

Along the New Keynesian Phillips Curve with Nominal and Real Rigidities

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Abstract

Phillips curves are used to explain price fluctuations. The new Keynesian Phillips curve (NKPC) adds forward-looking price expectations, which yields a present value model (PVM) that predicts permanent shocks dominate price level movements at all forecast horizons. When the PVM of the NKPC is applied to U.S. GDP deflator and nominal unit labor cost data, it reveals important business cycle movements in the price level. Our goal is to understand this rejection of the NKPC by solving, calibrating, and simulating monetary business cycle models that contain different combinations of nominal or sticky price and real or labor market search rigidities. Our simulations show a model that combines labor market search with *flexible* prices comes closer to matching actual price level fluctuations, than sticky price models do. This challenges claims sticky prices are a key part of the monetary transmission mechanism.

Key Words : New Keynesian Phillips Curve; Sticky Prices; Labor Market Search; Common Trend; Common Cycle.

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1. Introduction

Of all the comebacks of the 1990s, it seems a revival in Phillips curve research was the least anticipated. Unlike earlier Phillips curve research that focused on the connection between aggregate demand shocks and price changes, recent work aims to identify the impact of price expectations on price movements. Price expectations formation matters for business cycle theory and monetary policy. For example, a Phillips curve dependent more on forward- than backward-looking expectations suggests policymakers can disinflate with few costs. This favorable trade-off appears at odds with empirical evidence and the views of policymakers.

Yun (1996) constructs a monetary business cycle model that includes a revivalist Phillips curve. He assumes monopolistically competitive firms maximize their expected discounted profit stream subject to a Calvo (1983) sticky price constraint that reflects a nominal rigidity. The solution to the firms' problem can be cast as the new Keynesian Phillips curve (NKPC) with forward-looking price expectations and aggregate demand measured by nominal marginal cost.

The NKPC imposes restrictions on the price level summarized with a present-value model (PVM), in which the fundamental is nominal marginal cost. The restrictions correspond to those of the permanent income hypothesis put on consumption, which for the NKPC are permanent nominal marginal cost shocks dominate price level fluctuations. If other shocks produce significant movements in the price level, it responds to factors beside the price setting behavior of firms.

This paper studies price level dynamics using the PVM of the NKPC. Its restrictions are used to construct a trend-cycle decomposition of the price level and nominal marginal cost using the last 40 years of U.S. data. Since the cyclical component of the U.S. price level peaks during each of the last seven NBER dated recessions and shocks to this cycle explain 60 percent of price level variation at a two-year forecast horizon, the PVM of the NKPC is rejected.

These rejections alone do not suggest abandoning the NKPC. Instead, we compare our empirical results to the implications for dynamic stochastic general equilibrium (DSGE) models of the PVM of the NKPC.¹ We study a basic sticky price DSGE model, add to it the real rigidity of labor market search, and also consider a labor market search-*flexible* price DSGE model.²

The PVM of the NKPC gives us three “moments”. These are not the standard variances and correlations usually employed to evaluate DSGE models. Instead, we work with three NKPC moments: *(i)* the fraction of price constrained firms, *(ii)* plots of the trend-cycle decomposition of the price level, and *(iii)* the associated forecast error variance decompositions (FEVDs). NKPC moment *(i)* is a measure of the ‘stickiness’ of the price level, *i.e.*, an estimate of the frequency at which firms change prices, on average. The persistence and volatility of the price level’s trend and cycle are revealed by NKPC moment *(ii)*. NKPC moment *(iii)* shows which shocks matter most for price level fluctuations in the short-, medium-, and long-run.

We follow practices often found in the real business cycle (RBC) literature to evaluate DSGE models. The process starts with a basic model, and in light of its successes and failures offer new specifications to better fit the data. The first model we study is a version of the Yun (1996) model. Given Yun’s results, it is no surprise his sticky price-DSGE model produces excess smoothness in the synthetic price level’s trend and cycle. This matches the PVM prediction, but places the Yun-sticky price model at odds with NKPC sample moments *(i) – (iii)*.

Chari, Kehoe, and McGratten (2000) also cast doubt on sticky prices being a source of

¹Greenwood and Huffman (1986), Chéron and Langot (1999), Cooley and Quadrini (1999), Ellison and Scott (2000), Walsh (2002), and Krause and Lubik (2003) test DSGE models with unconditional Phillips curve moments.

²There are other models to fit different aspects of price dynamics. Ireland (1999) and Ruge-Murcia (2003) estimate game-theoretic monetary policy models to capture long-run inflation. Nason and Cogley (1994) show a flexible price-DSGE model mimics short-run price dynamics under a long-run monetary neutrality identification.

aggregate fluctuations. Ball and Romer (1990), Jeanne (1998), Gali and Gertler (1999), Dotsey and King (2001), Walsh (2002), Trigari (2003b), and Ireland (2003) argue real rigidities can solve this problem. For example, Ireland shows that a persistent, exogenous real demand shock is needed for his sticky price model to match U.S. business cycle fluctuations.

Solow (1976) points out traditional Phillips curves invoke the real rigidity of labor market search, rather than sticky prices, to identify unemployment with the state of aggregate demand. This motivates us to meld the Yun-sticky price model with Mortensen and Pissarides (1994) labor market search using elements of the RBC models of Andolfatto (1996), Merz (1995), and den Haan, Ramey, and Watson (2000). Labor market search is reported to create persistent and volatile aggregate fluctuations that resemble actual business cycles. The combination of nominal and real rigidities fail at this because excess smoothness is evident in the trend and cycle of the theoretical price level of the labor market search-sticky price model. Thus, labor market search is unable to move the Yun-sticky price model closer to NKPC sample moments (i) – (iii).³

The last DSGE model we study includes labor market search and a *flexible* price level. According to King and Watson (1997), traditional Phillips curves are consistent with flexible prices and sticky employment. This structure is also embedded in the equilibrium of our labor market search-flexible price model. This model produces synthetic NKPC moments (i) – (iii) that match their sample counterparts. Thus, our results indicate the real rigidity of labor market search likely contributes to observed price level fluctuations.

³Walsh (2002), Trigari (2003b), and Krause and Lubik (2003) obtain economically interesting results with labor market search-sticky price DSGE models, but examine a different set of moments. Besides difference in preferences and technologies, Walsh, Trigari, and Krause and Lubik also assume complete insurance exists to support the representative household. We construct the set of complete insurance arrangement needed for the representative household from the underlying primitives of the labor market search-sticky price DSGE model.

The next section presents the PVM of the NKPC, its trend-cycle decomposition, and reports empirical results. Section 3 reviews the Yun-sticky price model. Model calibration, labor market search, and Monte Carlo results are discussed in section 4. Section 5 concludes.

2. The New Keynesian Phillips Curve

NKPC estimates are controversial. Sbordone (2002), Gali and Gertler (1999), and Rabanal and Rubio-Ramírez (2003) report empirical success with the NKPC. Fuhrer and Moore (1995), Fuhrer (1997), Roberts (1995, 1997, and 2001), and Rudd and Whelan (2001), among others, test backward-looking Phillips curves against the forward-looking NKPC and reject it. We come at this debate differently because we identify the trend and cyclical components of the NKPC.

2.1 A NKPC Specification

Roberts (1995) shows that several sticky-price models yield the NKPC. Typical is the Calvo (1983) staggered price setting mechanism. Sticky prices arise because only a fraction, $1 - \mu$, of monopolistically competitive final goods firms are able to set and commit to a new price, $P_{C,t}$, between dates $t - 1$ and t . Aggregate price, P_t , dynamics are restricted by

$$(1) \quad P_t = \left[(1 - \mu)P_{C,t}^{1-\xi} + \mu \left(\frac{m^*}{y^*} P_{t-1} \right)^{1-\xi} \right]^{1/(1-\xi)}, \quad 1 < \xi,$$

where ξ , m^* , and y^* are the demand elasticity, steady state money growth, and non-stochastic growth rate of labor augmenting technology change, respectively. Assume the aggregator of final demand (in physical units) firms face is $Y_{D,t} = \left[\int_0^1 \mathcal{Y}_{D,j,t}^{(\xi-1)/\xi} dj \right]^{\xi/(\xi-1)}$, where $\mathcal{Y}_{D,j,t}$ represents the demand firm j faces. This implies the demand schedule of the j th firm is

$$(2) \quad \mathcal{Y}_{D,j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\xi} Y_{D,t},$$

where firm j charges $P_{j,t}$ for its output and all firms take $Y_{D,t}$ and P_t as given. Subsequent to minimizing costs, profit maximization leads to the forward-looking optimal commitment price

$$(3) \quad P_{C,t} = \left(\frac{\xi}{\xi - 1} \right) \left[\frac{\mathbf{E}_t \left\{ \sum_{i=0}^{\infty} \left(\beta \mu \left[\frac{m^*}{y^*} \right]^{-\xi} \right)^i \Gamma_{t+i} \phi_{t+i} Y_{D,t+i} P_{t+i}^{\xi} \right\}}{\mathbf{E}_t \left\{ \sum_{i=0}^{\infty} \left(\beta \mu \left[\frac{m^*}{y^*} \right]^{1-\xi} \right)^i \Gamma_{t+i} Y_{D,t+i} P_{t+i}^{\xi-1} \right\}} \right], \quad 0 < \beta < 1,$$

where $\mathbf{E}_t\{\cdot\}$, $\beta^i \Gamma_{t+i}$, and ϕ_t represent the mathematical expectations operator conditional on date t information, the date $t + i$ (stochastic) discount rate firms face, and real marginal cost.

Sticky prices force monopolistically competitive firms to be forward-looking when price setting. This gives the NKPC its forward-looking character, which is developed by linearizing the price aggregator (1) and the optimal price rule (3), subsequent to detrending, to construct

$$(4) \quad \ln[P_t] = \mu \ln[P_{t-1}] + (1 - \mu) \left(1 - \frac{\mu}{\mathcal{B}} \right) \sum_{j=0}^{\infty} \left(\frac{\mu}{\mathcal{B}} \right)^j \mathbf{E}_t \ln[\Phi_{t+j}], \quad \mathcal{B} \equiv \frac{m^*}{\beta y^*},$$

where constants are ignored. The trend or “annuity value” of the price level is the expected future path of *nominal* marginal cost, according to the equilibrium law of motion (4). It also shows that real marginal cost is a cointegration relation, $\ln[\phi_t] = \varpi_{CI}(\ln[P_t] \ln[\Phi_t])'$, where $\varpi_{CI} = [-1 \ 1]$, if nominal marginal cost is $I(1)$. Since ϕ_t is a cointegration relation, it helps drive a vector error correction model (VECM) of inflation and nominal marginal cost growth.

Engle and Issler (1995) call cointegration relations cycle generators, which ties real marginal cost to the NKPC cycle. We can recover its restrictions by subtracting $\ln[\Phi_t]$ from both sides of (4), applying the usual present-value algebra, and rearranging terms to produce

$$\ln[\phi_t] = \left(\frac{\mu}{1 - \mu} \right) \Delta \ln[P_t] - \sum_{j=1}^{\infty} \left(\frac{\mu}{\mathcal{B}} \right)^j \mathbf{E}_t \Delta \ln[\Phi_{t+j}].$$

Thus, the NKPC cycle is forward-looking because real marginal cost equals (a multiple of) inflation net of the expected present-value of nominal marginal cost growth.

The NKPC also predicts that inflation and nominal marginal cost growth share a serially correlated common feature, in the sense of Engle and Kozicki (1993). The NKPC common feature exists if inflation and nominal marginal cost growth form a linear combination whose residual is unpredictable. The PVM relation (4) produces the linear combination

$$(5) \quad \Delta \ln[P_t] - \mu_B \Delta \ln[\Phi_t] = \mu^{-1} \mathbf{E}_t \Delta \ln[P_{t+1}] - \zeta_{\Phi,t}, \quad \mu_B \equiv \frac{(1-\mu)(\mathcal{B}-\mu)}{\mu^2},$$

where $\zeta_{\Phi,t} = \mu_B \{ \mathbf{E}_{t-1} \Delta \ln[\Phi_t] + (\mathbf{E}_t - \mathbf{E}_{t-1}) \ln[\Phi_t] + \sum_{j=1}^{\infty} (\mu/\mathcal{B})^j (\mathbf{E}_t \Delta \ln[\Phi_{t+j}] - (\mu/\mathcal{B}) (\mathbf{E}_t - \mathbf{E}_{t-1}) \ln[\Phi_{t+j-1}] \}$. Regression (5) yields the NKPC common feature relation if $\zeta_{\Phi,t}$ annihilates serial correlation in expected inflation leaving only its unpredictable innovations.

2.2 NKPC Cointegration Relation: Estimates and Tests

The proxy for nominal marginal cost is *nominal* unit labor costs, ULC_t , which equals the ratio of hourly compensation to output per hour.⁴ The price level, P_t , is the GDP deflator. The sample period is 1960Q1–2001Q4, $T = 168$, with lags available beginning with 1955Q1.

We test for a cointegration relation in P_t and ULC_t using Johansen (1988, 1991) likelihood ratio (LR) tests based on a third-order VECM, case 1* model of Osterwald-Lenum (1992).⁵ The LR-max and trace statistics are [8.23, 12.98] and [8.23, 21.20], respectively. The former test rejects a common trend in P_t and ULC_t and the latter does not based on MacKinnon, Haug, and Michelis (1999) five percent critical values of [9.17, 15.88] for the LR-max test and [9.17, 20.25] for the trace test. The maximum likelihood estimate (MLE) of ϖ_{CI-MLE} is [1 -1.08 6.82].⁶

⁴Sbordone (2002) and Gali and Gertler (1999) show marginal cost equals ULC using the labor demand elasticity of a Cobb-Douglas technology of a monopolistic competitive firm. The Federal Reserve Bank of St.Louis' FRED databank labels the index of hourly compensation (output per hour), non-farm business sector `compnfb` (`ophnfb`).

