\right) = \frac{\left[C_{i,t}^{\nu}\left(\frac{M_{i,t}}{P_{i,t}}\right)^{(1-\nu)}\right]^{(1-\kappa)}}{1-\kappa}, \quad 0 < \nu < 1, \quad 0 < \kappa,$$

where $C_{i,t}$ and $M_{i,t}$ denote the ith country's consumption and the ith country's holdings of its money stock. The resource constraint of the home country is

$$(9)\quad B_{h,t}^{h} + s_t B_{h,t}^{f} + P_{h,t}C_{h,t} + M_{h,t} = (1+r_{h,t-1})B_{h,t-1}^{h} + s_t(1+r_{f,t-1})B_{h,t-1}^{f} + M_{h,t-1} + P_{h,t}Y_{h,t},$$

where $B_{i,t}^{i}$, $B_{i,t}^{\ell}$, $r_{i,t-1}$, $r_{\ell,t-1}$, $Y_{i,t}$, and s_t represent the ith country's nominal holding of its own bonds at the end of date t, the ith country's nominal holding of the ℓth country's bonds at the end of date t, the return on the ith country's bond, the return on the ℓth country's bond, the output level of the ith country, and the level of the exchange rate. The two-country DSGE model is closed with $B_{h,t}^{h} + B_{h,t}^{f} + B_{f,t}^{h} + B_{f,t}^{f} = 0$. This condition forces the world stock of nominal debt to be in zero net supply, period-by-period, along the equilibrium path.

Analysis of the standard PVM assumes that fundamentals are $I(1)$. We satisfy this requirement for the DSGE-PVM with processes for labor-augmenting technical change, $A_{i,t}$, or total factor productivity (TFP), and money stocks that satisfy

Assumption 3: $ln[A_{i,t}]$ and $ln[M_{i,t}] \sim I(1)$, $i = h, f$.

Assumption 4: Cross-country TFP and money stock differentials are $I(1)$.

Assumptions 3 and 4 impose stochastic trends on the two-country DSGE model.

3b. DSGE-Based UIRP and Money Demand

The home country maximizes its expected discount lifetime utility,

$$\mathbf{E}_t\left\{\sum_{j=0}^{\infty}(1+\rho)^{-j}\mathcal{U}\left(C_{h,t+j},\ \frac{M_{h,t+j}}{P_{h,t+j}}\right)\right\}, \quad 0 < \rho,$$

subject to (9). The first-order necessary conditions of economy i yield optimality conditions that describe UIRP and money demand. The utility-based UIRP condition of the home country is

$$\mathbf{E}_t \left\{ \frac{\mathcal{U}_{C,h,t+1}}{P_{h,t+1}} \right\} (1 + r_{h,t}) = \mathbf{E}_t \left\{ \frac{\mathcal{U}_{C,h,t+1}}{P_{f,t+1}} \right\} \frac{(1 + r_{f,t})}{s_t}, \tag{10}$$

where $\mathcal{U}_{C,h,t}$ is the marginal utility of consumption of the home country at date t. Given the utility specification (8), the exact money demand function of country i is

$$\frac{M_{i,t}}{P_{i,t}} = C_{i,t} \left(\frac{1 - \nu}{\nu} \right) \frac{1 + r_{i,t}}{r_{i,t}}, \quad i = h, f. \tag{11}$$

The consumption elasticity of money demand is unity, while the interest elasticity of money demand is a nonlinear function of the steady state bond return.

The UIRP condition (10) and money demand equation (11) can be stochastically detrended and then linearized to produce a DSGE model version of the law of motion of the exchange rate. Begin by combining the utility function (8) and the UIRP condition (10) to obtain

$$\mathbf{E}_t \left\{ \frac{\mathcal{U}_{h,t+1}}{P_{h,t+1} C_{h,t+1}} \right\} (1 + r_{h,t}) = \mathbf{E}_t \left\{ \frac{\mathcal{U}_{h,t+1}}{P_{f,t+1} C_{i,t+1}} \right\} \frac{(1 + r_{f,t})}{s_t},$$

where $\mathcal{U}_{i,t}$ is the utility level of country i at date t. Prior to stochastically detrending the previous expression, define $\widehat{\mathcal{U}}_{i,t} = \mathcal{U}_{i,t}/A_{i,t}$, $\widehat{P}_{i,t} = P_{i,t}A_{i,t}/M_{i,t}$, $\widehat{C}_{i,t} = C_{i,t}/A_{i,t}$, $\gamma_{A,i,t} = A_{i,t}/A_{i,t-1}$, $\gamma_{M,i,t} = M_{i,t}/M_{i,t-1}$, $\hat{s}_t = s_t A_t/M_t$, $A_t = A_{h,t}/A_{f,t}$, and $M_t = M_{h,t}/M_{f,t}$. Note that $\widehat{C}_{i,t}$ is the transitory component of consumption of the ith economy, $\gamma_{A,i,t}(\gamma_{M,i,t})$ is the TFP (money) growth rate of country i, and the cross-country TFP (money stock) differential A_t (M_t) are $I(1)$. Applying the definitions, the stochastically detrended UIRP condition becomes

$$\mathbf{E}_t \left\{ \frac{\widehat{\mathcal{U}}_{h,t+1} \gamma_{A,h,t+1}^{1-\kappa}}{\gamma_{M,h,t+1} \widehat{P}_{h,t+1} \widehat{C}_{h,t+1}} \right\} (1 + r_{h,t}) = \mathbf{E}_t \left\{ \frac{\widehat{\mathcal{U}}_{h,t+1} \gamma_{A,f,t+1}}{\gamma_{A,h,t+1}^{\kappa} \gamma_{M,f,t+1} \widehat{P}_{f,t+1} \widehat{C}_{h,t+1}} \right\} \frac{(1 + r_{f,t})}{\hat{s}_t},$$

where $i = h, f$. A log linear approximation of the stochastically detrended UIRP condition yields

$$\mathbf{E}_t\tilde{e}_{t+1} \; - \; \tilde{e}_t \quad = \quad \frac{r^*}{1+r^*}\tilde{r}_t \; + \; \mathbf{E}_t\left\{\tilde{\gamma}_{A,t+1} \; - \; \tilde{\gamma}_{M,t+1}\right\}, \tag{12}$$

where, for example, $\tilde{e}_t = \ln[\hat{s}_t] - \ln[s^*]$ and $r^*(= r_h^* = r_f^*)$ denotes the steady state (or population) world real rate, for example.