⁵A LR test for the lag length – beginning with 12 lags – of the VAR of the logs of the price level and ULC cannot reject a four lag model. The AIC gives the same result.

⁶The Johansen (1991) test of the theoretical ϖ_{CI} against ϖ_{CI-MLE} is not rejected at standard significance levels.

We report two more cointegration tests because Johansen's are inconclusive and can be unreliable. The Engle and Granger (1987) cointegration test yields a t -ratio of -3.34, which rejects its null at the five percent level according to MacKinnon (1991). Boswijk (1994) constructs a Wald test from a simultaneous equations VECM, based on a two-stage least squares (2SLS) estimator. The Wald statistic of 11.82 falls between the ten and five percent critical values Boswijk tabulates (his table B.3). These tests support a cointegration relation for P_t and ULC_t .

2.3 NKPC Common Feature Relation: Estimates and Tests

Two tests for a serially correlated common feature employ the canonical correlations, λ , of inflation and ULC growth, conditional on the VECM(3) information set. Inflation and ULC growth share a serially correlated common feature if the smallest $\lambda = 0$. Vahid and Engle (1993) develop a χ^2 -common feature test, but a F -test exists due to Rao (1973) that has superior small sample properties, according to Engle and Issler (1995). The λ^2 s equal 0.0513 and 0.8243, with associated p-values of 0.19 (0.21) and 0.00 (0.00) for the χ^2 (F -)test.

Vahid and Engle (1993) show a 2SLS regression provides a common feature test and recovers $\varpi_{CF} = [1 \quad -\mu_B]$.⁷ The 2SLS estimate of μ_B equals 0.8203, with a standard error of 0.0800, given the VECM(3) regressors are instruments. The LM test of instrument validity cannot reject the null, given a p-value of 0.16 for the statistic 9.32. Another test Vahid and Engle discuss compares a VECM(3) restricted by the common feature relation (5) against an unrestricted VECM(3). This LR test has ten degrees of freedom and a p-value of 0.55. In light of three common feature tests, the evidence favors the right side of (5) is unpredictable.

We have to calibrate \mathcal{B} to compute μ_{2SLS} from $\mu_{\mathcal{B},2SLS}$. We set $\beta = 1.03^{-0.25}$, $\gamma = 0.0047$,

⁷Gali and Gertler (1999) use generalized method of moments to estimate the NKPC, which imputes inflationary expectations to the instrument vector. Sbordone (2002) minimizes the distance between price level dynamics restricted by a NKPC and the actual price level. This is akin to the instrumental variables estimator of West (1989).

and $m^* = \exp\{0.0167\}$, where m^* and γ are taken from U.S. data; see section 4.1 for details. The calibration implies NKPC sample moment (i), μ_{2SLS} , is 0.5292 with a standard error of 0.0081.⁸ Thus, firms change prices twice a year, on average.

2.4 The Trend-Cycle Decomposition of the NKPC

Vahid and Engle (1993) show how to construct a simple trend-cycle decomposition, when the dimension of the VAR equals the number of cointegration and common feature relations. For the NKPC, the decomposition relies on the (log) levels of the price level and ULC , ϖ_{CF} , and ϖ_{CI} . Partition the columns of the inverse of the stack of these vectors into the matrix

$$[\pi_{.,1} \quad \pi_{.,2}] = \begin{bmatrix} \varpi_{CF} \\ \varpi_{CI} \end{bmatrix}^{-1}.$$

Since $(\ln[P_t] \quad \ln[\Phi_t])' = [\pi_{.,1} \quad \pi_{.,2}]^{-1} [\pi_{.,1} \quad \pi_{.,2}](\ln[P_t] \quad \ln[\Phi_t])'$, the NKPC cycle is $\pi_{.,2} \times \varpi_{CI}(\ln[P_t] \quad \ln[ULC_t])'$. Note that the NKPC cycle equals the cointegration relation of real unit labor cost, up to the vector $\pi_{.,2}$. Likewise, the NKPC trend is a scalar multiple of the common feature relation, $\pi_{.,1} \times \varpi_{CF}(\ln[P_t] \quad \ln[ULC_t])'$.

Figure 1 contain the NKPC trend and cycle, or NKPC sample moment (ii). The top panel of figure 1 shows that the NKPC trend supports traditional views of recent U.S. price history. The tight labor markets of the mid-1960s coincide with an increase in the NKPC trend. The trend falls with the recession that begins in late 1969. The 1970s sees a rising trend at the time of the first oil price shock. The contractionary monetary policy initiated late in 1979 pushes the NKPC trend below the price level from 1980 until the economic expansion of the mid-1990s. The NKPC trend dips below the price level prior to the NBER peak dated 2001Q1.

⁸Vahid and Engle describe a MLE that stacks the common feature regression on top of the ECM of ULC growth. The MLE of μ_B is 0.8927, with a standard error of 0.1080, which gives $\mu = 0.5192$ and a standard error of 0.0153.

The NKPC cycle and NBER dated business cycle peaks (vertical dash lines) and troughs (vertical dot-dash lines) appear in the bottom panel of figure 1. Peaks in the NKPC cycle peaks occur during the last seven NBER dated recessions, which signals a recovery coincides with rising real unit labor costs. Thus, the NKPC cycle is economically important, consistent with prior views of the Phillips curve, but at odds with the PVM predictions of the NKPC.

The NKPC sample moment (*iii*) employs the NKPC trend-cycle decomposition to gauge the contribution of the identified trend shock to movements in the GDP deflator.⁹ The FEVDs with respect to the trend are 2.70, 8.80, 26.38, 60.12, 78.37, 86.55, 91.44, and 98.05 percent at 1, 2, 4, 8, 12, 16, 20, and 40 quarter forecast horizons, respectively.¹⁰ Trend innovations are responsible for about a quarter of price level fluctuations at a one-year horizon and 60 percent at the end of two years. It takes four years to reach 85 percent.¹¹ Thus, cyclical shocks matter for the price level, at least for a two year horizon, which rejects NKPC sample moment (*iii*).

To summarize, we study three NKPC moments: (*i*) the 2SLS estimate of the sticky price parameter, μ_{2SLS} , (*ii*) the NKPC common trend-common cycle decomposition, and (*iii*) its FEVD. Our evidence lends only weak support to the NKPC because cyclical shocks matter for price level moments. This raises the question of the role of nominal rigidities for price level fluctuations. The next two sections study this question using DSGE monetary models.

⁹It is not possible to identify the trend (cyclical) shock as a supply (demand) shock.

¹⁰Engle and Issler (1995) and Issler and Vahid (2001) outline methods to compute FEVDs, under a common trend-common cycle decomposition. Trend innovation are a function of the common trend growth rate, lagged appropriately. Innovations to the cyclical component are the residuals of the cyclical component regressed on the information set of the VECM, lagged j times. Trend and cyclical innovations are orthogonalized by ‘regressing’ the latter on the former, which asserts trend innovations are prior to cyclical innovations.

¹¹The trend shock takes longer to dominate ULC fluctuations, because its FEVDs are 0.28, 2.01, 10.79, 41.34, 65.00, 77.52, 85.41, and 96.65 percent. However, the NKPC places no restrictions on these FEVDs

3. A Sticky Price DSGE Model

This section reviews the sticky-price DSGE model of Yun (1996). This model combines cash and credit goods, a cash-in-advance (CIA) constraint, and a Calvo-staggered price mechanism into a one-sector growth model.¹² Section 4 integrates the labor market-search structure that Merz (1995), Andolfatto (1996), and den Haan, Ramey, and Watson (2000) use in a RBC setting into Yun's DSGE model. We also study a flexible price version of this DSGE model.

3.1 The Final Goods Sector

Monopolistically competitive final goods firms take addresses on the unit interval. Producing a differentiated good employs a constant returns to scale (CRS) technology, $F(k - \bar{K}, hZ) \equiv (k - \bar{K})^\theta (hZ)^{1-\theta}$, $\theta \in (0, 1)$, where k is capital, \bar{K} is an exogenous minimum capital threshold (*i.e.*, infra-structure) common to all final goods firms, and hZ is productivity augmented hours.¹³

Monopolistic competition in the final goods market forces the associated prices to depend on nominal marginal cost, Φ . The j th final good firm sets its price by minimizing its total cost, $\mathcal{T}C_j = R_K k_j + W h_j$, subject to the CRS technology, where R_K is the nominal rental rate of capital. The first-order necessary conditions (FONCs) are $R_K = \Phi \theta y_j / k_j$ and $W = \Phi(1 - \theta) y_j / h_j$. Place these optimality conditions into the cost function and exploit the CRS technology to show, $\mathcal{T}C_j = \Phi y_j - R_K \bar{K}$. Hence, the net profit function of this firm is

$$(6) \quad \frac{D_j}{P} = \left(\frac{P_j}{P} - \phi \right) \left(\frac{P_j}{P} \right)^{-\xi} Y_D - \frac{R_K}{P} \bar{K},$$

given demand schedule (2).

¹²Examples of the variety of sticky price specifications are found in King and Wolman (1996), Nelson (1998), Ireland (2001), Kozicki and Tinsley (2001), Sbordone (2001), and Smets and Wouters (2002).

¹³Monopolistically competitive final good firms face fixed costs period-by-period. Below, we outline a labor market structure that precludes fixed labor costs, as in Yun (1996), because hours are not priced in a spot market.

We study economies in which final goods prices are sticky and flexible. When final goods prices are flexible, real marginal cost is constant, $\phi = (\xi - 1)/\xi$, and prices are a constant markup over marginal costs. A final good firm whose behavior is restricted by the Calvo sticky price mechanism (1) solves the intertemporal profit maximization problem

$$\mathbf{E}_t \left\{ \sum_{i=0}^{\infty} (\beta\mu)^i \Gamma_{t+i} \left[\left(\left[\frac{m^*}{y^*} \right]^i \frac{P_{C,t}}{P_{t+i}} - \phi_{t+i} \right) \left(\left[\frac{m^*}{y^*} \right]^i \frac{P_{C,t}}{P_{t+i}} \right)^{-\xi} Y_{D,t+i} - \frac{R_{K,t+i} \bar{K}_{t+i}}{P_{t+i}} \right] \right\}.$$

The FONC of $P_{C,t}$ leads to the forward-looking price-setting optimality condition (3).

Construction of aggregate dividend and production functions closes the final goods sector. Yun (1996) ties aggregate demand to aggregate supply with the supply price aggregator, $P_{A,t}^{-\xi} \equiv \int_0^1 P_{j,t}^{-\xi} dj$, where $P_{A,t}^{-\xi} = (1 - \mu)P_{C,t}^{-\xi} + \mu(P_{A,t-1} m^* / \exp\{\gamma\})^{-\xi}$. Aggregate output, $Y_{A,t} \equiv \int_0^1 \mathcal{Y}_{A,j,t} dj$, and the demand schedule (2) set $Y_{D,t} = (P_t/P_{A,t})^{-\xi} Y_{A,t}$.¹⁴ These facts yield aggregate real dividends, $D_t/P_t = (P_t/P_{A,t})^{-\xi} [1 - \theta\phi_t] Y_{A,t} - (R_{K,t}/P_t)K_t - (W_t/P_t)h_t$. The aggregate production function $Y_{A,t} = (K_t - \bar{K}_t)^\theta (h_t Z_t)^{1-\theta}$ follows from the CRS technology, the relative prices $R_{K,t}/P_t$ and W_t/P_t , and the definitions of aggregate capital and hours.

3.2 The Household

Households decisions cover consumption, leisure, capital accumulation, and financial portfolios (to hold cash and government bonds). Felicity is summarized by

$$(7) \quad u(c_{M,t}, c_{L,t}, \ell_t) \equiv \psi_1 \ln[c_{M,t}] + (1 - \psi_1) \ln[c_{L,t}] + \psi_3 \frac{\ell_t^{1-\psi_2}}{1-\psi_2},$$

where $0 < \psi_1 < 1$, $\psi_2 \neq 1$, $0 \leq \psi_3$, $c_{M,t}$, $c_{L,t}$, and $\ell_t (= 1 - h_t)$ are cash consumption, credit consumption, and leisure, respectively. The household faces the budget constraint

$$(8) \quad \begin{aligned} D_t + R_{K,t} k_t + W_t h_t + (1 + R_{B,t}) B_{G,t} + M_t - A_{t+1} \\ = P_t [c_{M,t} + c_{L,t} + k_{t+1} - (1 - \delta_K) k_t + T_t], \end{aligned}$$

¹⁴This eliminates $P_{C,t}$ from the state vector of the economy leaving only current and lagged aggregate prices.

the CIA constraint

$$(9) \quad M_t \geq P_t c_{M,t},$$

and the wealth constraint

$$(10) \quad A_t \geq B_{G,t} + M_t - X_{t-1},$$

where $0 < \delta_K < 1$, and D_t , $B_{G,t}$, M_t , A_t , T_t , and X_{t-1} denote dividends the household receives from final good firms, government bonds the household owns at the beginning of date t , cash the household carries over to date t from the end of date $t - 1$, nominal wealth it takes from the end of date $t - 1$ into the beginning of date t , a lump-sum tax levied on all households, and the total cash injection, respectively. The government pays $R_{B,t}$ on its one-period unit discount bond.

3.3 The Government

The government engages in monetary and fiscal operations. The latter activities involve expenditures, G_t , lump-sum tax collecting, T_t , and issuing one-period unit discount bonds, $B_{G,t+1}$. The monetary operation injects X_t units of cash into the household sector. Hence, the intertemporal budget constraint of the government is

$$(11) \quad P_t T_t + (B_{G,t+1} - B_{G,t}) + (M_{t+1} - M_t) = P_t G_t + R_{B,t} B_{G,t} + X_t.$$

We let $T_t = G_t$ at each date t and assume the government spending-output ratio, $g_t = G_t/Y_{D,t}$, evolves exogenously. Government bonds are restricted to be in *zero* net supply, $B_{G,t+1} = 0$, along the equilibrium path. Cash injections obey $X_t = M_{t+1} - M_t$ and monetary base growth, $m_t (= M_{t+1}/M_t)$, is assumed to be an exogenous stochastic process to avoid entangling the predictions of our DSGE models with an arbitrary monetary policy rule.

3.4 Household Optimality

The household maximizes its expected lifetime utility subject to (8) – (10). Lifetime utility is the expectation of the infinite discounted sum of felicity,

$$\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u(c_{M,t+j}, c_{L,t+j}, \ell_{t+j}) \right\}, \quad \beta \in (0, 1).$$

This problem yields the consumption-based money demand function

$$\frac{M_t}{P_t} = C_t \left[\frac{\psi_1}{1 + (1 - \psi_1)R_{B,t}} \right],$$

where $C_t \equiv c_{M,t} + c_{L,t}$. Another implication is the household's stochastic discount factor (SDF)

$$(12) \quad \frac{\Gamma_t}{P_t} = \beta \mathbf{E}_t \left\{ \frac{\psi_1}{P_{t+1} c_{M,t+1}} \right\}.$$

Firms and the government take the sequence of SDFs, $\{\Gamma_{t+j}\}_{j=0}^{\infty}$, as given, when they discount using (12). The SDF, the CIA constraint (9), and the FONC with respect to $c_{L,t}$ produces

$$(13) \quad C_t = \frac{M_t}{P_t} + \frac{(1 - \psi_1)}{\Gamma_t},$$

which is the household “consumption function”.

Optimal choice of employment hours by the typical household involves the usual trade-off between leisure and the rewards of labor market activity. The optimality condition of h_t is

$$(14) \quad \frac{\psi_3}{(1 - h_t)^{\psi_2}} = \Gamma_t \frac{W_t}{P_t}.$$

The household supplies labor up to the point at which the dis-felicity of work equals the discounted real wage according to (14). This wage is determined in a perfectly competitive spot market.

The dynamic program the household solves produces two intertemporal optimality conditions. The Euler equation, $\Gamma_t/P_t = \beta E_t \{(\Gamma_{t+1}/P_{t+1})[1 + R_{B,t+1}]\}$, describes optimal intertemporal choice in the money market. It shows the interaction of the CIA constraint and next period's liquidity preference trade-off between consumption and the government's unit discount bond. The intertemporal trade-off between consumption and capital accumulation is given by

$$(15) \quad \Gamma_t = \beta E_t \left\{ \Gamma_{t+1} \left[\frac{R_{t+1}}{P_{t+1}} + (1 - \delta_K) \right] \right\},$$

which is determined by the FONC of K_{t+1} and the envelope condition for K_t . Euler equation (15) shows the household is willing to postpone a unit of date t consumption for the return additional capital is expected to yield during date $t + 1$ for date $t + 2$ consumption.

3.5 Aggregate Equilibrium and Optimality

Equilibrium requires the goods, capital, money, government bond, and labor markets to clear. Goods market equilibrium relies on the aggregate resource constraint

$$(16) \quad \left(\frac{P_t}{P_{A,t}} \right)^{-\xi} Y_t = C_t + K_{t+1} + (1 - \delta_K)K_t + G_t.$$

The aggregate resource constraint (16) adds together the budget and wealth constraints, (8) and (10), of the household, the government's budget constraint (11), and the firm's dividend flow (6). Since the rental market for capital, the money market, the government bond market, and the labor market are perfectly competitive, agents treat the joint stochastic process that generates returns and the nominal wage, $\{R_{K,t+j}, R_{B,t+j}, W_{t+j}\}_{j=0}^{\infty}$, as given. The same holds for the exogenous shock process $\{Z_{t+j}, \bar{K}_{t+j}, G_{t+j}, X_{t+j}\}_{j=0}^{\infty}$.

Any candidate equilibrium of the Yun-sticky price must satisfy the optimality conditions and the aggregate resource constraint for C_t , h_t , P_t , ϕ_t , and K_{t+1} . The optimality condition for consumption is the aggregate resource constraint (16). Optimal labor market activity ties the

labor supply schedule embedded in (14) with a firm's FONC with respect to hours

$$(17) \quad \frac{\psi_3}{(1-h_t)^{\psi_2}} = \Gamma_t \left(\frac{P_t}{P_{A,t}} \right)^{-\xi} \phi_t (1-\theta) (K_t - \bar{K}_t)^\theta h_t^{-\theta} Z_t^{1-\theta}.$$

Optimal price behavior requires the consumption function (13), the SDF (12), and $c_{M,t} = M_t/P_t$, which is the CIA constraint (9) in equilibrium and defines money market equilibrium. A flexible price regime equates aggregate consumption to real balances plus (the present-value of) the purchasing power of a dollar. The law of motion of the price level (1) and the optimal commitment price condition (3) restricts the optimal path of ϕ_t in a sticky-price economy. This forces money market adjustment onto C_t and Γ_t . Optimal capital accumulation arises from the Euler equation

$$(18) \quad \Gamma_t = \beta \mathbf{E}_t \left\{ \Gamma_{t+1} \left[\theta \phi_{t+1} (K_{t+1} - \bar{K}_{t+1})^{\theta-1} (h_{t+1} Z_{t+1})^{1-\theta} + (1 - \delta_K) \right] \right\},$$

which rests on the Euler equation (15) and the nominal rental rate of capital. The transversality conditions of the endogenous state variables are sufficient conditions of any candidate equilibrium, where for capital $\lim_{j \rightarrow \infty} \beta^j \mathbf{E}_t \{ \Gamma_{t+j} K_{t+1+j} \} = 0$.

4. Comparing Sample and Theoretical NKPCs

This section reports on the calibration, solution strategies, and Monte Carlo experiments. Next, labor market search is added to the Yun-sticky price model to compare the implications for the PVM of the NKPC of this real rigidity with the nominal rigidity of sticky-prices. We complete this study of the NKPC with a flexible price version of the DSGE with labor market search.

4.1 The Calibration and Numerical Solution

We generate an approximate numerical solution of the Yun-sticky price model from the linearized stochastically detrended variants of its optimality conditions, laws of motion, and equilibrium conditions. Stochastic detrending is necessary because labor augmenting technology evolves as a random walk with drift, $Z_{t+1} = Z_t \exp\{\gamma + \varepsilon_{t+1}\}$, $0 < \gamma$, $\varepsilon_{t+1} \sim \mathbf{N}(0, \sigma_\varepsilon^2)$, and

money growth is a AR(1), $m_{t+1} = m^* (1-\rho_m) m_t^{\rho_m} \exp\{\eta_{m,t+1}\}$, $|\rho_m| < 1$, $\eta_{m,t+1} \sim \mathbf{N}(0, \sigma_{\eta,m}^2)$, where $\ln[m_t] = \ln[M_{t+1}/M_t]$ and $\mathbf{E}\{\varepsilon_{t+i}\eta_{m,t+j}\} = 0$, $\forall i, j$.¹⁵ Real aggregates and prices are detrended as $\hat{U}_{Y,t} = U_{Y,t}/Z_t$ and $\hat{U}_{P,t} = U_{P,t}Z_t/M_t$, respectively, where $U_{Y,t} = [Y_{D,t} Y_{A,t} C_t K_{t+1} G_t]$ and $U_{P,t} = [P_t P_{A,t} P_{C,t}]$. Detrending $\hat{W}_t = W_t/M_t$, $\hat{\Gamma}_t = \Gamma_t Z_t$, and $\hat{R}_{K,t} = R_{K,t}/P_t$, follows.

The numerical solution is obtained by linearizing the detrended aggregate resource constraint (16), the labor market optimality condition (17), the SDF (12), the consumption function (13), the law of motion of the price level that underlies the PVM relation (4), and the Euler equation of K_{t+1} , (18). The solution we conjecture is

$$(19) \quad \mathcal{K}_{t+1} = \mu_{\mathcal{K}} \mathcal{K}_t + \mu_{\mathcal{E}} \mathcal{E}_t,$$

where $\mathcal{K}_{t+1} = [\tilde{K}_{t+1} \ \mathfrak{N}_{t+1} \ \mathfrak{Y}_{t+1}]'$, $\tilde{K}_{t+1} = \ln[\hat{K}_{t+1}/K^*]$, $\mathfrak{N}_{t+1} = \mathbf{E}_t \tilde{P}_{t+1}$, $\mathfrak{Y}_{t+1} = P_t$, and the exogenous state vector, $\mathcal{E}_t = [\varepsilon_t \ \eta_{m,t}]'$. We seek the unknown elements of the three-by-three matrix $\mu_{\mathcal{K}}$ and the three-by-two matrix $\mu_{\mathcal{E}}$ using methods Zdrozny (1998) and Sims (2000) develop to compute approximate numerical solutions.¹⁶ Given solutions for $\mu_{\mathcal{K}}$ and $\mu_{\mathcal{E}}$, the control system is

$$(20) \quad C_t = \pi_{\mathcal{K}} \mathcal{K}_t + \pi_{\mathcal{E}} \mathcal{E}_t,$$

where $C_t = [\tilde{C}_t \ \tilde{h}_t \ \tilde{P}_t \ \tilde{\phi}_t]'$, $\pi_{\mathcal{K}}$ is a four-by-three matrix, and $\pi_{\mathcal{E}}$ is a four-by-two matrix. The approximate linearized solution drives \tilde{K}_{t+1} and C_t , which includes \tilde{P}_t , with lagged price expectations. This imposes the PVM restrictions of the NKPC on the Yun-sticky price model solution.

We employ sample data and previous RBC studies to calibrate model parameters. Pref-

¹⁵We assume the transitory components of \bar{K}_{t+1} and g_{t+1} are non-stochastic.

¹⁶A singular leading coefficient matrix often arises in the stochastic difference equation system of a linearized sticky price model. Sims (2000) describes a solution and provides software.

erence parameters β and ψ_1 are 0.9950 and 0.8428, respectively. The latter implies an interest elasticity of money demand of one percent, given the federal funds rate sample mean. Values for the remaining preferences parameters are from Andolfatto (1996), $\psi_2 = 2.0$ and $\psi_3 = 2.08$. The technology parameter $\theta = 0.35$ and $\delta_K = 0.0195$. The steady state markup is 1.10, which yields $\xi = 11.0$. The sticky price parameter is calibrated to NKPC sample moment (i), $\mu = 0.5292$.

The calibration of the impulse structure relies on sample data from 1960Q1 – 2001Q4. The deterministic growth rate $\gamma = 0.0047$ is the sample mean of measured total factor productivity growth and its sample standard deviation is $\sigma_\varepsilon = 0.0117$. The parameters of the AR(1) process of money growth are based on the Federal Reserve Bank of St. Louis' monetary base series. Its mean growth rate is 0.0166. Estimation of the AR(1) regression of money growth yield $\rho_m = 0.4456$ and $\sigma_{\eta,m} = 0.0068$.¹⁷

4.2 Monte Carlo Design

We generate 5000 replications of the monetary DSGE models. A replication is 168 observations of the price level and ULC .¹⁸ At each replication, the MLE-cointegrating vector, ϖ_{CI} , of the case 1* VECM(3) is estimated. Conditional on a lag of the cointegration relation and three lags of artificial inflation and ULC growth, the 2SLS regression of inflation on a constant and ULC growth is computed to produce the common feature vector, ϖ_{CF} , and a synthetic estimate of μ_{2SLS} . These estimates are used to construct the NKPC trend-cycle decomposition and its

¹⁷As previously mentioned, the transitory components of the fixed capital component and government spending are assumed to be nonstochastic. We set g^* at its sample mean, 0.1878. Calibration of \bar{K}^* is problematic. It cannot be constructed without observations on fixed capital. The closest notion is structures, but U.S. capital stock data reveals the ratio of structures to total capital is about 0.23 for the 1960 – 2000 sample. We assume $\bar{K}^* = 0.025$. Experiments with values between 0.25 and 25 percent alter the steady state, but have little impact on theoretical NKPC sample moments.

¹⁸We compute 372 artificial observations, but drop the first 204 to remove dependence on initial conditions.

FEVD. We report theoretical FEVDs in table 1. Figure 2 contains nonparametric densities of the ensemble of synthetic estimates of μ_{2SLS} and the asymptotic 95 percent confidence interval of the sample μ_{2SLS} . Theoretical NKPC trends and cycles appear in figures 3, 4, and 5.

4.3 *Yun-Sticky Price Model Experiments*

Simulations of the Yun (1996) model reveal it to be at odds with NKPC sample moments (i) – (iii). The mean of theoretical μ_{2SLS} estimates is 0.8264, which places its density (the dashed curve) to the right of the asymptotic 95 percent confidence interval (vertical dotted lines) of the sample μ_{2SLS} (= 0.5293) in figure 2. Although the Yun-sticky price model is calibrated to NKPC sample moment (i), this model predicts more stickiness in the price level.

Figure 3 has the evidence the Yun (1996) sticky price DSGE model fails to replicate NKPC sample moment (ii). The top window of figure 3 shows that the theoretical NKPC trend falls on top of the sample GDP deflator. Thus, the theoretical NKPC cycle exhibits excess smoothness, which explains the theoretical one-standard deviation confidence bands of figure 3.

Table 1 contains the sample and theoretical FEVDs of the price level with respect to trend shocks and theoretical one-standard deviation confidence intervals of the latter FEVDs (in brackets). The FEVDs of the Yun-sticky price model and its one-standard deviation confidence intervals are all greater than 97.8 percent, which indicate little uncertainty about these FEVDs. The price level FEVDs of this sticky price model matches the PVM prediction, but is far away from NKPC sample moment (iii). Thus, trend shocks dominate price level movements in the Yun-sticky price model, which is not observed for the U.S. GDP deflator.