The DSGE model produces a log linear approximate law of motion of the exchange rate (12) which includes an unobserved time-varying risk premium, the expected money and TFP growth differentials. Thus, transitory deviations from unobserved fundamentals are attributed by the DSGE model to changes in money growth and fluctuations in multi-factor productivity disparities across the domestic and foreign economies.

3c. A DSGE-Based PVM of the Exchange Rate

We use the linear approximate law of motion of the exchange rate (12), and a stochastically detrended version of the money demand equation (11) to produce the PVM of the exchange of our DSGE model. The unit consumption elasticity-money demand equation (11) implies

$$-\tilde{p}_t \quad = \quad \tilde{c}_t \; - \; \frac{1}{1+r^*}\tilde{r}_t. \tag{13}$$

Impose PPP on the stochastically detrended version of the money demand equation (13) and combine it with the law of motion (12) of the transitory component of the exchange rate to find

$$\left[1 \; - \; \frac{1}{1+r^*}\mathbf{E}_t\mathbf{L}^{-1}\right]\tilde{e}_t \quad = \quad \frac{1}{1+r^*}\mathbf{E}_t\left\{\tilde{\gamma}_{M,t+1} \; - \; \tilde{\gamma}_{A,t+1}\right\} \; - \; \frac{r^*}{1+r^*}\tilde{c}_t,$$

with the DSGE-PVM

$$\tilde{e}_t \;\; = \;\; \sum_{j=1}^{\infty}\left(\frac{1}{1+r^*}\right)^j \mathbf{E}_t\left\{\tilde{\gamma}_{M,t+j} \; - \; \tilde{\gamma}_{A,t+j}\right\} \; - \; \frac{r^*}{1+r^*}\sum_{j=0}^{\infty}\left(\frac{1}{1+r^*}\right)^j \mathbf{E}_t\tilde{c}_{t+j}, \tag{14}$$

where the tranversality conditions are implied by long-run behavior of $\tilde{y}_{M,t}$, $\tilde{y}_{A,t}$, and $\tilde{c}_t$. The DSGE-PVM relation (14) is the equilibrium law of motion of transitory component of the exchange rate. It equates exchange rate fluctuations to the future discounted expected path of cross-country money and TFP growth and the (negative of the) annuity-value of the transitory component of cross-country consumption. The latter two unobserved factors suggest additional sources of exchange rate fluctuations.

3d. DSGE Cointegration Restrictions

The DSGE model produces an ECM of the exchange rate. The cointegrating relation follows from a balanced growth restrictions of the DSGE model, $e_t \equiv \ln[s_t] = \ln[\hat{s}_t] + m_t - \ln[A_t]$, where $m_t = \ln[M_t]$. Thus, the DSGE model yields the cointegrating relation

$$(15) \qquad \mathcal{X}_{DSGE,t} = \tilde{e}_t + \tilde{c}_t, \quad \mathcal{X}_{DSGE,t} \equiv e_t - (m_t - c_t),$$

where constants are ignored, $c_t = \ln[C_t]$, and stochastic detrending implies $\ln[A_t] = c_t - \tilde{c}_t$.

The ECM reflects the forces that push the exchange rate toward long-run PPP plus sources of short- and medium-run PPP deviations. The persistence of PPP deviations rely on the forward-looking component $\tilde{e}_t$ and transitory date t cross-country consumption, $\tilde{c}_t$. Nonetheless, the DSGE model restricts PPP deviations to be stationary, which suggests

Proposition 6: *If m_t and A_t satisfy Assumptions 3 and 4, $\mathcal{X}_{DSGE,t} = \beta'_{DSGE} q_{DSGE,t}$ forms a cointegrating relation with cointegrating vector $\beta'_c = [1 \;\; -1 \;\; 1]$, where $q_{DSGE,t} \equiv [e_t \;\; m_t \;\; c_t]'$.*

The DSGE model predicts a forward-looking cointegration relation, but with new sources of transitory dynamics. Unobserved $\tilde{e}_t$ and $\tilde{c}_t$ movements create persistence and volatility in the "*cycle generator*" $\mathcal{X}_{DSGE,t}$ of (15). Thus, the DSGE-PVM model places unobserved sources of serial correlated short- and medium-run PPP deviations not found in the standard PVM.

3e. DSGE Equilibrium Currency Return Dynamics

The DSGE model produces an equilibrium currency return generating equation that departs from the standard PVM (6). The same process that produced the PVM equilibrium currency return generating equation (6) takes us from the DSGE-PVM (14) to the equilibrium currency return generating equation

$$(16) \qquad \Delta e_t - (\Delta m_t - \Delta c_t - \mathcal{X}_{DSGE,t-1}) = \sum_{j=1}^{\infty}\left(\frac{1}{1+r^*}\right)^j \Big[\mathbf{E}_t - \mathbf{E}_{t-1}\Big]\Big\{\gamma_{M,t+j} - \gamma_{A,t+j}\Big\} - \frac{r^*}{1+r^*}\sum_{j=0}^{\infty}\left(\frac{1}{1+r^*}\right)^j \Big[\mathbf{E}_t - \mathbf{E}_{t-1}\Big]\tilde{c}_{t+j} + \tilde{e}_t + \tilde{c}_t,$$

of the DSGE model.