4.4 *Labor Market Search-Sticky Price Models Experiments*

The failure of the Yun-sticky price model indicates the nominal rigidity of sticky prices *alone* cannot explain the NKPC sample moments (i) – (iii). Gali and Gertler (1999), among

others, suggest a real rigidity may resolve this problem. We add the real rigidity of Mortensen and Pissarides (1994) job-search that Andolfatto (1996), Merz (1995), and den Haan, Ramey, and Watson (2000) successfully place in RBC models. An appeal of labor market search is the restrictions it places on the Phillips curve, as discussed, for example, by Solow (1976).

Labor market search ties real and nominal activity together with the matching and search technologies available to firms and households. They engage in job search because hours are bought and sold in the presence of labor market externalities related to the costs of posting vacancies and looking for work. Firms post $v_{j,t}$ plant-job vacancies at a cost of v per vacancy.¹⁹ The not-employed devote S_t hours to job search to generate felicity and pecuniary costs.

We assume a final good firm operates multiple plants and identify an active plant with a job.²⁰ Firms with empty-plant jobs and the not-employed are brought together randomly, according to the den Haan, Ramey, and Watson (2000) CRS matching technology

$$(21) \quad \mathcal{M}(V_t, (1 - N_t)S_t) = \frac{V_t[(1 - N_t)S_t]}{(V_t^\vartheta + [(1 - N_t)S_t]^\vartheta)^{1/\vartheta}}, \quad 0 < \vartheta,$$

where $V_t (\equiv \int_0^1 v_{j,t} dj)$ is total plant-job vacancies and N_t denotes aggregate employment (or the measure of active plant-jobs). However, the probability of a successful match is influenced indirectly by variation either in posted vacancies or in not-employed search effort. This reflects the labor market externalities associated with search; see the appendix for details.

Job search alters aggregate household felicity (7). When a not-employed household gives up leisure to search a fraction S_t of its one unit of date t time endowment, this household suffers a felicity loss equal to $\psi_5 (1 - S_t)^{1 - \psi_4} / (1 - \psi_4)$, where $\psi_4 \neq 1$ and $0 \leq \psi_5$. Since complete income

¹⁹Total recruitment costs represent a drain on aggregate output. This forces us to assume that v shares the technology trend, but has a non-stochastic transitory component.

²⁰Andolfatto (1996) points out that a CRS production technology in the presence of job search ties a plant-job to an operating plant. Hence, the aggregate measure of plant-jobs and the measure of the employed are equivalent.

and wealth insurance creates an aggregate household that is a weighted average of employed and not-employed households, the leisure component of aggregate household felicity becomes $N_t \psi_3 (1 - h_t)^{1-\psi_2} / (1 - \psi_2) + (1 - N_t) \psi_5 (1 - S_t)^{1-\psi_4} (1 - \psi_4)$; see the appendix for details.

The wealth constraint of the employed and not-employed differ because the not-employed face job search transactions costs. We assume these costs rise with search effort at rate $\varphi (> 0)$ and that the only resource available to pay these costs is the cash injection the not-employed receive from the government. For the not-employed, this adds $-\varphi S_t X_{t-1}$ to the wealth constraint (10). Thus, the employed and not-employed respond differently to the cash injection. Combine the employed and not-employed wealth constraints to obtain the aggregate wealth constraint

$$(22) \quad A_t \geq B_{G,t} + M_t - [1 - \varphi(1 - N_{t-1})S_{t-1}]X_{t-1},$$

where the weights are N_{t-1} and $1 - N_{t-1}$. Within the aggregate household, complete wealth insurance requires the employed to transfer cash to the not-employed to hold the latter harmless for their search costs. The appendix discusses these issues.

Firm and not-employed search frictions place demands on aggregate output. Given complete income and wealth insurance, these costs enter the aggregate resource constraint additively

$$(23) \quad \left(\frac{P_t}{P_{A,t}} \right)^{-\xi} Y_t = C_t + K_{t+1} + (1 - \delta_K)K_t + G_t + (1 - N_t)\varphi \frac{X_t}{P_t} S_t + u_t V_t,$$

from the wealth constraint of the aggregate household and the aggregate dividend process.²¹

²¹The appendix outlines the insurance schemes that give rise to the felicity function of the aggregate household and the rest of the economy-wide optimality and equilibrium conditions. Aggregation rests on the capital stocks, dividends received, cash held, and bonds owned by these households to be equal date-by-date. This assumes that employed and not-employed households hold equal endowments of capital and financial wealth at date zero. Further, we assume away any wealth disparities caused by ownership claims on final goods firms. If employed and not-employed households are initially given equal equity in final goods firms, the dividend flows will be equalized. Also, these results depend on the additive separability of felicity. Sims (1998) discusses related issues.

The last two terms on the right of the aggregate resource constraint (23) reflect the real resource loss that arises from job search by households and firms, respectively.

Job search negates a spot market for labor. Rather than a Walrasian auctioneer, a firm and the aggregate household negotiate a labor contract over hours and the real wage to split match surplus at each date the employment relationship exists. Match surplus is the sum of the capitalized value of an active plant-job and the net benefits the aggregate household receives from the ongoing job match. Merz (1995), Andolfatto (1996), and Cooley and Quadrini (1999) assume the aggregate household receives a fixed fraction, ζ , of the surplus at each date t . Thus, the aggregate household's contribution to match surplus equals ζ times the capitalized value of an active plant-job.

The surplus splitting rule together with optimal firm and aggregate household behavior produces the (Nash) equilibrium real wage function

$$(24) \quad \Gamma_t \frac{W_t}{P_t} h_t = \zeta \Gamma_t \left[\left(\frac{P_{A,t}}{P_t} \right)^\xi \left(1 - \left[\frac{\theta \phi_t}{1 - \bar{K}^*} \right] \right) \frac{Y_{A,t}}{N_t} + \frac{v_t V_t}{1 - N_t} \right] + (1 - \zeta) \mathcal{H}_{X,t},$$

where $\mathcal{H}_{X,t} = \psi_3 (1 - \psi_2)^{-1} (1 - h_t)^{1-\psi_2} - \psi_5 (1 - \psi_4)^{-1} (1 - S_t)^{1-\psi_4} - \varphi \Gamma_t X_t S_t / P_t$ and \bar{K}^* is the steady state of \bar{K}_t .²² Along the equilibrium path, discounted real labor income is a weighted average of that match's value-added to aggregate output and the alternative activities (e.g., non-employment and search) available to the aggregate household. The marginal product of labor plus the foregone costs of fixed capital and firm job search represent the former. The latter is the net impact on felicity of an ongoing plant-job match and foregone transactions job-search costs. Note that the real wage is a function of real marginal cost, ϕ_t , in a sticky price regime. Unlike a spot market in which the real wage equals the intersection of labor supply and

²²The appendix constructs the equilibrium real wage process (24).

labor demand (productivity) schedules at each date t , labor market search creates persistence and volatility in the real wage that differs from labor productivity. This creates a monetary transmission mechanism in a labor market search model.

Calibration of the labor market search models follows the process described in section 4.1. The not-employed preference parameters ψ_4 and ψ_5 equal two and 1.37, respectively. The exogenous fixed separation rate is set at 0.0848, which places δ_N within the range Merz (1995), den Haan, Ramey, and Watson (2000), and Andolfatto (1996) use. The calibration of ψ_4 , ψ_5 , and δ_N help guarantees aggregate hours and employment match their sample counterparts. Cooley and Quadrini (1999) fix $1/\vartheta = 1 - \zeta$ at 0.6. We do the same.²³ The vacancy cost parameter $v = 0.1050$ is taken from Andolfatto.²⁴ We assume job-search transactions costs impose a 0.1 percent loss on velocity at the steady state (in terms of sample GDP and the monetary base). This yields $\varphi = 0.3060$.

We solve the labor market search models using methods described in section 4.1. The linearized aggregate household and firm job-search optimality conditions, the aggregate resource constraint (23), and the law of motion of aggregate employment, $N_{t+1} = (1 - \delta_N)N_t + \mathcal{M}_t$, add \tilde{N}_{t+1} to the state vector \mathcal{K}_{t+1} and \tilde{S}_t to the control vector C_t . The transversality condition for employment is $\lim_{j \rightarrow \infty} \beta^j \mathbf{E}_t \{ \Lambda_{t+j} N_{t+1+j} \} = 0$, where Λ_t is the shadow price of a job match.

The theoretical density of μ_{2SL5} generated by the labor market search-sticky price model (dot-dash curve) appears in figure 2. This density is to the right of NKPC sample moment (i). Only 58 of the 5000 estimates reside within the 95 percent asymptotic confidence interval of

²³Since $1 - \zeta = 1/\vartheta$, the power the aggregate household exerts on contract negotiations equals the its share of the match surplus. Thus, the equilibrium real wage is the same as the socially optimal wage; see Hosios (1990).

²⁴The steady state is also constructed to make the probabilities that a vacant plant-job is filled and that someone not-employed finds work consistent with the den Haan, Ramey, and Watson calibration.

the sample estimate of μ_{2SLS} and the mean of the synthetic estimates of μ_{2SLS} equals 0.6607. Thus, the labor market search-sticky price model cannot explain NKPC sample moment (i).

Figure 4 presents NKPC moment (ii), the common trend and common cycle, of the labor market search-sticky price model. The theoretical NKPC trend (the top window) closely follows the actual GDP deflator. This explains the smoothness of the one-standard deviation confidence bands of the theoretical NKPC cycle (the bottom window).

The FEVDs of the price level with respect to the trend shock and their one-standard deviation coverage intervals generated by the labor market search-sticky price model appear in the fourth column of table 1. Since the one-standard deviation coverage interval of the FEVD runs from 35 to 96.5 percent at the one-quarter forecast horizon, it suggests a short-run role for cyclical shocks. However, the theoretical one-quarter horizon FEVD of 69 percent and the one year-horizon of nearly 90 percent are closer to the NKPC prediction than to the relevant sample FEVDs. Thus, the labor market search-sticky price model finds it difficult to reproduce NKPC sample moment (iii).

In summary, NKPC sample moments (i) – (iii) fail to be replicated by the Yun- and labor market search-sticky price models. A common element across the two models is that the state vector, \mathcal{K}_{t+1} , of their linearized solutions contain price expectations, as in the state system (19) and (20) of the Yun-sticky price model. Since this is the way the PVM restrictions are imposed in the sticky price models, it explains the upward bias in synthetic estimates of the sticky price parameter, the excess smoothness in the NKPC trend and cycle, and the dominate trend response of the theoretical FEVDs. This suggests the theoretical link between the price level and price expectations needs to be broken for monetary DSGE models to fit NKPC sample moments (i) – (iii).

4.5 Flexible Price-Labor Search Model Experiments

This section studies the PVM restrictions using labor market search, but replaces the sticky price mechanism (1) with a flexible price regime. The labor market search-flexible price model eliminates the stochastically detrended demeaned price expectation, $E_t \tilde{P}_{t+1}$, and price level, \tilde{P}_t , from the equilibrium state system

$$(25) \quad \begin{bmatrix} \tilde{K}_{t+1} \\ \tilde{N}_{t+1} \end{bmatrix} = \mu_{\mathcal{K}} \begin{bmatrix} \tilde{K}_t \\ \tilde{N}_t \end{bmatrix} + \mu_{\mathcal{E}} \begin{bmatrix} \varepsilon_t \\ \eta_{m,t} \end{bmatrix},$$

and their lags from the control system that generates the equilibrium price level process

$$(26) \quad \tilde{P}_t = \pi_{\mathcal{K},P} \begin{bmatrix} \tilde{K}_t \\ \tilde{N}_t \end{bmatrix} + \pi_{\mathcal{E},P} \begin{bmatrix} \varepsilon_t \\ \eta_{m,t} \end{bmatrix},$$

The solution (25) – (26) emphasize the persistence or stickiness in employment created by labor market search and its contracting process.

Figure 2 shows that the 95 percent asymptotic confidence interval of μ_{2SL5} falls within the density of theoretical μ_{2SL5} estimates produced by the labor market search-flexible price model (the solid line). More than 46 percent of these estimates are contained in the 95 percent asymptotic confidence interval, [0.5133, 0.5450]. The theoretical mean of μ_{2SL5} is 0.5300, compared to a sample mean of 0.5293. Thus, an econometrician who studies the labor market search-flexible price model would recover NKPC sample moment (*i*).

The FEVDs of the labor market search-flexible price model appear in the last column of table 1. Theoretical FEVDs are larger than sample FEVDs at 1, 2, 4, and 8 quarter forecast horizons, but smaller beyond a two-year horizon. However, one-standard deviation confidence intervals of the theoretical FEVDs cover the sample FEVDs, except at one- and two-quarter horizons. Although only a random walk technology shock and a money growth process whose AR1

coefficient is 0.45 drives the labor market search-flexible price model, it produces a synthetic FEVD of the price level that matches much of NKPC sample moment (*iii*).

The theoretical NKPC trend and cycle decomposition of the labor market search-flexible price model appear in figure 5. A weakness of the labor market search-flexible price model is that the theoretical NKPC trend (top panel) and cycle (bottom panel) are more volatile than their sample counterparts. The relative volatility of the NKPC cycle (trend) is 1.5098 (1.3622). This model is able to mimic the persistence of the sample NKPC cycle. The AR1 coefficients from the actual data and the theoretical ensemble of NKPC cycles are 0.9335 and 0.9476, respectively. Figure 5 also shows the labor market search-flexible price model successfully matches NKPC sample moment (*ii*) because the one-standard deviation confidence bands cover the sample NKPC cycle for much of the sample. Thus, sticky employment from labor market search combined with flexible prices replicates the PVM restrictions the NKPC places on the U.S. GDP deflator.