Proposition 7: *The equilibrium currency return generating equation* (16) *predicts* Δe_t, Δm_t, Δc_t, and $\mathcal{X}_{DSGE,t-1}$ *share a weak form common feature*, $\mathcal{F}_{DSGE,t} = \overline{\beta}'_{DSGE}[\Delta q'_{DSGE,t} \;\; \mathcal{X}_{DSGE,t-1}]'$, *where* $\overline{\beta}'_{DSGE} = [1 \;\; -1 \;\; 1 \;\; 1]$, *only if* $\tilde{e}_t$ *and* $\tilde{c}_t$ *are serially uncorrelated.*

Proposition 7 places restrictions on Δe_t, Δm_t, Δc_t and $\mathcal{X}_{DSGE,t-1}$ in the spirit of the weak form common feature of Hecq, Palm, and Urbain (2006). A weak form common feature is unpredictable, conditional on the relevant history, as is the strong form common feature of Engle and Kozicki (1993) and Vahid and Engle (1993). The point of departure between the strong and weak form common features is that the latter contains the lagged ECM.

The long-run properties of the exchange rate (*i.e.*, PPP) are decoupled from its short-run dynamics, according to Proposition 7. Fluctuations in short-run currency returns and fundamentals growth are tied to movements in the lagged ECM, $\mathcal{X}_{DSGE,t-1}$. The common cycle of currency returns, money growth, and consumption growth share the serial correlation of $\mathcal{X}_{DSGE,t-1}$ because it is not annihilated by the weak form common feature vector $\overline{\beta}_{DSGE}$. For the same reason, the exchange rate is predictable in the long-run by the levels of cross-country money and consumption which is consistent with Mark (1995). Nonetheless, no transitory serial correlation can exist in fundamentals for the restrictions of Proposition 7 to hold.

The previous section reports tests for the lag length of levels VARs of exchange rates and fundamentals. The tests select VARs of order greater than one because the transitory component of fundamentals drive higher-order serial correlation in exchange rates. The equilibrium generating process of currency returns suggests the source of the serial correlation.

Proposition 8: *Given* $\widetilde{e}_t \sim ARMA(k_{e1}, k_{e2})$ *and* $\widetilde{c}_t \sim ARMA(k_{c1}, k_{c2})$ *with maximum lag length* k_{DSGE}*, the linear combination* $\mathcal{F}_{DSGE,t}$ *is unpredictable beyond lag* k_{DSGE}*. It follows that the impulse response function of* $\Delta q_{DSGE,t}$ *is linearly independent at horizons greater than* k_{DSGE}*.*

Vahid and Engle (1997) and Schleicher (2006) develop the idea of a common feature that creates imperfectly synchronized or co-dependent cycles in VARMAs, VARs, and VECMs. Perfectly synchronized cycles imply impulse response functions that are white noise subsequent to impact and are associated with strong and weak form common features. The impulse response functions of imperfectly synchronized time series are collinear only after a finite forecast horizon.

Propositions 6, 7, and 8 rest on two assumptions. One is that the world real interest rate is non-zero, $r^* > 0$. The other is that the transitory components of cross-country money, $\widetilde{m}_t$, and consumption, $\widetilde{c}_t$, have Wold representations. Given the balanced growth restriction, A3 and A4 endow M_t and A_t with unit roots (with drift). These trends are $\mu_{t+1} = \mu^* + \mu_t + \varepsilon_{\mu,t+1}$, $\varepsilon_{M,t+1} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_M})$, and $\ln[A_{t+1}] = a^* + \ln[A_t] + \varepsilon_{A,t+1}$, $\varepsilon_{A,t+1} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_A})$. Since stochastic detrending of cross-country money and cross-country consumption sets $m_t = \mu_{M,t} + \widetilde{m}_t$ and $c_t = \ln[A_t] + \widetilde{c}_t$ (ignoring constants), the DSGE-PVM (14) becomes

$$(17) \qquad \widetilde{e}_t = \sum_{j=1}^{\infty} \left(\frac{1}{1+r^*}\right)^j \mathbf{E}_t \left\{\widetilde{m}_{t+j} - \widetilde{m}_{t+j-1}\right\} - \frac{r^*}{1+r^*} \sum_{j=0}^{\infty} \left(\frac{1}{1+r^*}\right)^j \mathbf{E}_t \widetilde{c}_{t+j}.$$

If the Wold assumption is maintained and $r^* \xrightarrow{p} 0$, we have

Proposition 9: *Assume* $\widetilde{m}_t = \alpha_m(\mathbf{L})\varepsilon_{m,t}$, $\alpha_m(\mathbf{Z}) = \sum_{j=0}^{\infty} \alpha_{m,j}\mathbf{Z}^j$, *and* $\sum_{j=0}^{\infty} \alpha_{m,j}^2 < \infty$. *As the PVM discount factor* $\frac{1}{1+r^*} \longrightarrow 1$, $\widetilde{e}_t$ *equals the negative of* $\widetilde{m}_t$. *Thus, the exchange rate is dominated by permanent shocks.*

As $r^* \xrightarrow{p} 0$, (17) becomes $\widetilde{e}_t = -\alpha_m(\mathbf{L})\varepsilon_{m,t}$ subsequent to applying the Wiener-Kolmogorov prediction formula .[10] Thus, the exchange rate is

$$e_t \;=\; \mu_t + \widetilde{m}_t - \ln[A_t] + \widetilde{e}_t \;=\; \mu_t - \ln[A_t].$$

subsequent to decomposing cross-country money and consumption into permanent and transitory components. The DSGE-PVM predicts e_t is driven only by permanent factors as $r^* \xrightarrow{p} 0$, given $\widetilde{m}_t$ has a Wold representation. If e_t, m_t, and A_t violate the balanced growth restriction, the exchange rate is an independent random walk. Thus, Proposition 9 generalizes the Engel and West (2005) hypothesis that the exchange rate mimics a random walk when the discount factor is near one and fundamentals have a unit root to the wider class of DSGE models.

This section develops a DSGE-PVM of the exchange rate. The DGSE-PVM creates short- and medium-run PPP deviations in equilibrium exchange rates with persistence in the transitory components of cross-country money growth and consumption. The next section examines whether these predictions matter for the data.

4. Econometric Methods

This section describes the empirical methods employed to estimate the DSGE-PVM model of the exchange rate. First, we develop a multivariate UC-model to connect the unobserved permanent and transitory components of the exchange rate and cross-country money, consumption

[10] Sargent (1987) provides the relevant formulas in chapters XI.24 and XII.3.

to the observed prices and aggregates. Next, the UC model is cast in state space form to evaluate the likelihood function of the data. This section also discusses the priors of the parameters of the UC models and outlines the procedure to draw from the posterior distribution.

4a. The UC Model and Its State Space

The linear approximate UIRP (14) places restrictions on the transitory exchange rate process. The cross-equation restrictions vary with the process that drive the transitory components of cross-country money, $\widetilde{m}_t$, and cross-country consumption, $\widetilde{c}_t$. We assume $\widetilde{m}_t$ is a MA(k_m), $\widetilde{m}_t = \sum_{j=0}^{k_m} \alpha_j \varepsilon_{m,t-j}$, where $\alpha_0 \equiv 1$ and $\varepsilon_{m,t} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_m})$. For $\widetilde{c}_t$, we specify a AR(k_c), $\widetilde{c}_t = \sum_{j=1}^{k_c} \theta_j \widetilde{c}_{t-j} + \varepsilon_{c,t}$, where $\varepsilon_{c,t} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_c})$.

The balanced growth restriction ties long-run exchange rate behavior to the permanent components of cross-country money and consumption. Given observed cross-country money and consumption are the sum of their permanent and transitory components, the UC model has a state space form. We combine the balanced growth restriction with the permanent and transitory decompositions of cross-country money, m_t, and cross-country consumption, c_t, the equilibrium currency return generating equation (14), and the MA(k_m) of $\widetilde{m}_t$ and AR(k_c) of $\widetilde{c}_t$ to obtain a restricted UC-model of the exchange rate driven by permanent shocks to cross-country money and TFP and transitory fluctuations in cross-country money and consumption.

The state space form of the UC model with transitory cycles in cross-country money and consumption consists of the observation equation

$$\text{(18)} \qquad \begin{bmatrix} e_t \\ m_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 & -1 & \delta_{m,0} & \delta_{m,1} & \dots & \delta_{m,k_m} & \delta_{c,0} & \dots & \delta_{c,k_c-1} \\ 1 & 0 & 1 & \alpha_1 & \dots & \alpha_n & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 1 & 0 & \dots \end{bmatrix} \mathcal{S}_t,$$

where $S_t = [\mu_t \;\; \ln[A_t] \;\; \varepsilon_{m,t} \;\; \varepsilon_{m,t-1} \; \ldots \; \varepsilon_{m,t-k_m} \;\; \widetilde{c}_t \;\; \widetilde{c}_{t-1} \; \ldots \; \widetilde{c}_{t-k_c}]'$, the factor loadings on $\widetilde{m}_t$ and its lags are

$$(19) \qquad \delta_{m,i} \;=\; -\frac{1}{1+r^*}\left[\alpha_i - \frac{r^*}{1+r^*}\sum_{j=i+1}^{k_m}\left(\frac{1}{1+r^*}\right)^j \alpha_j\right], \quad i \;=\; 0, \ldots, k_m,$$

and the factor loadings on $\widetilde{c}_t, \ldots, \widetilde{c}_{t-k_c}$ are elements of the row vector

$$(20) \qquad \delta_{c,i} \;=\; s_c \frac{r^*}{1+r^*}\left[\mathbf{I}_{k_c} \;-\; \frac{1}{1+r^*}\Theta\right]^{-1}, \quad s_c \;=\; [1 \;\; \mathbf{0}_{1\times k_c-1}],$$

and Θ is the companion matrix of the AR(k_c) of $\widetilde{c}_t$. The system of first-order state equations is

$$(21) \quad S_{t+1} \;=\; \begin{bmatrix} \mu^* \\ a^* \\ 0 \\ \vdots \\ 0 \\ \vdots \end{bmatrix} + \begin{bmatrix} 1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \mathbf{I}_{k_m} & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & 0 & 0 & \theta_1 & \ldots & \theta_{k_c} \\ \vdots & \vdots & & \vdots & \vdots & & \mathbf{I}_{k_c-1} & \vdots \end{bmatrix} S_t + \begin{bmatrix} \varepsilon_{\mu,t+1} \\ \varepsilon_{A,t+1} \\ \varepsilon_{m,t+1} \\ \mathbf{0}_{k_m\times 1} \\ \varepsilon_{c,t+1} \\ \mathbf{0}_{(k_c-1)\times 1} \end{bmatrix}.$$

4b. The UC Model and Its Likelihood Function

Equations (18) and (21) define the state space model. Harvey (1989) and Hamilton (1994) map state space models into the Kalman filter to evaluate the likelihood.[11] Denote the likelihood $\mathcal{L}(\mathcal{Y}_t \,|\, \Gamma, \; UC(i))$ where $\mathcal{Y}_t = [e_t \;\; m_t \;\; c_t]'$,

$$\Gamma \;=\; [\beta \;\; \alpha_1 \; \ldots \; \alpha_{k_m} \;\; \theta_1 \; \ldots \; \theta_{k_c} \;\; \mu^* \;\; a^* \;\; \sigma_\mu \;\; \sigma_A \;\; \sigma_m \;\; \sigma_c \;\; \varrho_{A,c}]'.$$

where the DSGE-PVM discount factor is $\beta = \frac{1}{1+r^*}$, σ_j is the standard deviation of shock