4.6 Price Level Fluctuations, Labor Market Search, and the NKPC

Labor market search models can have difficulties with several business cycle facts. Cole and Rogerson (1999) discuss why these models suffer from incorrect predictions about job creation, job destruction, and unemployment flows.²⁵ Likewise, Walsh (2002) and Trigari (2003a) report that labor market search-monetary DSGE model produce too much nominal volatility, which we confirm with the labor market search-flexible price model.²⁶ Nonetheless, the labor market search-flexible price model is better at replicating the PVM restrictions embodied by NKPC sample moments (*i*) – (*iii*), than are the sticky price models we study.

The labor market search-flexible price model generates theoretical NKPC moments that

²⁵Trigari (2003b) argues that a combination of nominal rigidities and labor market search solves these problems.

²⁶Ireland (2003) finds a model with flexible prices and an interest rate rule to define monetary policy mimics inflation volatility in pre-1979 U.S. data. If prices are sticky, the model fits inflation volatility in post-1979 data.

help to resolve several open question. Sticky wages are often shown to matter more than sticky prices, for example, by Jeanne (1998), Erceg, Henderson, and Levin (2000), and Rabanal and Rubio-Ramírez (2003). Our results suggest sticky wages represent a reduced-form of the labor market search contracting process, where the latter creates “stickiness” in employment.

Our results are also related to the structural VAR analysis of King and Watson (1997). They show U.S. data supports a Phillips curve that allows for flexible prices, but in which labor market variables are sticky. According to King and Watson, their evidence fits the sort of Phillips curve Solow (1976) describes. It is also consistent with the state system (25) – (26) of the labor market search-flexible price model because the equilibrium decision rule for N_{t+1} responds to shocks dated t , not $t + 1$, and the current price level responds to all date t shocks.

Price level fluctuations depend on changes in the endogenous state of the economy. Under a sticky price regime, short- to long-run theoretical price level movements in the Yun and labor market search models are “excessively” smooth because the endogenous state vector, \mathcal{K}_t includes price expectations, whose dynamics are restricted by the PVM relation (4) of the NKPC. The costly adjustment inherent in the NKPC flattens out the response of \tilde{P}_t to changes in $E_{t-1}\tilde{P}_t$ and \tilde{P}_{t-1} in the (linearized) solution of the labor market search-sticky price model.

The impact of the PVM restrictions of the NKPC on the labor market search models can be explained with the real wage generating equation (24). Note that measured real unit labor cost, $W_t h_t / P_t Y_{A,t}$, equals the right side of the real wage equation (24), subsequent to dividing by $\Gamma_t Y_{A,t}$. Given models have balanced growth conditions, $W_t h_t / P_t Y_{A,t}$ is a cointegration relation that represents the NKPC cycle (up to a scalar). An approximation to real unit labor cost

$$(27) \quad \ln \left[\frac{W_t h_t}{P_t Y_{A,t}} \right] = \Xi_{\mathcal{K}} \mathcal{K}_t + \Xi_{\mathcal{E}} \mathcal{E}_t,$$

uses the linearized solution (19) – (20) and the real wage equation (24), where the coefficients

are nonlinear functions of model parameters. According to (27), measured real unit labor cost exhibits excess smoothness because the PVM restrictions dominate changes in the price level and its expectation, which are contained in the state vector \mathcal{K}_t of the labor market search-sticky price model.

A flexible price regime does not drive \tilde{P}_t with price expectations, but instead by capital, employment, and shock impulse as described by the state system (25) – (26). Although the PVM restrictions no longer govern \tilde{P}_t in the labor market search-flexible price model, the price level exhibits stickiness because labor market search imparts it to employment. This stickiness is enough for the labor market search-flexible price model to mimic NKPC sample moments (i) – (iii). Thus, the real rigidity of labor market search is a prime candidate for an economically useful monetary transmission mechanism.

5. Conclusion

This paper studies the present-value model (PVM) of the new Keynesian Phillips curve (NKPC), in which nominal marginal cost is the fundamental. The PVM of the NKPC restricts the price level to respond only to trend shocks. We also show the PVM of the NKPC has a common cycle-common trend decomposition, based on Beveridge and Nelson (1981), Stock and Watson (1988), and Vahid and Engle (1993). The NKPC trend-cycle decomposition is used to do the first job of a Phillips curve: provide a good description of price level dynamics.

The last 40 years of U.S. GDP deflator and nominal unit labor cost data offers weak support of the NKPC. We estimate about half of U.S. final goods firms are price constrained, which is close to estimates reported elsewhere. The NKPC cycle is economically important because it peaks during the last seven NBER dated recessions. Forecast error variance decompositions

reveal that trend shocks only begin to account for more than 60 percent of price level movements at forecast horizons of two years or more. Thus, the NKPC prediction that trend shocks dominate price level fluctuations at all forecast horizons is not supported by the data.

We study the implications of the PVM of the NKPC prediction for the theoretical price level of several dynamic stochastic general equilibrium monetary models. Simulation experiments show that the Yun (1996) model with Calvo (1983) staggered price setting reproduce the PVM predictions. Hence, a model with only the nominal rigidity of sticky prices generates excess smoothness in the NKPC trend-cycle decomposition.

Earlier Phillips curve models invoke labor market imperfections to explain price level movements. We pursue this idea by adding labor market search to the Yun-sticky price model. Monte Carlo experiments of the labor market search-sticky price model yield NKPC moments that are not qualitatively different from the Yun-sticky price model. Unlike the labor market search-sticky price model, its flexible price cousin is better able to match price level fluctuations.

This paper takes up the Chari, Kehoe, and McGratten (2000) challenge to new Keynesian notions that nominal rigidities generate short-run monetary non-neutralities. We broaden Chari, Kehoe, and McGratten's research agenda by studying theoretical price dynamics that arise from the real rigidity of labor market search. We identify the real rigidity with labor market search because its externality suggests a role for monetary policy. Labor market search is only one specification within a large class of real rigidity-DSGE flexible price models, which indicates the need to continue the search for models with an economically meaningful monetary transmission mechanism that can be used for policy analysis. We leave this task for future research.

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Table 1. Forecast Error Variance Decomposition

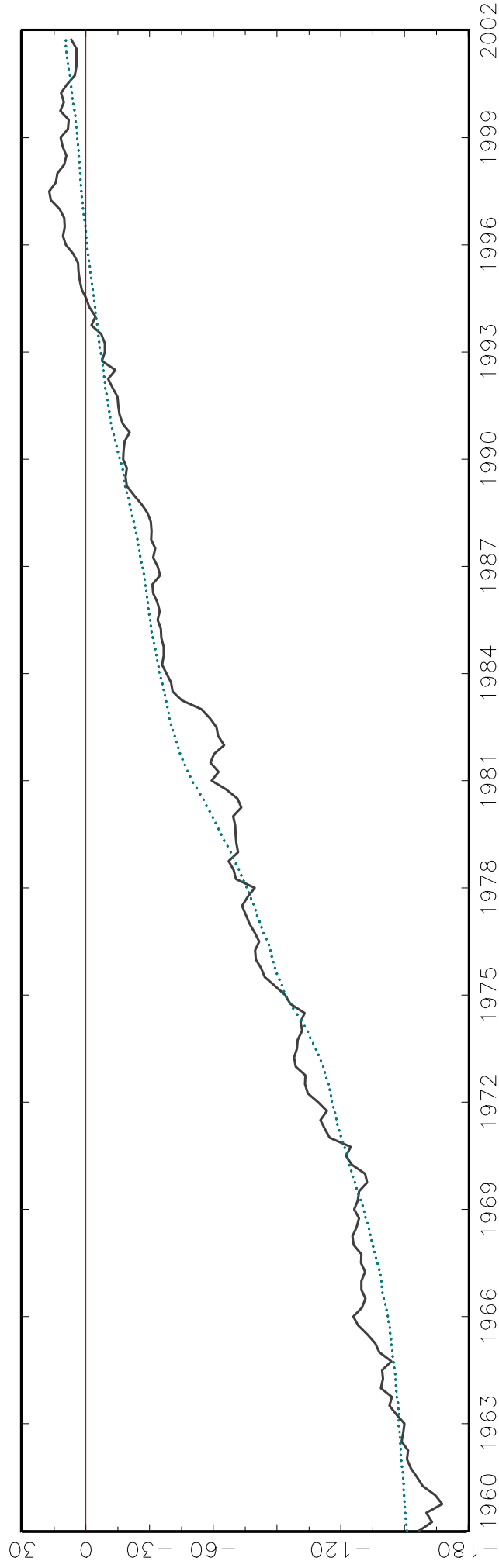
One Standard Deviation Confidence Intervals
 FEVDs of Price w/r/t Trend, Generated by DSGE Models

Horizon	Sample PGDP	Yun-Sticky Price Model	Search-Sticky Price Model	Search-Flexible Price Model
1	2.70	98.90 [97.81 99.93]	69.22 [34.77 96.53]	24.94 [6.92 48.03]
2	8.80	99.33 [98.67 99.96]	80.87 [61.21 98.72]	37.90 [13.09 69.48]
4	26.38	99.65 [99.31 99.98]	89.32 [84.05 99.60]	52.90 [23.01 85.63]
8	60.13	99.85 [99.70 99.99]	94.32 [94.80 99.88]	67.48 [38.70 94.08]
12	78.37	99.91 [99.82 100.00]	96.17 [97.40 99.95]	74.80 [49.05 96.68]
16	86.55	99.94 [99.88 100.00]	97.15 [98.48 99.97]	79.18 [56.35 97.77]
20	91.44	99.96 [99.92 100.00]	97.75 [99.01 99.98]	82.12 [61.93 98.42]
40	98.05	99.99 [99.97 100.00]	98.97 [99.76 99.99]	89.21 [76.92 99.51]

The values in brackets are the 16th and 84th percentiles of the FEVDs generated from 5000 replications of the DSGE models.

Figure 1: The U.S. Phillips Curve

The Phillips Curve Trend (solid line) and GDP Price Level (dotted line)



The Phillips Curve Cycle and NBER Business Cycle (Peak dash line, Trough dot-dash line)