[11] Harvey, Trimbur, and van Dijk (2005) use Bayesian methods to estimate trends and cycles of aggregate time series, but their analysis is not based on rational expectations models.

innovation to $j = \mu$, A, $\widetilde{m}$, and $\widetilde{c}$, $\varrho_{A,c}$ is the correlation coefficient of innovations to cross-country TFP trend and transitory component of cross-country consumption, and UC_i denotes UC model i with $\widetilde{m}$ and $\widetilde{c}$ cycles, only the $\widetilde{m}$ cycle, or only the $\widetilde{c}$ cycle.

4c. The Data

The sample runs from $1976Q1$ to $2004Q4$, $T = 116$. We have observations on the Canadian dollar – U.S. dollar exchange rate (average of period). The Canadian monetary aggregate is equated with M1 in current Canadian dollar, while for the U.S. we use the Board of Governors Monetary Base (adjusted for changes in reserve requirements) in current U.S. dollars. Consumption is the sum of non-durable and services expenditures in constant local currency units for both economies.[12] The aggregate data is seasonally adjusted and converted to per capita units. The data is logged and multiplied by 400, but neither demeaned nor detrended.

4d. Priors

The second column of table 1 lists the priors of Γ. The parameter vector is appended with three parameters, μ_e, τ_e, and δ_A. The first two parameters account for the level and determinist growth rate of the exchange rate, e_t. The priors of μ_e and τ_e are set to capture the deterministic features of the exchange rate. The parameter δ_A is the factor loading on cross-country TFP, $\ln[A_t]$. The balanced growth restriction predicts $\delta_A = -1$, the $(1,\ 2)$ element of the matrix of the observer equation (18). However, there is little information about δ_A. Thus, we select a prior uniform distribution that contains -1.0, as well as values as small as negative ten. If δ_A is small it indicates the inadequacy of the theoretical balanced growth restriction and the impact of permanent fluctuations in cross-country TFP on the exchange rate. Note that the factor loading on the permanent component of cross-country money m_t remains (normalized to) one.

[12]This includes Canadian semi-durable expenditures.

We choose priors of the MA(k_m) of $\widetilde{m}_t$ and AR(k_c) for $k_m = k_c = 2$. These lag lengths admit transitory cycles in cross-country money and consumption that allow for power at the business cycle frequencies, if the data wants. Normal priors for θ_1, θ_2, α_1, and α_2 allow for disparate transitory behavior in $\widetilde{m}_t$ and $\widetilde{c}_t$. The prior means of θ_1, θ_2, α_1, and α_2 are set to guarantee the relevant eigenvalues are strictly less than one. When a draw generates an eigenvalue greater than one for either the MA or AR parameters, the draw is discarded.

Priors on the standard deviations of the shock innovations reflect the lack of good information about these shocks. This explains the uniform priors on σ_μ, σ_A, σ_m, and σ_c. However, we attach a normally distributed prior to the correlation of innovations to $\ln[A_t]$ and $\widetilde{c}_t$, $\varrho_{A,c}$. Its mean is negative to capture our prior that $\ln[A_t]$ is smoother than c_t. Since we have no information about the extent of the smoothness, the mean is -0.5 with a standard deviation of 0.2 that allows for values close to negative one or zero. Draws less than negative one are ignored. The correlation of innovations to μ_t and $\widetilde{m}_t$ is fixed at zero because our belief that the sources and causes of permanent and transitory monetary shocks are unrelated.

The UC model has only one 'economic' parameter, the discount factor $\beta = \dfrac{1}{1 + r^*}$. We adopt the Engel and West (2005) prior for β. They conjecture that for $\beta \in [0.9,\ 0.999]$ to generate an exchange rate process observationally equivalent to a random walk depends crucially on the data. Hence, our prior on β is constructed to provide information about this conjecture. This is reflected by centering the mean of the prior of the normal distribution at 0.95 with a standard deviation 0.025. We toss out draws of $\beta \notin [0.9,\ 0.999]$.

4e. Estimation Methods

The likelihood function of the UC models do not have analytic solutions. We approximate the likelihood $\mathcal{L}(\mathcal{Y}_t |\ \Gamma,\ UC(i))$ with numerical methods based on the Metropolis-Hastings simulator. Our approach follows Rabanal and Rubio-Ramírez (2005). They exploit Bayesian estimation tools Geweke (1999) develops. The idea is to evaluate $\mathcal{L}(\mathcal{Y}_t \mid \Gamma,\ UC(i))$ from the random walk Metropolis-Hastings simulator. The result is the posterior distribution of Γ, which is proportion to the likelihood multiplied by the prior. For this draft, we draw $\mathcal{J} = 200,000$ replications from the posterior of a UC-Model.

5. Results

This section reports on the results of our empirical strategy. This draft presents parameter estimates of the UC model with independent transitory components in cross-country money and consumption. In the future, we plan on estimating models UC models with only a common cycle tied either to $\widetilde{m}_t$ or $\widetilde{c}_t$. Given posterior distributions, Rabanal and Rubio-Ramírez show how to use the posterior distribution to construct the marginal likelihood to conduct inference across competing models based on a proposal of Geweke (1999).

5a. Parameter Estimates

Table 3 contains the posterior means of Γ, along with standard deviations of the posterior in parentheses. The key economic parameter is the discount factor β. Its posterior mean of 0.96 is economically sensible. However, a standard deviation of 0.02 suggests a lack of precision in the data about β, as filtered through the UC-model. It is not unreasonable to believe that β is as large as 0.99 or as small as 0.92, according to its 95 percent coverage interval. Thus, the posterior of β suggest the data will find it difficult to distinguish between the UC model and an independent random walk as the source of exchange rate dynamics. This provides support for the Engel and West (2005) conjecture.

The estimates of table 3 indicate that the MA(2) process of $\widetilde{m}_t$ and AR(2) process of $\widetilde{c}_t$ generate persistence. The posterior means of $\theta_1 = 0.96$, and $\theta_2 = 0.04$ yield a leading eigenvalue of 0.95 from the associated companion matrix. An eigenvalue of 0.91 is produced by the posterior means of $\alpha_1 = 0.54$ and $\alpha_2 = 0.33$. However, the smaller root is -0.36, which points to substantial short-run reversion in $\widetilde{m}_t$ to an own shock. Shock innovations to $\widetilde{m}_t$ are more volatile than to $\widetilde{c}_t$, according to the estimates of $\sigma_m = 1.67$ and $\sigma_c = 0.70$.

The random walk trends of cross-country money and TFP reveal the former to be more persistent than the latter by a factor of five. Table 3 shows that cross-country TFP is a relatively smooth process, $\sigma_A = 0.30$, which suggests permanent income dynamics are at work. Since $\varrho_{A,c} = -0.60$, it reinforces the view of a smooth $\ln[A_t]$ process. Canadian TFP growth lags behind U.S. TFP growth by 0.7 percent per year, on average, because $a^* = 0.18$. The U.S. money stock grows more slowly in Canadian, but $\sigma_\mu = 1.53$ makes the permanent component of cross-country money volatile.

The deterministic components of the exchange rate show the Canadian dollar was far from par and, on average, depreciated from 1976Q1 to 2004Q4, according to table 3. Estimates of μ_e and τ_e are 125.28 and 1.65, respectively. The former estimate sets the level of the Canadian dollar-U.S. dollar exchange rate at 1.37.

The posterior distribution of table 3 provides a large (in absolute value) factor loading, δ_A, on cross-country TFP. Although σ_{mu} is larger than σ_A, the response of the exchange rate to fluctuations in $\ln[A_t]$ is large, $\delta_A = -8.07$, and far away from the balanced growth restriction. The estimate of δ_A also shows 'excess' sensitivity in the Canadian dollar-U.S. dollar exchange rate, which suggests the importance of real factors in driving its low frequency movements.

Table 4 presents posterior means of the factor loadings on the shocks to $\widetilde{m}_t$, $\varepsilon_{m,t}$ and its lags, and on $\widetilde{c}_t$ and $\widetilde{c}_{t-1}$. The estimated factor loadings reveal that the Canadian dollar-U.S. dollar exchange rate responds more to movements in $\varepsilon_{m,t}$ and its lags than to fluctuations in the transitory component of $\varepsilon_{c,t}$. The implication is that transitory monetary shocks matter more for the exchange rate than real side shocks.

5b. Permanent-Transitory Decompositions

The permanent-transitory decomposition of cross-country money is found in figure 1. Actual cross-country money is plotted as the solid (blue) in the top window of figure 1. Its trend is the (red) dot-dot line computed as the posterior mean by the passing the 200,000 draws of the vector of Γ and the data through the Kalman smoother.[13] The posterior mean of the cross-country money trend is smoother than its observed counterpart. The standard deviation of the growth rate of μ_t is 1.13 compared to 2.37 for m_t.

The bottom window of figure 1 presents the posterior mean of $\widetilde{m}_t$. Rather than generating a cycle in $\widetilde{m}_t$, its posterior mean exhibits sharp short-run reversion in response to an own shock. For example, the first element of the autocorrelation function (ACF) of the posterior mean of $\widetilde{m}_t$ is -0.09. Note also that table 5 reports that σ_μ is smaller than σ_m.

The UC model generates a permanent-transitory decomposition of cross-country consumption with an economically significant cycle. The top window of figure 2 plots observed cross country consumption as the solid (blue) line and smoothed cross-country TFP as the dot-dot (red) line. Not surprisingly, the volatility of cross-country consumption dominates cross-country TFP fluctuations. The standard deviation of the latter is 0.59 compared to 0.27 for the latter.

Nonetheless, the posterior mean of cross-country TFP has an economically interesting story to tell. Cross-country TFP is flat in the latter 1970s, which reflects the productivity slowdown in the U.S. and catch up by Canada. By the 1980s, U.S. TFP is growing more rapidly than in Canada. This continues until the early 1990s, when Canadian TFP again recovers relative to

[13] The Kalman smoother as described in Hamilton (1994).

U.S. TFP. At the end of the sample, the U.S.-Canadian TFP differential is expanding once more.

The plot of smoothed $\widetilde{c}_t$ appears in the bottom window of figure 2. The cycle in $\widetilde{c}_t$ is apparent and shows the impact of movements in cross-country TFP. The posterior mean of $\widetilde{c}_t$ is persistent and volatile. Its standard deviation is 2.01, while the leading term of the ACF gives a half-life to an own shock for $\widetilde{c}_t$ of nearly ten quarters.

The cycle of $\widetilde{c}_t$ has peaks and troughs that coincide with several U.S.-Canadian business cycles dates. For example, troughs in the posterior mean of $\widetilde{c}_t$ appear in 1981 and 1990 which also represent recessions dates in the U.S. and Canada. Since the end of the 1990 – 1991 recession, the rise in $\widetilde{c}_t$ points to persistent, but transitory, increase in U.S. consumption relative to Canada. However, $\widetilde{c}_t$ has been falling rapidly since a peak in $2001Q3$, which corresponds to the end of the U.S. recession of 2001.