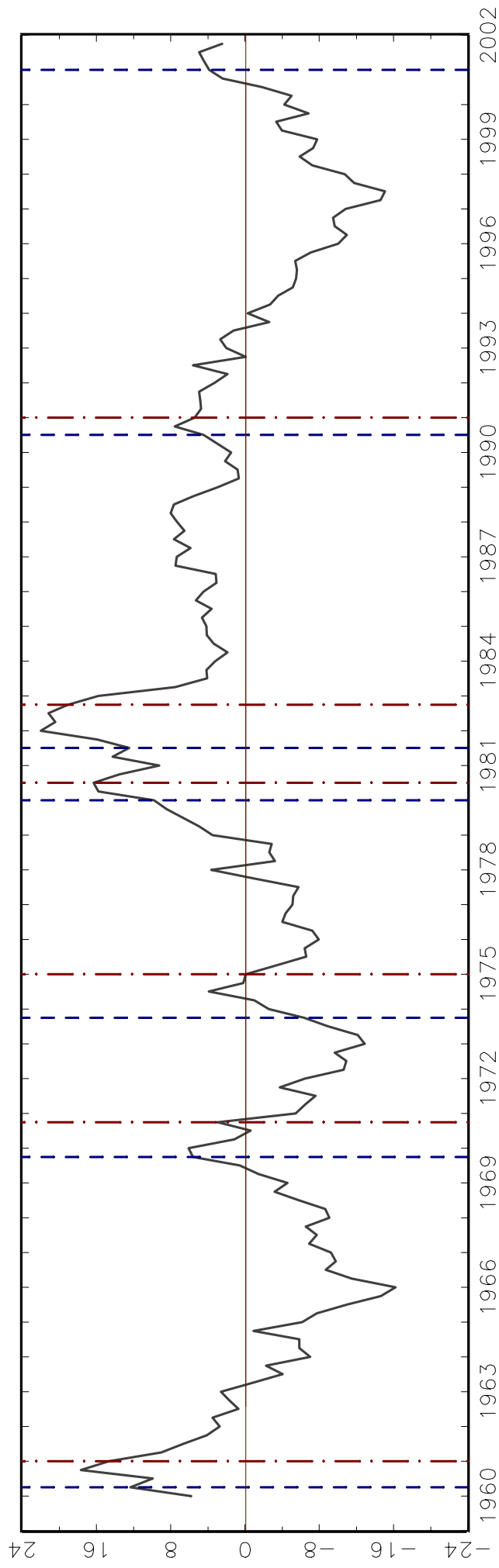


Figure 2: Theoretical Densities of μ

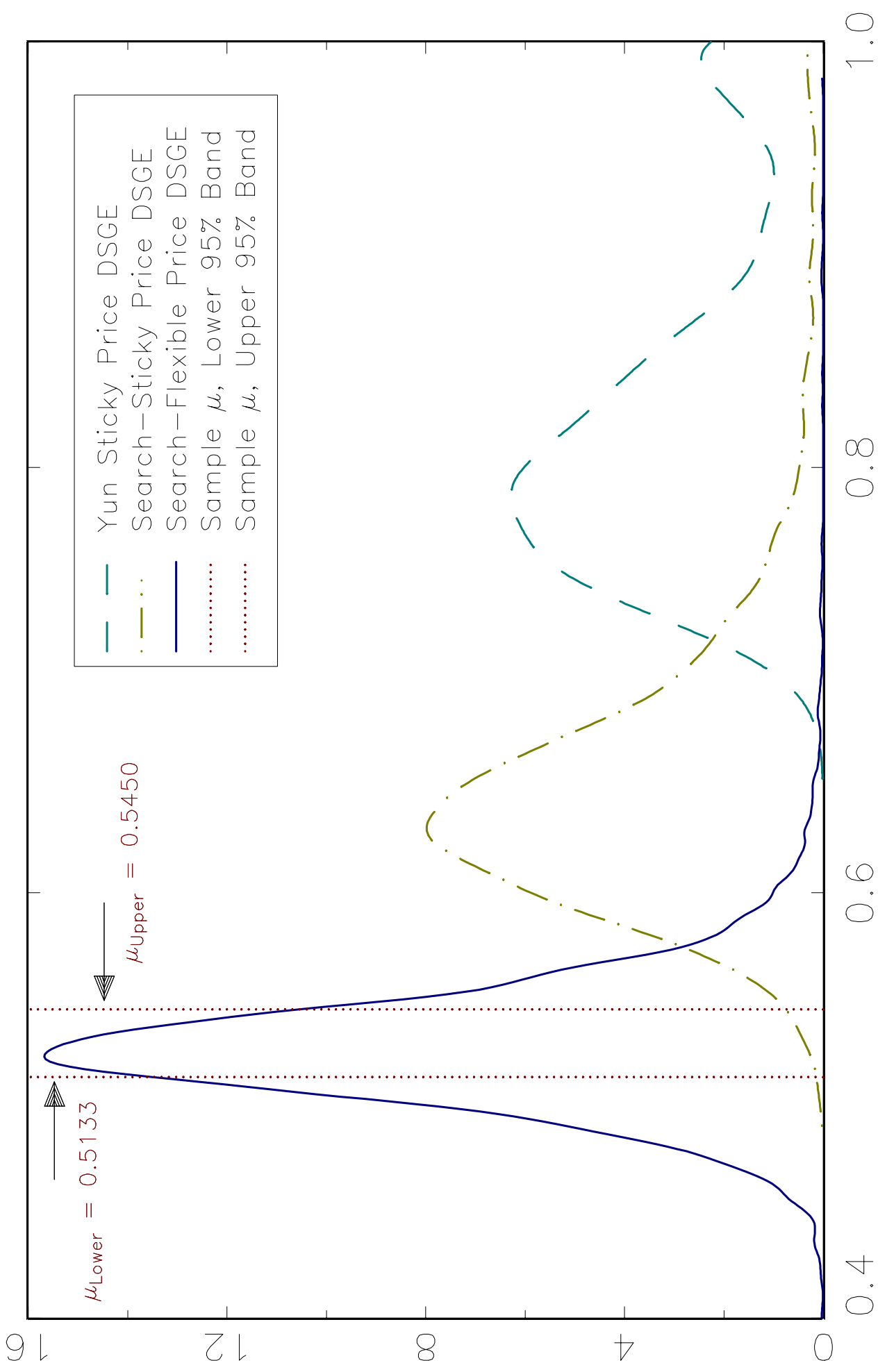
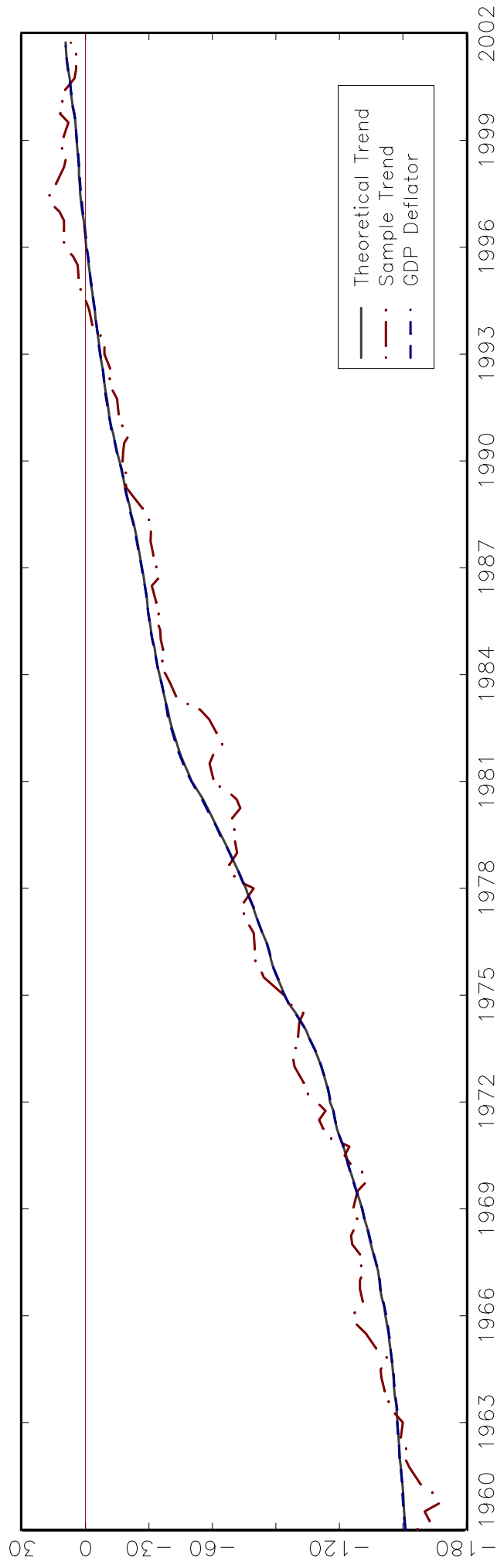


Figure 3: Yun–Sticky Price DSGE Model

Yun–Sticky Price Phillips Curve Trend, Sample Phillips Curve Trend, and Actual Price Level



Sample Phillips Curve Cycle and 1–Standard Deviation Band of Yun–Sticky Price Model

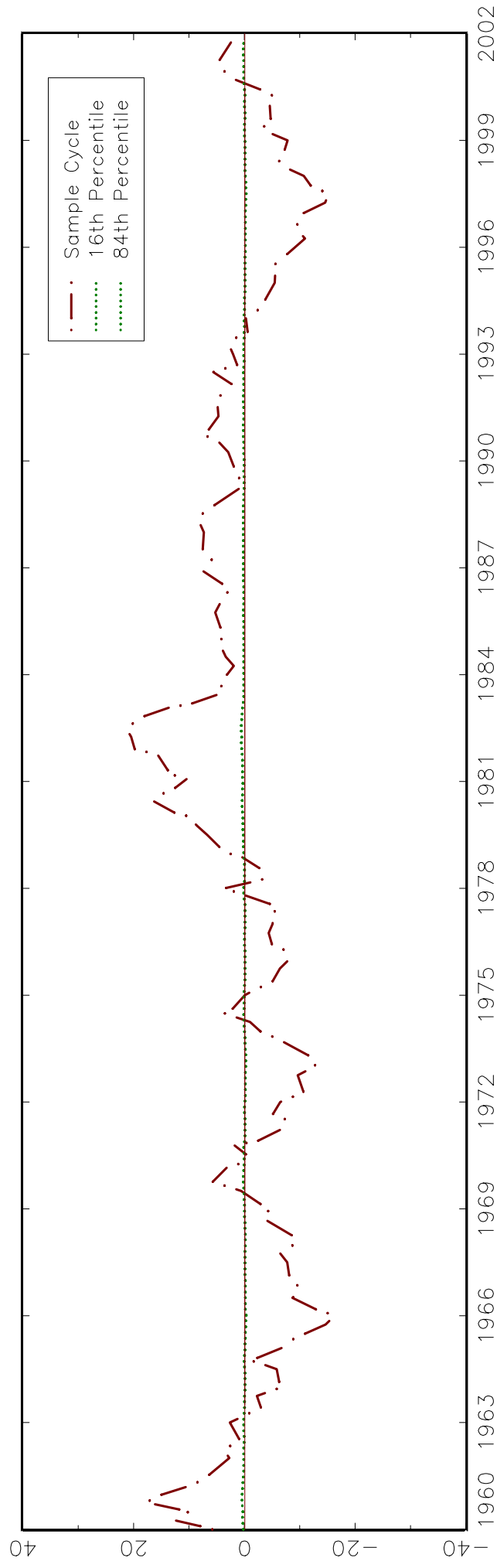
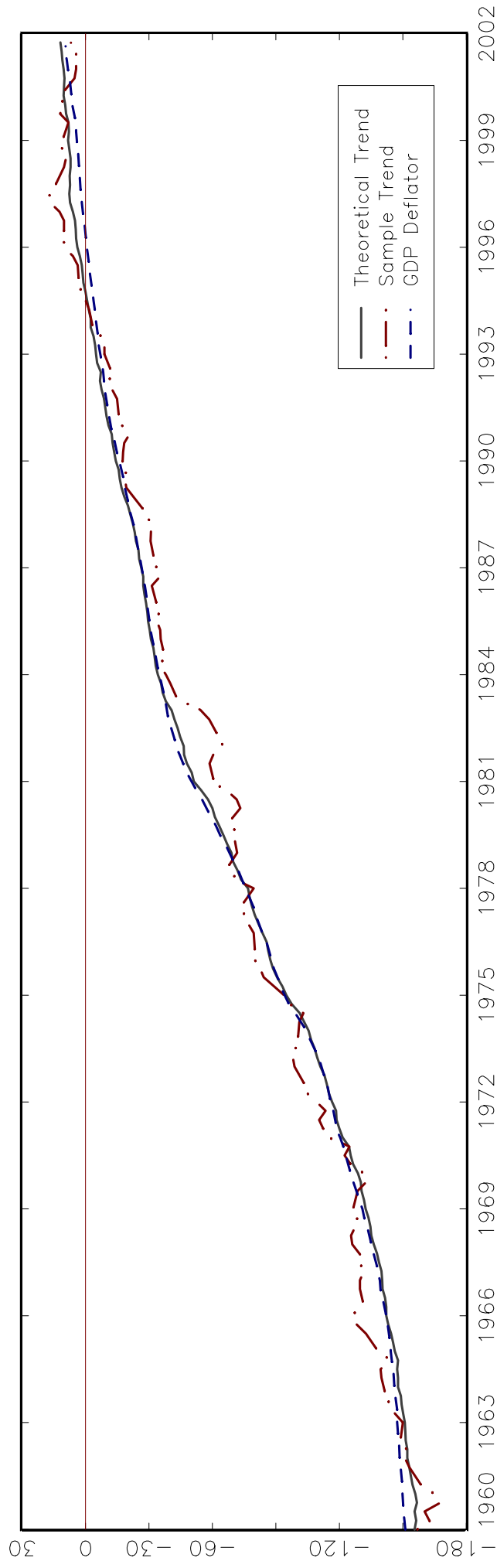


Figure 4: Search–Sticky Price DSGE Model

Search–Sticky Price Phillips Curve Trend, Sample Phillips Curve Trend, and Actual Price Level



Sample Phillips Curve Cycle and 1–Standard Deviation Band of Search–Sticky Price Model

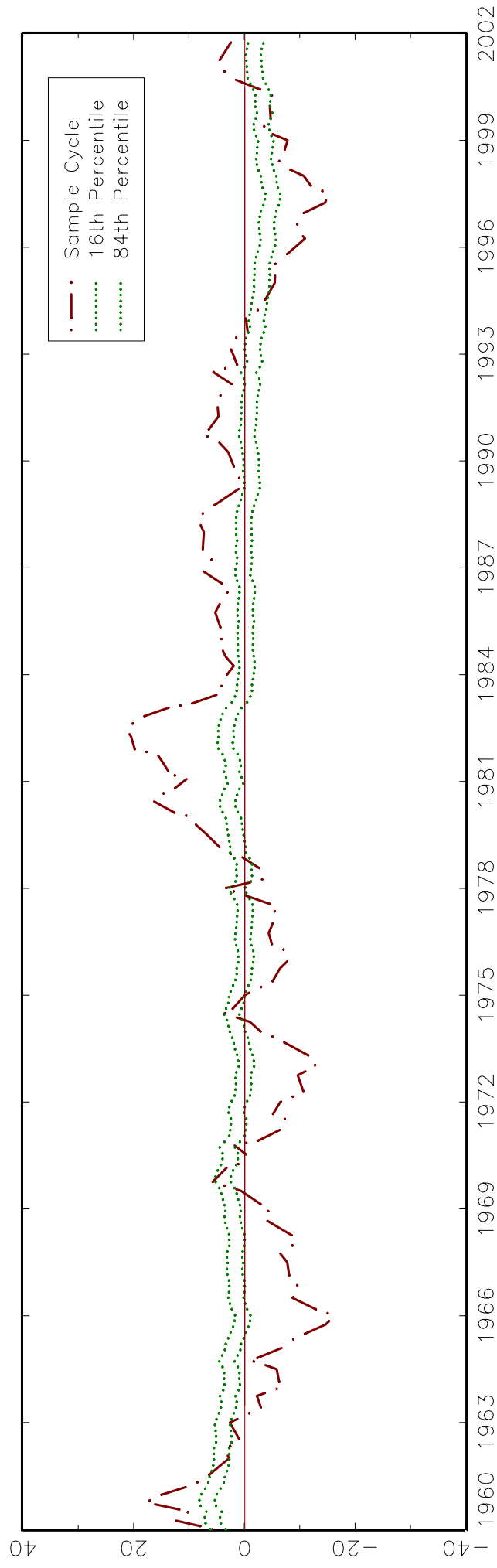
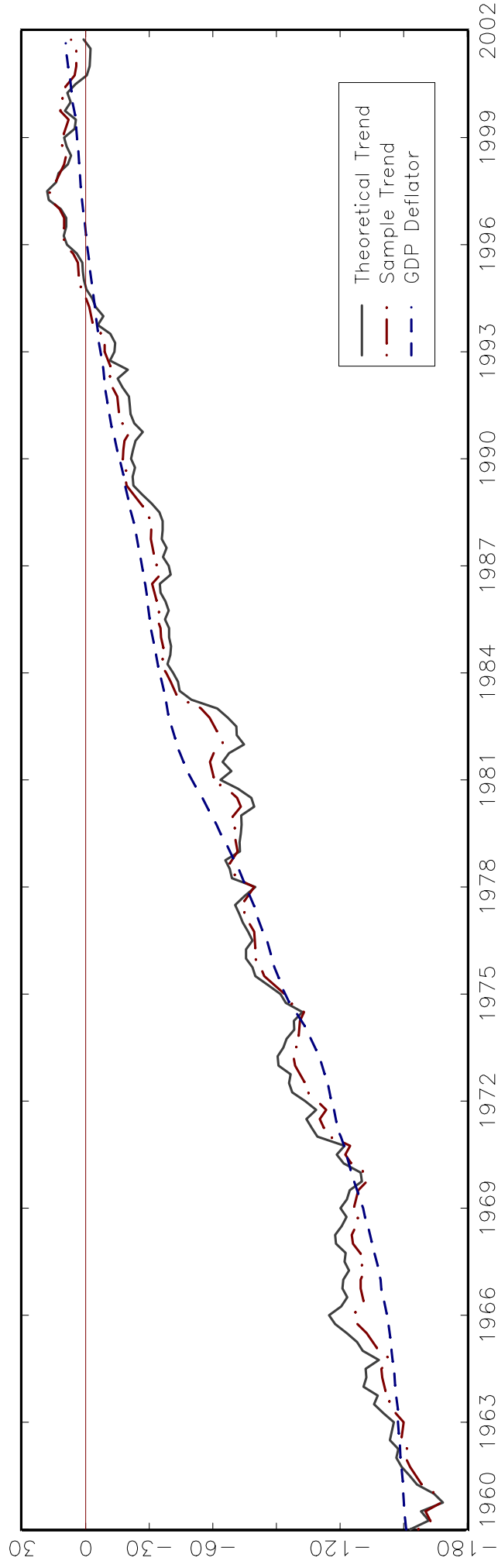
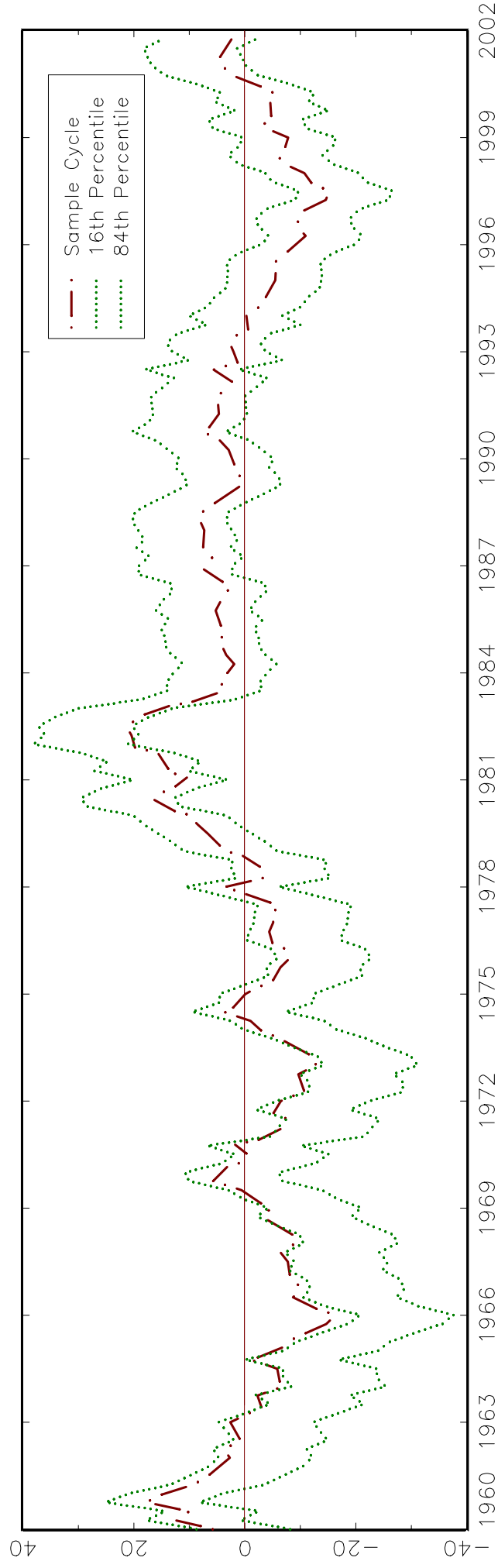


Figure 5: Search—Flexible Price DSGE Model

Search—Flexible Price Phillips Curve Trend, Sample Phillips Curve Trend, and Actual Price Level



Sample Phillips Curve Cycle and 1—Standard Deviation Band of Search—Flexible Price Model



Appendix

This appendix contains details of the timing of the DSGE models, the labor market search structure, and the income and wealth insurance scheme we develop to construct the aggregate household of the labor market search DSGE models of our paper.

A1. Events and Decisions in Monetary DSGE Models with Labor Market Search

The timing of the economy follows. Households enter date t with the physical and financial assets accumulated at the end of date $t - 1$. Date t begins with the exogenous shocks of the economy being realized. Successfully matched firms and households strike labor contracts over real wages and hours. Firms with active plant-job rent capital from households at this moment to combine with contracted hours and the exogenous level of technology to generate output to sell into the final goods market. When the law of motion (1) guides aggregate price dynamics, those firms able to alter their output price do so just prior to production because supply adjusts to meet demand in Keynesian model. Next, the money and bonds market open given households have made decisions about accumulating capital, financial wealth, and cash to carry into date $t + 1$. At this moment, firms with latent plant-jobs and not-employed households search for one another. Along with existing job matches (that do not separate), the flow of new job matches are carried forward as date $t + 1$ aggregate employment.

A2. Elements of a Monetary DSGE Model with Labor Market Search

Section A2.1 develops the optimal search decisions a firm employs to fill its job-plant vacancies. We construct the complete insurance that is the underpinning of the representative household in section A2.2. Section A2.3 presents the optimality conditions of the aggregate household with respect to labor market search. Aggregate optimality and equilibrium conditions appear in section A2.4

A2.1 *The Firm and Labor Market Search*

Firms either produce with a plant-job, are engaged in search to fill an empty plant-job, or leave it latent. If the final good firm fills a vacancy, the benefit to this firm equals the capitalized value of the active plant-job at date $t + 1$, J_{t+1} , with probability $\omega_{V,t}$. An unsuccessful search forces the final good firm to consider the future capitalized value of the latent plant-job, Q_{t+1} , given the probability it is not filled, $1 - \omega_{V,t}$. Hence, the capitalized value of a latent job-plant evolves according to $Q_t = -\Gamma_t Z_{V,t} + \beta \mathbf{E}_t \{ \omega_{V,t} J_{t+1} + (1 - \omega_{V,t}) Q_{t+1} \}$, where $Z_{V,t}$ denotes date t job-recruitment costs.^{A.1} Free entry into the final good sector requires $Q_t = 0$ because firms have an incentive to activate a vacancy when $Q_t > 0$, or to close a plant down when $Q_t < 0$. Thus, along an equilibrium path the optimality condition

$$(A2.1.1) \quad \omega_{V,t} \mathbf{E}_t J_{t+1} = \Gamma_t Z_{V,t},$$

equates the expected capitalized value of a future active plant-job to the discounted cost of filling the vacancy.

Labor market search forces firms to recognize that an active plant-job represents an ongoing employment relationship. Since job matches last more than one date, firms treat active plant-jobs as capitalized assets. This implies the law of motion of aggregate plant-jobs faced by final goods firms is $N_{t+1} = (1 - \delta_N)N_t + \omega_{V,t}V_t$, where the exogenous non-stochastic job separation rate is $\delta_N \in (0, 1)$ and, on average, $\omega_{V,t}V_t$ vacancies are filled at date t .

An active plant-job's capitalized value can also be measured with its expected discounted profit flow, $J_t = \mathbf{E}_t \{ \sum_{i=0}^{\infty} (1 - \delta_N)^i \prod_{j=0}^i \Gamma_{t+j} D_{t+i} \}$, where discounting involves $(1 - \delta_N)$ because active matches separate at the non-stochastic rate δ_N . This yields the law of motion

$$(A2.1.2) \quad J_t = \Gamma_t D_t + (1 - \delta_N) \mathbf{E}_t \{ \Gamma_t J_{t+1} \},$$

where the aggregate real dividend process is $D_t/P_t = (P_{A,t}/P_t)^\xi [1 - \theta \phi_t] Y_{A,t} - (R_{K,t}/P_t)K_t - (W_t/P_t)N_t h_t - Z_{V,t}V_t$, in the symmetric equilibrium.

A2.2 *Risk Sharing in the Household Sector*

The employed and not-employed comprise the household sector. These households make decisions about capital accumulation, financial portfolios (to hold cash and government bonds), labor supply (employment, hours, and wages), and job search (effort). The penultimate decisions are associated with the employed while the latter activity pertains to the not-employed.

^{A.1}We assume that the transitory component of $Z_{V,t}$ is non-stochastic.

A2.2.1 Risk Sharing in the Household Sector

An employed household enjoys the benefits and suffers the costs of an active job match. The non-pecuniary benefits and costs are summarized by this household's felicity function

$$u(c_{E,M,t}, c_{E,L,t}, \ell_t) \equiv \psi_1 \ln[c_{E,M,t}] + (1 - \psi_1) \ln[c_{E,L,t}] + \psi_3 \frac{\ell_t^{1-\psi_2}}{1-\psi_2},$$

where $0 < \psi_1 < 1$, $\psi_2 \neq 1$, $0 \leq \psi_3$, $c_{E,M,t}$, $c_{E,L,t}$, and $\ell_t (= 1 - h_t)$ are cash consumption, credit consumption, and leisure of the employed household, respectively. The employed household faces the budget constraint

$$\begin{aligned} D_{E,t} + R_{K,t} k_{E,t} + W_t h_t + (1 + R_{B,t}) B_{E,G,t} + M_{E,t} - A_{E,t+1} \\ = P_t [c_{E,M,t} + c_{E,L,t} + k_{E,t+1} - (1 - \delta_K) k_{E,t} + \tau_{N,t} T_{N,t} + T_t], \end{aligned}$$

the cash-in-advance (CIA) constraint $M_{E,t} \geq P_t c_{E,M,t}$, and the wealth constraint

$$(A2.2.1) \quad A_{E,t} \geq B_{E,G,t} + M_{E,t} - \tau_{X,t-1} T_{X,t-1} - X_{t-1},$$

where $\delta_K \in (0, 1)$, and $D_{E,t}$, $B_{E,G,t}$, $M_{E,t}$, $A_{E,t+1}$, $\tau_{N,t}$, T_t , $\tau_{X,t-1}$, and X_{t-1} are the dividends the employed household receives from final goods firms, the government bonds this household owns at the beginning of date t , the cash the employed household carries over from the end of date $t - 1$, the nominal wealth the employed household takes from the end of date t into the beginning of date $t + 1$, the tax levied on employed households to pay for income insurance, $T_{N,t}$, a lump-sum tax, the tax levied on the cash injection received by employed households to pay for the wealth insurance, $T_{X,t}$, that not-employed households receive, and the total cash injection, respectively. Cash earns a zero nominal return. The government pays $R_{B,t}$ on its one-period unit discount bond.

An employed household enjoys an ongoing relationship with a plant-job of a final good firm. The ongoing nature of this relationship occurs because the job match continues from date t into date $t + 1$ with probability $1 - \delta_N$. Given a not-employed household exerts efforts to move into employed status, the probability a job match occurs is $\omega_{S,t}$. In this case, the law of motion of the measure of employed households becomes

$$(A2.2.2) \quad N_{t+1} = (1 - \delta_N) N_t + \omega_{S,t} (1 - N_t) S_t,$$

where $\omega_{S,t} (1 - N_t) S_t$ equals the measure of successful job searches by the not-employed.

Besides the obvious difference, an employed household differs from a not-employed household because the latter puts effort into finding a match and employment. This implies the felicity function of the typical not-employed household is

$$u(c_{S,M,t}, c_{S,L,t}, \ell_t) \equiv \psi_1 \ln[c_{S,M,t}] + (1 - \psi_1) \ln[c_{S,L,t}] + \psi_5 \frac{\ell_{S,t}^{1-\psi_4}}{1 - \psi_4},$$

where $\psi_4 \neq 1$, $0 \leq \psi_4$, $c_{S,M,t}$, $c_{S,L,t}$, and $\ell_{S,t} (= 1 - S_t)$ are cash consumption, credit consumption, and leisure of the not-employed household, respectively.

In all respects save one, the budget constraint of the not-employed household is the same as the budget constraint of the employed household. The disparity between the budget constraints is the not-employed household receives a government income transfer that replaces wage income. As a result, the budget constraint of the not-employed household is

$$\begin{aligned} D_{S,t} + R_{K,t} k_{S,t} + (1 + R_{B,t}) B_{S,G,t} + M_{S,t} - A_{S,t+1} \\ = P_t [c_{S,M,t} + c_{S,L,t} + k_{S,t+1} - (1 - \delta_K) k_{S,t} + (1 - \tau_{N,t}) T_{N,t} + T_t], \end{aligned}$$

where the subscript S denotes the not-employed household. However, the CIA constraint of the not-employed household maintains the form $M_{S,t} \geq P_t c_{S,M,t}$.

The wealth constraint of the not-employed household differs from that of a employed household. The not-employed faces transactions costs when it searches. We assume these transactions costs rise with search effort in a linear fashion at rate $\varphi (> 0)$. Hence, the not-employed household requires cash to engage in job search. Since the not-employed household faces a CIA constraint, the cash injection from the government represents the only available cash to pay the transactions search cost. In this case, the wealth constraint becomes

$$(A2.2.3) \quad A_{S,t} \geq B_{S,G,t} + M_{S,t} + (1 - \tau_{X,t-1}) T_{X,t-1} - (1 - \varphi S_{t-1}) X_{t-1},$$

where the not-employed household receives a nominal wealth transfer of $1 - \tau_{X,t-1} T_{X,t-1}$. The next section discusses the set of government policies necessary for this transfer and the income transfer to make the distribution of capital and financial wealth independent of household employment histories.