Figure 3 contain plots of the Canadian dollar-U.S. dollar exchange rate, its smoothed trend, and its smoothed cycle. Exchange rate fluctuations are dominated by trend – solid (blue) line – in the upper window of figure 3. Trend volatility is almost 2.5 times greater than observed in $\widetilde{e}_t$, as shown in table 5. Although the volatility of $\widetilde{e}_t$ is relatively small, it is persistent. For example, $\widetilde{e}_t$ has the smallest standard deviation found in table 5, while the leading term of the ACF of $\widetilde{e}_t$ is 0.92. This persistence is directly tied to $\widetilde{c}_t$ because its correlation with $\widetilde{e}_t$ the exchange rate equals -0.99. Exchange rate trend growth and cross-country TFP growth are also negatively correlated at -0.87. Replacing $\widetilde{c}_t$ and cross-country TFP growth with $\widetilde{m}_t$ and m_t, yields correlations only of 0.22 and 0.31, respectively.

The strong negative correlation of the transitory component of the exchange rate with $\widetilde{c}_t$ help to interpret the Canadian dollar-U.S. dollar exchange rate cycle. Peaks in the transitory component of the Canadian dollar-U.S. dollar exchange rate occur either at or shortly after the end of recession. For example, the transitory component of the exchange rate peaks during the 1990 – 1991 recession, which is the last time the Canadian dollar approached par against the U.S. dollar. An exception is the end of the 2001 recession at which the Canadian reached a low of nearly 0.6 to the U.S. dollar. Thus, the transitory component of the exchange has economic content at the posterior mean of Γ, which includes $\beta = 0.96$.

5c. Exchange Rate Dynamics as $\beta \longrightarrow 1$

Engel and West (2005) argue that the exchange rate will approximate a random walk when the discount factor is close to one and fundamentals have a unit root. Proposition 9 also predicts that $\tilde{e}_t$ will collapse to zero pointwise in the 1976Q1 – 2004Q4 sample, as $\beta \longrightarrow 1$. The posterior distribution of Γ contains information about how close to one β needs to be to generate an approximate random walk in the exchange rate.

Figure 4 plots the smoothed $\tilde{e}_t$ conditional on a draw from the posterior distribution of Γ. The draws are conditioned on the smallest, 16th percentile, 84th percentile, and largest draws of β. These are β = [0.906 0.944 0.978 0.999] and are represented by the solid (orange), dot-dot (green), dot-dash (pink), and dash-dash (black) lines, respectively. The plots of $\tilde{e}_t$ exhibit similar behavior with two exceptions. First, the volatility of $\tilde{e}_t$ is compressed as β moves toward one. This is reflected in the standard deviations of $\tilde{e}_t$ that are 1.78, 0.93, 0.45, and 0.07 for β = [0.906 0.944 0.978 0.999], respectively. Second, $\tilde{e}_t$ is smooth and never strays far from zero at $\beta = 0.999$. For example, $\tilde{e}_t$ is no larger than 0.116 and no smaller than -0.14 for $\beta = 0.999$, while it varies between 3.332 and 3.568 given $\beta = 0.906$. This suggests plots of the transitory component of the exchange rate are economically interesting when draws from Γ produce a β below the posterior mean. Thus, it is most likely difficult for the data to distinguish between an independent random walk and the restrictions the DSGE-PVM model imposes.

6. Conclusion

Economists have little to say about the impact of policy on currency markets without a theory of exchange rate determination that is empirically relevant. According to Engel and West (2005), the near random walk behavior of exchange rates explain the failure of equilibrium models to fit the data or to find any model that systematically beats it at out-of-sample forecasting. They produce a random walk in the exchange rate by restricting the standard present-value model (PVM) with a unit root in a fundamental and a discount factor close to one.

This paper complements, extends, and generalizes Engel and West (2005). We find that the standard PVM places *common trend* and *common cycle* restrictions on the exchange rate and its fundamental. Under the former restriction and a large interest (semi-)elasticity of money demand, the exchange rate collapses to a martingale. We also show that the exchange rate approximates a random walk when only the common cycle restriction holds.

We also construct a PVM of exchange rates from a dynamic stochastic general equilibrium (DSGE) model. The DSGE-PVM places restrictions on the exchange rate and its fundamental similar to those of the standard PVM. For example, the exchange rate is dominated by permanent shocks in the DSGE-PVM, as its discount factor approaches one. Thus, we extend and generalize the Engel and West (2005) random walk result to a wider class of DSGE models.

Our empirical results support the view that it is difficult for the data to choose between exchange rate models when the discount factor is close to one. Preliminary estimates of the DSGE-Model suggest that the Candians-U.S. data place similar weight on discount factors of 0.99 as on 0.96. At the latter estimate, the transitory component of the exchange has economic and statistical significance, while at the former it does not. This challenges future research to develop DSGE models that are superior to the random walk in- and out-of-sample.

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Table 1: Summary of Propositions

Standard-PVM	
Proposition 1:	PVM Predicts Exchange Rate and Fundamentals Cointegrate; Campbell and Shiller (1987).
Proposition 2:	Currency Returns Are a ECM(0).
Proposition 3:	VECM(0) Imply Common Trend and Common Cycle for Exchange Rate and Fundamental.
Proposition 4:	Exchange Rate Approximates a Martingale as $\frac{1}{\phi} \longrightarrow 0$.
Proposition 5:	If Currency Returns and Fundamental Growth Share a Co-Feature and $\frac{1}{\phi} \longrightarrow 0$, Verify EW's (2005) Hypothesis.
DSGE-PVM	
Proposition 6:	DSGE Model Produces PVM to Replicate **Proposition 1**.
Proposition 7:	DSGE Model Imposes Co-Feature on Currency Returns and Fundamental Growth when No Serial Correlation in Fundamental Growth.
Proposition 8:	Currency Returns and Fundamental Growth Are Co-Dependent with Serial Correlation in Fundamental.
Proposition 9:	Generalize EW Hypothesis **Proposition 5** to Wider Class of Open Economy DSGE Models.

Table 2: Tests of Propositions 1, 3, and 5

Sample: $1976Q1 - 2004Q4$

	Canada & U.S.	Japan & U.S.	U.K. & U.S.
Proposition 3: VECM(0)			
Levels VAR Lag Length	8	5	4
LR statistic $p-$value	(0.02)	(0.01)	(0.09)
Proposition 1: Common Trend			
Cointegration Tests			
$\lambda-$Max statistic	4.86	0.20	2.27
	17.28	4.64	12.32
Trace statistic	4.86	0.20	2.27
	12.42	4.43	10.04
Proposition 5: Common Cycle			
Sq. Canonical Correlations	0.30	0.44	0.19
	0.09	0.08	0.07
χ^2 statistic $p-$value	(0.01)	(0.00)	(0.00)
	(0.69)	(0.21)	(0.12)
$F-$statistic $p-$value	(0.00)	(0.00)	(0.00)
	(0.61)	(0.19)	(0.11)

The level of fundamentals equals cross-country money netted with cross-country output calibrated to a unitary income elasticity of money demand. The money stocks (outputs) are measured in current (constant) local currency units and per capita terms. A constant and linear time trend are included in the level VARs. The LR statistics employ the Sims (1980) correction and have standard asymptotic distribution according to results in Sims, Stock, and Watson (1990). For the Canada-U.S. sample, the cointegration tests are based on the case $2*$ model of Osterwald-Lenum (1992). MacKinnon, Haug, and Michelis (1999) provide five percent critical values of 8.19 (8.19) and 18.11 (15.02) for the $\lambda-$max (trace) statistics. The cointegration evidence for the Japanese-U.S. and U.K.-U.S. samples are computed for the case 1 model. The associated MacKinnon, Haug, and Michelis five percent critical values are (3.84, 15.49) and (3.84, 14.26) for the trace and $\lambda-$max tests, respectively. The common feature tests compute the canonical correlations of Δe_t and $\Delta m_t - \Delta y_t$. The common feature null is all or a subset of the canonical correlations are zero. See Engle and Issler (1995) and Vahid and Engle (1993) for details.

Table 3: Estimates of the UC-Models

Posterior Means

Parameter	Priors	Two Cycles	Money Cycle	Consumption Cycle
β	Normal $[0.95,\ 0.025]$	0.96 (0.02)		
θ_1	Normal $[0.7,\ 0.2]$	0.91 (0.05)	–	
θ_2	Normal $[-1.0,\ 0.3]$	0.04 (0.05)	–	
α_1	Normal $[0.4,\ 0.2]$	0.54 (0.05)		–
α_2	Normal $[0.2,\ 0.1]$	0.33 (0.05)		–
μ^*	Normal $[-0.2,\ 0.1]$	-0.17 (0.07)		
a^*	Normal $[0.1,\ 0.1]$	0.18 (0.23×10^{-2})		
σ_μ	Uniform $[0.0,\ 2.0]$	1.53 (0.14)		
σ_A	Uniform $[0.0,\ 1.0]$	0.30 (0.03)		
σ_m	Uniform $[0.0,\ 2.0]$	1.67 (0.13×10^{-2})		
σ_c	Uniform $[0.0,\ 1.0]$	0.70 (0.01)		
$\varrho_{A,c}$	Normal $[-0.5,\ 0.2]$	-0.60 (0.06)	–	
μ_e	Normal $[100.0,\ 15.0]$	125.28 (6.89)		
τ_e	Normal $[1.0,\ 0.5]$	1.65 (0.15)		
δ_A	Uniform $[-10.0,\ 0.0]$	-8.07 (0.31)		

For the parameters with a normal prior, the first value in brackets is the degenerate prior and the second the prior standard deviation. Priors for the θs are on the unconstrained AR coefficients. The associated posterior means are for constrained AR coefficients.

Table 4: Estimates of the UC-Models

Posterior Means

Parameter	Two Cycles	Money Cycle	Consumption Cycle
$\delta_{m,0}$	−0.93 (0.03)		
$\delta_{m,1}$	−0.50 (0.05)		
$\delta_{m,2}$	−0.32 (0.04)		
$\delta_{c,0}$	0.43 (0.16)		
$\delta_{c,1}$	0.02 (0.02)		

Table 5: Summary of the Posterior of the UC-Models

	Two Cycles	Money Cycle	Consumption Cycle
$STD(\Delta e^{trend})$	2.24		
$STD(\tilde{e})$	0.94		
$AR1(e^{cycle})$	0.92		
$Corr(\Delta e^{trend},\ e^{cycle})$	-0.17		
$STD(\Delta\mu)$	1.13		
$STD(\widetilde{m})$	1.35		
$AR1(\widetilde{m})$	-0.09		
$Corr(\Delta\mu,\ \widetilde{m})$	0.38		
$STD(\Delta \ln[A])$	0.27		
$STD(\tilde{c})$	2.01		
$AR1(\tilde{c})$	0.93		
$Corr(\Delta \ln[A],\ \tilde{c})$	-0.27		
$Corr(\Delta e^{trend},\ \Delta\mu)$	0.31		
$Corr(\Delta e^{trend},\ \Delta \ln[A])$	-0.87		
$Corr(e^{cycle},\ \widetilde{m})$	0.22		
$Corr(e^{cycle},\ \tilde{c})$	-0.99		

The summary statistics are taken from the mean of the posterior distributions of the trends and cycle of the exchange rate, cross-country money, and cross-country consumption.