A.2.2.2 *The Government*

The government engages in monetary, fiscal, (real) income and (nominal) wealth insurance operations. Besides its expenditure, G_t , and tax collecting, T_t , activities, the government

injects a total of X_t units of cash into the household sector, and conducts open market operations (OMOs) by issuing one-period unit discount bonds, $B_{G,t+1}$. Governmental social insurance policy provides actuarially fair income and wealth insurance to households. This implies household resource allocations arise without consideration of past employment by any household. The intertemporal budget constraint

$$\begin{aligned} P_t T_t + P_t N_t \tau_{N,t} T_{N,t} + \tau_{X,t} T_{X,t} + (B_{G,t+1} - B_{G,t}) + (M_{t+1} - M_t) \\ = P_t G_t + P_t (1 - N_t) (1 - \tau_{N,t}) T_{N,t} + (1 - \tau_{X,t}) T_{X,t} + R_{B,t} B_{G,t} + X_t, \end{aligned}$$

records government accounts across this range of activities.

Complete insurance requires the government chooses $\tau_{N,t}$ and $\tau_{X,t}$ to equate the (shadow) prices of the budget constraint and wealth constraint across employed and not-employed households. When complete income insurance prevails, the government sets $\tau_{N,t} = 1 - N_t$ to yield $P_t T_t = W_t h_t$. The income (insurance) tax rate households face equals the probability they will need this insurance. Thus, employed and not-employed households enjoy the same level of consumption.^{A.2}

Similarly, the government achieves complete wealth insurance when $\tau_{X,t} = 1 - N_t$ so that $T_{X,t} = -\varphi S_t X_t$.^{A.3} For the wealth of the not-employed to be fully insured, government transfers more of the cash injection employed households receive to not-employed households as their search hours rise, taking the cash injection X_{t-1} parametrically.

The government's intertemporal budget constraint becomes

$$(A2.2.4) \quad P_t T_t + (B_{G,t+1} - B_{G,t}) + (M_{t+1} - M_t) = P_t G_t + R_{B,t} B_{G,t} + X_t,$$

in the presence of complete income and wealth insurance. Equation (A2.2.4) is the government's intertemporal budget constraint, equation (11), of the paper.

A.2.2.3 *The Aggregate Household*

The outcome of the government employment insurance program yields the aggregate household. Aggregation of a typical employed and not-employed household produces the value function, \mathcal{V}_t , and its Bellman's equation

^{A.2}This results rests on the felicity of the employed and not-employed being separable in consumption and leisure and log in consumption. Andolfatto (1996) lays out further assumptions and restrictions necessary and sufficient for this type result. In particular, the aggregate household is assumed to be engaged in randomly handing out job matches to its members period-by-period. This separates the worker flows from the flow of ongoing matches at plant-job level. Also, the value function of the aggregate household has to be concave in its arguments.

^{A.3}This follows from complete wealth insurance imposing equality on the shadow prices of wealth of employed and not-employed households which requires $\tau_{X,t} T_{X,t} + X_t = (1 - \varphi S_t) X_t - (1 - \tau_{X,t}) T_{X,t}$.

$$\begin{aligned} \mathcal{V}_t = & \mathbf{Max}_{\{c_{M,t}, c_{L,t}, h_t, h_t, M_t, B_{G,t}, K_{t+1}, N_{t+1}\}} \left[\psi_1 \ln[c_{M,t}] + (1 - \psi_1) \ln[c_{L,t}] \right. \\ & \left. + N_t \psi_3 \frac{(1 - h_t)^{1 - \psi_2}}{1 - \psi_2} + (1 - N_t) \psi_5 \frac{(1 - S_t)^{1 - \psi_4}}{1 - \psi_4} \right] + \beta \mathbf{E}_t \{ \mathcal{V}_{t+1} \}, \end{aligned}$$

where $\mathcal{V}_t \equiv \mathcal{V}(K_t, N_t, M_t, B_{G,t}, Z_t, m_t)$. The constraints the aggregate household faces are

$$\begin{aligned} \text{(A2.2.5)} \quad D_t + R_{K,t} K_t + W_t h_t + (1 + R_{B,t}) B_{G,t} + M_t - A_{t+1} \\ = P_t [c_{M,t} + c_{L,t} + K_{t+1} - (1 - \delta_K) K_t + T_t], \end{aligned}$$

and the CIA constraint

$$\text{(A2.2.6)} \quad M_t \geq P_t c_{M,t}.$$

which follow from the government provision of full income and wealth insurance.^{A.4} Given complete wealth insurance, aggregation of the wealth constraints (A2.2.1) and (A2.2.3) produces

$$\text{(A2.2.7)} \quad A_t \geq B_{G,t} + M_t - [1 - (1 - N_{t-1})\varphi S_{t-1}] X_{t-1}.$$

Along with (A2.2.5), (A2.2.6), and (A2.2.7), the aggregate household faces the law of motion of the measure of employed households, (A2.2.2), given the laws of motion of the exogenous shocks.

^{A.4}In an equilibrium with complete income and wealth insurance, it becomes apparent that $K_{t+1} = k_{E,t+1} = k_{S,t+1}$, $D_{t+1} = D_{E,t+1} = D_{S,t+1}$, $M_{t+1} = M_{E,t+1} = M_{S,t+1}$, and $B_{G,t+1} = B_{E,G,t+1} = B_{S,G,t+1}$. This assumes that employed and not-employed households hold equal endowments of capital and financial wealth at date zero. Further, we assume away any wealth disparities that are caused by ownership claims on final goods firms. However, if employed and not-employed households are initially given equal equity stakes in final goods firms, the dividend flows will be equalized. Also, these results depend on the additively separable character of the felicity functions, as already noted.

A2.3 *The Household and Labor Market Search*

The impact of labor market search on aggregate household felicity, the aggregate wealth constraint, and the law of motion of aggregate employment leads to the optimal job-search condition

$$(A2.3.1) \quad \frac{\psi_5}{(1 - S_t)^{\psi_4}} + \varphi \Gamma_t \frac{X_t}{P_t} = \omega_{S,t} \Lambda_t,$$

where Λ_t is the shadow price of an active job-match. The optimality condition (A2.3.1) equates the dis-felicity of job-search plus the associated transaction-search costs to the probability, $\omega_{S,t}$, of a successful job match, valued at the shadow price of the marginal match, Λ_t .

The aggregate household's optimal choice of aggregate employment, N_{t+1} , yields the Euler equation

$$(A2.3.2) \quad \Lambda_t = \beta \mathbf{E}_t \left\{ \frac{\partial \mathcal{V}_{t+1}}{\partial N_{t+1}} \right\}.$$

The Euler equation (A2.3.2) sets the value of an additional job match equal to the discounted expected value of continuing the match. Labor market optimality also depends on the envelope condition

$$(A2.3.3) \quad \frac{\partial \mathcal{V}_t}{\partial N_t} = \mathcal{H}_t + \Gamma_t \frac{W_t}{P_t} h_t + \varphi \Gamma_t \frac{X_t}{P_t} S_t + [(1 - \delta_N) - \omega_{S,t} S_t] \Lambda_t,$$

where $\mathcal{H}_t \equiv \psi_3(1 - \psi_2)^{-1}(1 - h_t)^{1-\psi_2} - \psi_5(1 - \psi_4)^{-1}(1 - S_t)^{1-\psi_4}$. The envelope condition (A2.3.3) is the sum of the benefits the aggregate household receives from an ongoing job match, the change in felicity from moving a household from not-employed to employed status, the discounted real labor income, the foregone transactions-search costs, and the discounted value of the job match. The latter discounting accounts for the fixed rate of job separation net of the probability that job search is successful given search effort is S_t .

A2.4 *Aggregate Optimality and Equilibrium*

Remember that firms with plant jobs not in operation and the not-employed meet randomly and that we borrow the den Haan, Ramey, and Watson (2000) CRS matching technology

$$(A2.4.1) \quad \mathcal{M}(V_t, (1 - N_t)S_t) = \frac{V_t[(1 - N_t)S_t]}{(V_t^\vartheta + [(1 - N_t)S_t]^\vartheta)^{1/\vartheta}}, \quad 0 < \vartheta,$$

to place the not-employed in available job vacancies, where $V_t \equiv \int_0^1 v_{j,t} dj$ is total plant-job vacancies. Note that the probability of a successful match is influenced indirectly either with variation in posted vacancies or by changes in not-employed search effort. Denote the probability a vacant plant-job is filled $\omega_{V,t} (= \mathcal{M}(V_t, (1 - N_t)S_t)/V_t)$ and that someone not-employed finds work $\omega_{S,t} (= \mathcal{M}(V_t, (1 - N_t)S_t)/[(1 - N_t)S_t])$. The CRS search technology (A2.4.1) bounds $\omega_{V,t}$ and $\omega_{S,t}$ between zero and one. Given these probabilities, the aggregate law of motion of employment is

$$(A2.4.2) \quad N_{t+1} = (1 - \delta_N)N_t + \mathcal{M}_t.$$

The stochastic flow of new job matches and non-stochastic rate of job destruction is a weakness of this class of labor market search model.

The optimality condition (A2.1.1) determines if a final good firm operates a plant-job. The surplus rule implies that the left-side of (A2.1.1) equals $[(1 - \zeta)/\zeta]E_t \partial \mathcal{V}_{t+1}/\partial N_{t+1}$. Equate this expression with the Euler equation (A2.3.2) to find

$$(A2.4.3) \quad \zeta \Gamma_t Z_{V,t} = (1 - \zeta) \omega_{V,t} \Lambda_t.$$

This optimality condition states that the discounted cost of posting job vacancies by firms equals the expected value of the job match to the household.

The optimality condition that completes the labor market substitutes for $\partial \mathcal{V}_{t+1}/\partial N_{t+1}$ in the Euler equation (A2.3.2) with the envelope condition (A2.3.3) and the optimality condition (14) of h_t using the equilibrium real wage generating process. This yields the Euler equation of the value of the marginal match

$$(A2.4.4) \quad \Lambda_t = \beta E_t \left\{ \zeta \Gamma_{t+1} \left[\left(\frac{P_{A,t+1}}{P_{t+1}} \right)^\xi \left(1 - \left[\frac{\theta \phi_{t+1}}{1 - \bar{K}^*} \right] \right) \frac{Y_{A,t+1}}{N_{t+1}} + \frac{u_{t+1} V_{t+1}}{1 - N_{t+1}} \right] \right. \\ \left. - \zeta \mathcal{H}_{X,t+1} + [(1 - \delta_N) - \omega_{S,t+1} S_{t+1}] \Lambda_{t+1} \right\},$$

The current value of a plant-job match is forward-looking. It equals the expected discounted value of the date $t + 1$ equilibrium wage process (24) plus the net probability the match continues into date $t + 1$. The persistence of the endogenous state variable N_{t+1} , its shadow price Λ_{t+1} , and the non-Walrasian equilibrium wage process (24) propagate productivity and money growth shocks.

As noted in the paper Merz (1995), Andolfatto (1996), and Cooley and Quadrini (1999) assume the rule to split match surplus is $S_t = \mathcal{J}_t + \partial \mathcal{V}_t/\partial N_t$ and that the aggregate household

receives a fixed fraction, ζ , of S_t during each date t .^{A.5} Since $\partial \mathcal{V}_t / \partial N_t = \zeta S_t$ and $(1 - \zeta) \partial \mathcal{V}_t / \partial N_t = \zeta J_t$, the surplus splitting rule, the envelope condition (A2.3.3), the law of motion of J_t , (A2.1.2), the aggregate dividend function (6), the rental rate of capital, and the optimality condition (A2.1.1) produces the Nash equilibrium real wage generating process

$$(A2.4.5) \quad \Gamma_t \frac{W_t}{P_t} h_t = \zeta \Gamma_t \left[\left(\frac{P_{A,t}}{P_t} \right)^\xi \left(1 - \left[\frac{\theta \phi_t}{1 - \bar{K}^*} \right] \right) \frac{Y_{A,t}}{N_t} + \frac{u_t V_t}{1 - N_t} \right] + (1 - \zeta) \mathcal{H}_{X,t},$$

where $\mathcal{H}_{X,t} = \psi_3 (1 - \psi_2)^{-1} (1 - h_t)^{1-\psi_2} - \psi_5 (1 - \psi_4)^{-1} (1 - S_t)^{1-\psi_4} - \varphi \Gamma_t X_t S_t / P_t$ and \bar{K}^* is the steady state of \bar{K}_t . Equation (A2.4.5) is the equilibrium real wage (24) of the paper.

A2.5 The Model Solution

Numerical solution of the labor market search models requires the linearization of the (detrended) optimality and equilibrium conditions: consumption function (13), the cost to firms of posting vacancies (A2.4.3), the search cost of the not-employed (A2.3.1), employed labor supply (14) evaluated using the real wage generating process (A2.4.5), the Euler equation of capital (18), and the law of motion of Λ_{t+1} , (A2.4.4), the law of motion of employment (A2.4.2), and the aggregate resource constraint (23). This adds N_{t+1} to the state vector \mathcal{K}_{t+1} and the shadow price of employment Λ_t to the control vector C_t . The solution algorithm of the labor market search-sticky price model is the same that is described in section 4.2 for the Yun-sticky price model. Note that this solution linearizes the law of motion that generates the NKPC-PV relation (4). Under the flexible price regime, price expectations drop out of the standard solution state vector \mathcal{K}_{t+1} and theoretical real marginal cost, ϕ_t , is eliminated from the control vector C_t . This permits standard solution methods to be engaged, as in Zdrozny (1998).

^{A.5}In the negotiations between a final good firm and the aggregate household, $1 - \zeta$ reflects the power the aggregate household exerts on equilibrium real wages and hours.